Robust optimization for location-routing problem with stochastic demand under facilities failure risk

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Abstract: Relief distribution is one of the key steps for emergency response after a major public emergency. Relief commodities requirements of demand node are presented intervals based on robust optimization. The facilities failure risk is processed by stochastic programming. The model of emergency location-routing problem is developed to minimize the total transportation time. An improved genetic algorithm is proposed to solve the model. Finally, the validity of the model and algorithm is demonstrated by a numerical example based on earthquake relief distribution.

Introduction

In recent years, global natural disasters occur frequently such as the Indonesian tsunami in 2004, the Pakistan earthquake in 2005, the Wenchuan earthquake in 2008, the Haiti earthquake in 2010, the earthquake of Japan in 2011, the United States snowstorm disasters in 2015 and so on. Frequent occurrence of natural disasters already affects the stability and economic development of human society, causing very serious casualties and huge economic losses. After the disaster, fast and accurate delivery of emergency relief commodities is a top priority of the relief work. It is urgent to achieve scientific optimization of emergency rescue facility location and emergency relief commodities distribution system with the constraints of time, space and resource.

Emergency rescue facility location problem and emergency rescue commodities distribution routing problem are the most important problem in emergency rescue system [1]. As a whole, there is an inevitable connection between emergency rescue facility location problem and emergency rescue commodities distribution routing problem. The Location-Routing Problem(LRP) is the Integration issues of two problems. The research of LRP is both to reduce delivery time and logistics costs, while also can improve the robustness of the facility location [2]. Scholars have done a research on the emergency logistics distribution system under certain information. Prodhon[3] proposed a hybrid intelligent algorithm for solving large scale emergency location - path. Zheng et al. [4] proposed a bi-level programming model of LRP with considering the total time of emergency relief commodities to reach the node and the balance of commodities distribution. Other scholars have done a research on the emergency logistics distribution system under uncertain information. Dai et al.[5] proposed a multi-objective LRP model, total costs and rescue time satisfaction as the goal, considering fuzzy variables demand and fuzzy emergency rescue time. Toro-Díaz et al.[6] proposed a comprehensive model that based on integer programming model for location and distribution decision and the hypercube model of the queuing and congestion, considering the uncertainty of the rescue time.

In this paper, considering the randomness and the risk of emergency logistics system, an emergency logistics system including the stochastic demand and facilities failure risk is developed to minimize the total transportation time. An improved genetic algorithm is designed to solve the model.

LRP Model Description

After natural disasters, it is urgent to select or form a number of emergency rescue facilities. Emergency relief commodities must be delivered to the various nodes safely and timely. Vehicles from emergency facilities node deliver relief commodities to demand node. Emergency facilities node failure is caused by secondary disasters, which affects the vehicle's delivery of emergency resource. The question is how to effectively avoid the facilities failure risk, to select the emergency facilities, and to determine the transport plan from the demand node to the node emergency facilities node. The total transportation time is required to be the minimum.

There are many candidates for emergency facilities. Each emergency facility has a number of the same emergency vehicles. The capacity of each emergency facility is greater than the load of any emergency vehicle. There is a failure risk of emergency facilities, and there is not more than a failure of emergency facilities. Once the facility is out of service, it cannot provide service to the demand node. We do not consider the possibility of providing partial services. The demand of each demand node does not exceed the capacity of the rescue vehicle. Only a rescue vehicle is responsible for the distribution of its material. Each vehicle starts from the node of emergency facilities and return to the node of emergency facilities after task is finished. The number and location of emergency demand node is certain, but the demand of each demand node is not determined.

We use the following notation throughout the paper. $I\{i|i=1,2\mathbf{L}m\}$ represents the set of all material requirements, $J\{j|j=1,2\mathbf{L}\,n\}$ denotes the set of candidate emergency facilities, $K\{k|k=1,2\mathbf{L},g\}$ is the set of all emergency facility j , Ω_j , Q_k represent the capacity of emergency facility j , the loading capacity of vehicle k , t_i denotes the time of vehicle arriving or departing , node i , q_i , X_{ji} , q_i denote the material demand at the node i ,the total quantity of material transported from the emergency facilities j to the disaster node i , the deficient demand of node i , v_k is the speed of vehicle k , d_{gh} represents the distance between node g to node h , b is the penalty coefficient, we need the penalty coefficient when relief resource cannot be properly served because of the failure of facilities, ; x_j is 0-1 integer variables, where 1 means emergency facility j is selected and 0 is otherwise, m_{ij} is 0-1 integer variables, where 1 means emergency facilities j service demand node i and 0 is otherwise, y_{jk} is 0-1 integer variables, where 1 means vehicle k is allocated to emergency facility j . z_{ghk} is also 0-1 integer variables, where 1 means vehicle k start node g to node h ,

Establish Mathematical Model

$$\min Z = \sum_{i\in I} c_i (t_i + \frac{b\, t_i q_i}{q_i}) \tag{1}$$

$$\sum_{i\in I}\sum_{k\in K} z_{jik} - x_j \geq 0, \forall j \in J \tag{2}$$

$$\sum_{i\in I} z_{ijk} - x_j \leq 0, \forall j \in J, k \in K \tag{3}$$

$$y_{jk} \leq x_j, \forall j \in J, k \in K \tag{4}$$

$$\sum_{i\in I} z_{ijk} = y_{jk}, \forall j \in J, k \in K \tag{5}$$

$$\sum_{k\in K} z_{jrk} = 0, \forall j, r \in J \tag{6}$$

$$\sum_{g\in(I\cup J)} z_{hgk} - \sum_{g\in(I\cup J)} z_{ghk} = 0, \forall k \in K, \forall h \in (I \cup J) \tag{7}$$

$$\sum_{g\in(I\cup J)}\sum_{k\in K} z_{gik}=1,\forall i\in I$$

(8)

$$\sum_{j\in J}\sum_{i\in I}\sum_{g\in(I\cup J)} X_{ji}z_{gik}+\sum_{i\in I}\sum_{g\in(I\cup J)} q_i z_{gik}\geq\sum_{i\in I}\sum_{g\in(I\cup J)} q_i z_{gik}, k\in K$$

(9)

$$\sum_{j\in J}\sum_{i\in I}\sum_{g\in(I\cup J)} X_{ji}z_{gik}\leq Q_k, k\in K$$

(10)

$$\sum_{i\in I} X_{ji}u_{ij}\leq\Omega_j, j\in J$$

(11)

$$T_i=\left(T_g+\frac{d_{gi}}{v_k}\right)z_{gik},\forall i\in I,\forall g\in(I\cup J),\forall k\in K$$

(12)

$$x_j, m_{ij}, y_{jk}, z_{ghk}\in\{0,1\} \quad (13)$$

Where formula (1) ensures that the total transportation time is minimum; formula (2) ensures that all selected emergency facilities have the rescue vehicles; formula (3) ensures that the emergency facilities that have not been chosen have no rescue vehicles; formula (4) and formula (5) ensure that the vehicles start from their selected emergency facilities; formula (6) shows that the same path cannot have two emergency facilities; formula (7) shows that the vehicle goes into the node and then leaves from it, which can ensure the right order of the access nodes; formula (8) ensures that there is only one vehicle serve the each node; formula (9) shows that vehicle's load must meet the requirements of its served node; formula (10) and formula (11) are the capacity constraints for vehicle and facility; formula (12) shows the computational method of transportation time.

LRP Robust Optimization

Firstly, we define the scenario set $S\in\{s|s=0,1,2,\ldots,n\}$, if $s=0$, then there was no facilities failure. When $s\geq 1$ represented that s seized up suddenly, causing the vehicle was unable to complete the delivery mission, r_s represented the probability of the scenario s. Secondly, we define strategy set V under the scenario s, $V=\{v=1,2,\ldots,N\}$, Z_s^v is the objective function value of strategy v, and Z_s is the optimal objective function value in this scenario.

According to the stochastic programming theory, the minimum expectations value is as follows:

$$F=\min_{v\in V}\sum_{s\in S} r_s(Z_s^v-Z_s)$$

(14)

After natural disasters, the demand fluctuation for the emergency relief commodities is large. Collection interval is used to describe the variation range of demands for emergency relief commodities: $Q_{q_i}=\left\{q_i|q_i\in\left[\tilde{q}_i-\hat{q}_i,\tilde{q}_i+\hat{q}_i\right]\right\}, i\in I$, where $\tilde{q}_i$ denotes the nominal value of the demand for emergency relief commodities, it can be determined according to the information such as affected population density, disaster level, disaster spread range and so on. $\hat{q}_i$ represents maximum perturbation value of deviation from nominal value. Relative robust model of Bertsima and Sim[7], in order to make the vehicle load capacity to meet actual demand, the control parameters Γ_k is introduced to $\hat{q}_i$ to describe the variation range of demand, $\Gamma_k\in[0,10], k\in K$. Γ_k is introduced to formula (9), then we get formula (15):

$$\sum_{j\in J}\sum_{i\in I}\sum_{g\in(I\cup J)} X_{ji} z_{gik} + \sum_{i\in I}\sum_{g\in(I\cup J)} q_i z_{gik} \ge \sum_{i\in I}\sum_{g\in(I\cup J)} \tilde{q}_i z_{gik} + \max_{\{A_k\cup\{t_k\}|A_k\subseteq B_k,|A_k|=\lfloor\Gamma_k\rfloor,t_k\in B_k\setminus A_k\}} \left\{\sum_{i\in A_k}\sum_{g\in(I\cup J)} \hat{q}_i z_{gik} + (\Gamma_k-\lfloor\Gamma_k\rfloor)\sum_{g\in(I\cup J)}\hat{q}_{t_h} z_{gik}\right\}, k\in K \qquad (15)$$

In order to solve the above model, the constraint (15) is transformed by the Lemma 1(Bertsimas & Sim). The above model can be translated into the optimization function:

$$\begin{aligned} &\sum_{j\in J}\sum_{i\in I}\sum_{g\in(I\cup J)} X_{ji} z_{gik} + \sum_{i\in I}\sum_{g\in(I\cup J)} q_i z_{gik} \ge \sum_{i\in I}\sum_{g\in(I\cup J)} \tilde{q}_i z_{gik} + h_k\Gamma_k + \sum_{i\in I} r_{ki}, k\in K \\ &h_k + r_{ki} \ge \sum_{g\in(I\cup J)} \hat{q}_i z_{gik}, k\in K, i\in I, \\ &h_k \ge 0, r_{ki} \ge 0, k\in K, i\in I \end{aligned} \qquad (16)$$

We get the model with stochastic demand under facilities failure risk: (1)~(8),(10)~(14),(16).

If $\Gamma_h = 0$, then the model is the Demand Certainty model. If $\Gamma_h = [1,9]$, then the model is the relative robust model. If $\Gamma_h = 10$, then the model is the absolute robust model.

Case Study

We assume that there are 4 emergency facilities, 5 vehicles, 11 disaster demand areas. Set demand disruption: 10% ($\hat{q}_i(t)=0.1\tilde{q}_i(t)$) .The vehicle speed is 40km/h. The maximum loading is 5000 units. The maximum capacity of emergency facilities is 10000 units. The emergency facilities and disaster demand areas related data as shown in table 1-2.

Table1 Data of disaster demand areas

No	Coordinate (km)	demand nominal value(unit)
Q1	(425,720)	1500
Q2	(383,587)	1200
Q3	(790,62)	2400
Q4	(735,560)	2000
Q5	(411,519)	600
Q6	(609,562)	1900
Q7	(405,500)	1000
Q8	(389,677)	1300
Q9	(728,603)	1100
Q10	(749,120)	1900
Q11	(708,388)	1800

Table2 Data of emergency facilities

No	Coordinate (km)	Failure probability
J1	(460,574)	0.15
J2	(430,383)	0.2
J3	(510,195)	0.25
J4	(677,305)	0.3

Due to space limitations, we only list the value of weighted regret of control coefficient =0, 5, 10, as shown in Table 3. As seen in Figure 1, regardless of different variation range, the value of weighted regret always tends to converge, which demonstrates the validity of stochastic programming in facilities failure treatment.

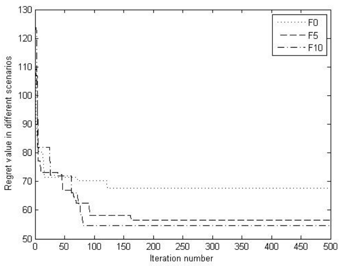

Figure1 Iterative weighted regret value in different scenarios

Table3 Comparison of regret values

Scenario	Control coefficient 0	Control coefficient 5	Control coefficient 10
s_0	55.61	69.19	59.77
s_1	49.74	50.19	37.59
s_2	19.71	25.88	35.81
s_3	19.71	19.87	23.37
s_4	15.49	4.66	16.74
Stochastic programming	4.72	3.77	2.84

As seen in Table 3 and Figure 1, in the same control coefficient, the regret values of stochastic programming are less than the regret values of the optimal solution of each scenario. Because the stochastic programming model take into account all the scenarios, demonstrating the ability of stochastic programming model of improving decision scheme with facilities failure.

Conclusion

In this paper, we analyze the influence of stochastic demand under facilities failure risk on location-routing problem in relief distribution. The facilities failure risk is processed by stochastic programming. Relief commodities requirements of demand point are presented intervals based on robust optimization. The model of emergency location-routing problem is developed to minimize the total transportation time. An improved genetic algorithm is proposed to solve the model. The results prove the effectiveness of the algorithm. It provides decision-making basis for policymakers making rescue plans with risk response capacity and quick response.

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References

[1]Zhang Qian. Logistics distribution routing optimization scheduling modeling and practice[M]. China Material Press, 2006.

[2]Salhi S, Nagy G. Consistency and robustness in location-routing[J]. Studies in Locational Analysis, 1999 (13): 3-19.

[3]Prodhon C. A hybrid evolutionary algorithm for the periodic location-routing problem[J]. European Journal of Operational Research, 2011, 210(2): 204-212.

[4]Zheng Bin, Ma Zujun, Li Shuanglin. Joint location-transportation problem in relief distribution systems based on bi-level programming[J]. Journal of System Science and Mathematical Science, 2013,33 (9): 1046-1060.

[5]Dai Ying Ma Zujun, Zheng Bin. Fuzzy multi-objective location-routing problem in emergency systems for unexpected public emergency[J].Management Review, 2010, 22(1):121-128.

[6]Toro-Díaz H, Mayorga M E, Chanta S, et al. Joint location and dispatching decisions for Emergency Medical Services[J]. Computers & Industrial Engineering, 2013, 64(4): 917-928.

[7]Bertsimas D, Sim M. Robust discrete optimization and network flows[J]. Mathematical Programming, 2003, 98(1-3): 49-71.