Electromagnetic Current Transformer Transfer Characteristics under harmonics load

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Key words: Harmonics Load, Current Transformer, Transfer Characteristics, Transfer Function **Abstract.** Due to the extensive use of power electronic devices and nonlinear loads, the resulting of distortion in power grid voltage and current waveform becomes more and more serious. There not only exist integer harmonics of larger containing rate, but also include non integer harmonics of abundant spectral components, what is called inter-harmonic. Through the experiment platform having built before, this paper measures the frequency characteristics of current transformer in the 10Hz-2.5kHz, at the same time, the frequency characteristics of current transformer is analyzed theoretically from the point of the transfer function. The actual measurement could proof mutually well with theory, and the study found that, in inter-harmonic wave and high harmonics, the error of current transformer are more serious than in frequency. In addition, by collecting data actually, and we found that the error of fundamental wave brought by current transformer and the error of harmonic power increased obviously when they were compared with the error of current transformer itself, besides, harmonic power error was affected more.

Introduction

With the development of modern power electronics technology, power devices and power electronic devices are widely used in power systems. However, a large number of harmonics, inter-harmonics and a variety of signals of impact, aperiodicity and high dynamic occur frequently in grid, thus, the problem of power quality is becoming increasingly serious. so it brings new challenges to electric energy measurement. Modern electric energy measurement technology usually focused on accuracy analysis[1], new method of electric energy measurement[2,3], and analysis algorithms of signal[4,5] in distortion signal, but there is not any research about the energy metering device and sensors.

As an energy metering system front-end signal acquisition device, Electromagnetic current transformer(Electromagnetic Current Transformer, referred to CT) has different transfer characteristics and frequency signals in the complex signals, and it will directly affect the reliability and accuracy of electric energy measurement [6].

Most of the existing literatures discuss transfer characteristics of current transformer in harmonics, rather than the study of inter-harmonics, in addition, the influence of current transformer of electric energy measurement in harmonic load is also a lack of systems research. In this paper, an experimental platform is sited up, while the frequency characteristics of the current transformer at 10Hz-2.5kHz are measured, and the frequency characteristics of the current transformer are analyzed by a transfer function theoretically, after measured, we find the measurement results and the analysis results in theory validate each other. In addition, actual data is used to analyzed the influence of current transformers on electric energy measurement under harmonic load.

Experiment Platform

A. Block diagram of the symstem

Not only the m language, but also the existing models can be used in building the Simulink models, and the signals can be imported into From Workspace module which is more powerful, so, abundant source model library can be built into the present experiment platform, which lay the foundation for the later test of simulation and real signals. After the establishment of the model library, signal source can be outputted to the CT measurement circuit through real-time simulation platform via the power amplifier, and then the input signal of CT of the primary side is formed. The block diagram of principle experiment platform is shown as Fig.1, from the Fig.1, we can see that the waveform can be replayed in laboratory by importing the collected data into the simulink.



Fig. 1 The block diagram of principle experiment platform

In this paper, dSPACE simulation platform is used as a real-time simulation platform, and it is produced to test and develop in accordance with the MATLAB / Simulink control systems and it can completely connect with the Simulink.

The D/A board DS2102 of Simulink with the 16-bit resolution and $1.6\mu s$ setting time is applied in this paper ,which can transfer digital signal into an analog signal.

B.Measurement circuit diagram



Fig.2 Measuring principle diagram of the transformer

The measurement principle of current transformer is shown in Fig.2. From Fig.2, R_1 is the primary sampling resistor, using high-power-and-Non-inductive aluminum shell resistance, with the resistance of 1 Ω ; R is the rated load of CT, which varies with the change of CT, and is made up with the same material as R_1 . The reason of using non-inductive aluminum shell resistance is to prevent the influence that resistor impedance angle brings error to the measurement of resistance phase displacement. C.Calculation method of error

According to GB1208-2006 for CT, errors are made up with phase displacement and ratio error[8]. Ratio error: error in the circuit measurement, exists because of the difference between actual current and ratio current, and its expression is shown as Eq.1 :

$$e_{\lambda}(\%) = \frac{(KI_2 - I_1) \times 100}{I_1}.$$
(1)

Where, *K* is the rated transformation ratio, and I_1 , I_2 are current of primary side and secondary side respectively.

Phase displacement: the phase difference between the primary current and the secondary current of transformer. If the secondary current phase is ahead of primary current phase, the phase displacement is positive. It is usually represented by(') or card.

During experiment, if the current clamp which is also one kind of transformer is directly used for the current measurement, the measurement results will be affected. In this paper, the voltage between R_1 and R_2 is be measured to indirectly calculate the current value of the primary and secondary side. so Eq.1 will be rewritten as Eq.2:

$$e_r = K \frac{I_2}{I_1} - 1 = K \frac{\frac{U_2}{R_2}}{\frac{U_1}{R_1}} - 1 = K \frac{U_2}{U_1} \times \frac{R_1}{R_2} - 1.$$
(2)

Similarly, the phase displacement is represented by β , and its value will be calculated by Eq.3:

$$\boldsymbol{b} = angle(\boldsymbol{b}_{2}^{\boldsymbol{k}}) - angle(\boldsymbol{b}_{1}^{\boldsymbol{k}}) .$$
⁽²⁾

Ratio error will be measured by current transformer calibrator under power frequency, rated current and rated load while the measurement results is usually given by the manufacturer factory. This parameter can be used to test the resistance correction, assuming current transformer ratio error under frequency, rated current and rated load is e_n , then:

$$e_n = K \frac{U_{2n}}{U_{1n}} \times \frac{R_1}{R_2} - 1.$$
(3)

where U_{1n} and U_{2n} represent the voltage RMS of R_1 and R_2 under rated current, frequency and load, then the correction factor expression can be shown as:

$$K' = K \frac{R_1}{R_2} = (1 + e_n) \frac{U_{1n}}{U_{2n}}.$$
(4)

According to Eq.5 and Eq.4, we can get the ratio error of transformer under any current signal compare to the ratio error of transformer under frequency current, which can be shown as:

$$e_r = K' \frac{U_2}{U_1} - 1$$
 (5)

And we can ge ε_h and β_h in the same way according to Eq.6 and Eq.3.

The Actual Measurement in Frequency Characteristics of the Current Transformer

Choosing a meter with micro-transformer of current, its parameters are shown in table 1

Table I CI	parameters
parameters	parameter values
transformation ratio	5:1500
rated current	1.5 (6) [A]
accuracy class	0.2
rated load	20[Ω]
core material	ultra microcrystal

According to the national standard GB/T 17215.302-2013, the maximum number of harmonic measurement is 41 times [9], in order to retain a certain margin, we analyze it in 2.5kHz. At the same time, considering the influence of the inter harmonics, the frequency section lower than 50Hz is analyzed.

A.The single harmonic method

By changing the frequency of the input current in primary side, using the current transformer measurement circuit based on Fig.2 to measure the ratio difference and the angle difference between the CT at the different frequency. The frequency characteristics of CT are shown in Fig.3.

From Fig.3 (a), the frequency range of 30-1000Hz can be obtained, which is within the range of -0.2%, and it conforms to the accuracy requirements. Although there is no requirement to make the error limit of current transformer at the frequency point except the power frequency, it can be seen from Fig.3 that the error is significantly increased in the frequency lower than 1000 Hz. When the frequency range to 2500Hz, the difference of ratio error is reached to -0.64% and the angular deviation is reached to -3.7, and the deviation is larger than the operating mode.

In Fig.3(b), phase angle difference of CT algebraic value is positive at at low frequency; while at high frequency, the phase angle difference of CT algebraic value is negative. At the same time, we can see that the angular difference is a linear function of the frequency, and the conclusion is consistent with the results from the delay perspective of the literature [10], which is mainly caused by the time effect[10].



(a) Frequency characteristic curve chart of ratio error
 (b) Frequency characteristic curve chart of angular deviation
 Fig. 3 CT error of frequency characteristic under a single frequency method

B.The method of single harmonic superposition

This method is proposed by A Cataliotti et al. The proposed method proposed in literature[7] shows that the actual signal is generally the fundamental wave superposition of harmonic, and it is considered that the current transformer is a nonlinear element, and the single frequency method may lead to inaccurate measurement, so the method of the fundamental wave superposition method is used to measure[7]. When the fundamental wave is under rated current, we consider to change the harmonic content, phase angle and frequency, and then observe and analyze the fundamental error and the harmonic error in various conditions.

C.Superimposed multiple harmonic method

In the single frequency method and the superposition method, the characteristics of the frequency are consistent, and the change of the harmonic phase and the harmonic content has little effect on the frequency characteristics. So it can be found that, although the error of current transformer is different in different frequency, the error relative to fundamental wave is increased in low frequency and high frequency, but according to the applicable conditions of the superposition theorem, the current transformer is still in the linear region, which is not saturated. Relative to dc, the effect of frequency on the current transformer is relatively small, and the single frequency is not enough to make the current transformer vary from linear region to saturation.

Based on the conclusions above, the article puts forward a method to measure the frequency characteristics of current transformer with multiple harmonic method, which is superposing multiple harmonics based on the fundamental wave with a certain content, and then, by selecting some of the key points, the frequency characteristics of current transformer can be obtained. This method is more simple than the single frequency method and the superposition method of single harmonic measurement. Figure 4 is a frequency characteristic diagram obtained by the superposition of several harmonic method. It can be found that the results obtained from the single harmonic measurements are generally consistent, and verify the validity of the method.



(a) Frequency characteristic of ratio error curve
 (b) Frequency characteristic of phase displacement curve
 Fig. 4 Error characteristic of CT in the method of multiple harmonics

Verification Based On Transfer Function

The theoretical derivation of transfer function definition of CT in transfer function is from the Eq.8:

$$G(s) = \frac{KI_2(s)}{I_1(s)} \,.$$
(8)

The transfer function of the amplitude-frequency characteristics and phase-frequency characteristics can be respectively determined from the ratio difference of frequency characteristics and the angle difference of frequency characteristics from the Eq.9-10.

$$|G(jw)| = \frac{K}{I_1(jw)} \cdot \left(\frac{1 + e(jw)}{K} I_1(jw)\right) = 1 + e(jw) .$$
(6)

$$\angle G(jw) = \arg(I_2(jw)) - \arg(I_1(jw)) = b(jw) \quad . \tag{10}$$

Using Eq.8, multiplied by the rated transformation ratio, is to make subtle changes in the amplitude-frequency characteristics which can be better reflected, otherwise it will be ignored by-20lg(K).



Fig.5 CT equivalent circuit diagram of the high-frequency The most common circuit of Current Transformer equivalent circuit is T-shaped and-shaped equivalent circuit as illustrated in Fig.5.

 C_1 and C_2 represent the primary and secondary electrode capacitance. Usually their value is very small, for why they aren't considered in a lot literature.

 R_{1} - primary winding resistance, L_{1} - primary winding leakage inductance, R_{2} - Secondary winding resistance, L_{2} - Secondary winding leakage inductance. $Z_{m} = R_{m} + j2p fL_{m}$, represents magnetizing impedance.

Seeking transfer function of CT is a gray box model problems. The relationship between input and output is known, and CT equivalent circuit is also known, but the circuit parameters are unknown. The structure and the order of transfer function can be roughly estimated based on CT equivalent model, then according to the measured error frequency characteristic curve, we can solve undetermined parameters.

The circuit in Fig.5 is divided into two parts in the part of ideal variable device, as shown in Fig.6. (a) represents the primary side portion, while (b) represents the secondary side portion.

Eq.11 given according to Fig.6(a), Obviously when C_1 is very small, $G_1(s)=1$, the amplitude doesn't decay, and the phase doesn't shift.

$$G_{1}(s) = \frac{I_{3}(s)}{I_{1}(s)} = \frac{1}{L_{1}C_{1}s^{2} + R_{1}C_{1}s + 1}$$
(7)

Only amplitude of ideal variable device changes ,there is:

$$I_3(s) = K \cdot I_3'(s)$$
 (8)

According to Fig6 (b), there is:

$$G_{2}(s) = \frac{I_{2}(s)}{I'_{3}(s)} = \frac{Z_{m}}{Z_{m} + Z_{2L}} \cdot \frac{\frac{1}{sC_{2}}}{\frac{1}{sC_{2}} + R_{L}}.$$
(9)

Substitute $Z_{2L} = (\frac{1}{sC_2} / R_L) + (R_2 + sL_2)$ into Eq.13, simply it to get Eq.14.

$$G_2(s) = \frac{R_m + sL_m}{(R_L C_2 s + 1)[(L_m + L_2)s + (R_m + R_2)] + R_L}.$$
(14)

According to (14), contains a single zero and a two-pol , obviously, $(R_m + R_2) \square R_L$, the two-pole can

be converted to two single-pole. And $w_0 = \frac{R_m}{L_m}$ is very closed with $w_1 = \frac{R_m + R_2}{L_m + L_2} (R_m \square R_2, L_m \square L_2).$

If further approximation, it becomes a single-pole system according to Eq.11, 12and 13 there is:

$$G(s) = G_1(s)G_2(s) . (10)$$

According to Eq.15, the transfer function of CT can be derived.

B.Based on the Frequency Characteristics Measure of the Transfer Function

There shows the ratio difference of frequency characteristic from Fig.3(a), and the curve is divided into three sections, where the slope are roughly 20dB / dec, 0dB / dec, and -20dB / dec. Its expression form of transfer function is:

$$G(s) = k \frac{s + W_0}{(s + W_1)(s + W_2)}.$$
(11)

Among them, $w_0 < w_1$; according to Eq.9 and 10, error frequency curve can be deduced through frequency characteristic curve of the transfer function.

Making $f_0 = 4.95Hz$, $f_1 = 5Hz$, $f_2 = 32kHz$, w = 2pf, $k = (1 + e_0)w_2$, e_0 is the contrast difference when CT gets to 50Hz. The error frequency curve derived by the transfer function compares with Fig.3, and the result is shown in Fig.7.

From Fig.7, we can find, Eq.16 greatly reflects the error frequency characteristic of CT and is equivalent to Eq.15 deduced by CT equivalent circuit. However, the deviations between the two exist because Eq.11 isn't considered and the two-pole is simplified to two single poles in Eq.14.

If only considering the error of integer harmonics, namely, Eq.16 can continue to simplify the type as below:

$$G(s) = \frac{1+e_0}{(s/w_2)+1} (\Re k = (1+e_0)w_2) .$$
(12)

$$\int_{0}^{0} \frac{1}{(s/w_2)+1} (\Re k = (1+e_0)w_2) .$$
(12)
(a) the contrast of the ratio difference of frequency characteristic curve (b) the contrast of the angular difference of frequency characteristic curve (b) the contrast of the angular difference of frequency characteristic curve (b) the contrast of the angular difference of frequency characteristic curve (b) the contrast of the angular difference of frequency (b) the contrast of the angular difference of frequency (b) the contrast of the angular difference of frequency (b) the contrast of the angular difference of frequency (b) the contrast of the angular difference of frequency (b) the contrast of the angular difference of frequency (b) the contrast of the angular difference of frequency (b) the contrast of the angular difference of frequency (b) the contrast of the angular difference of frequency (b) the contrast of the angular difference of frequency (b) the contrast of the angular difference of frequency (b) the contrast of the angular difference of frequency (b) the contrast of the angular difference of frequency (b) the contrast of the angular difference of frequency (b) the contrast of the angular difference of frequency (b) the contrast of the angular difference of frequency (b) the contrast of the angular difference of (b) the contrast of the angular differ

Fig.7 CT frequency characteristics derived by transfer function

Based on Eq.14,

$$G(s) \approx \frac{1}{R_L C_2 s + 1} \quad . \tag{13}$$

Combining Eq.17 with Eq.18, there have:

$$f_2 = \frac{1}{2p\sqrt{R_L C_2}} \ . \tag{14}$$

From Eq.17, there have:

$$\angle G(jw) = -\frac{180}{p} \cdot \arctan\frac{w}{w_2} = -\frac{180}{p} \cdot \arctan\frac{f}{f_2} = -k_n f + b_n$$
(15)

The derivation on both sides of (20):

$$k_{n} = \frac{180}{p} \cdot \frac{\frac{1}{f_{2}}}{1 + (f/f_{2})^{2}} \approx \frac{180}{p f_{2}} \left(\frac{f}{f_{2}} \Box 1\right)$$
(16)

From Eq.21, the angle difference varies linearly with frequency, which verifies the reliability of the results data. Mutually, that verifies the correctness of the transfer function of the current transformer, and lays the foundation for the error compensation of current transformer.

The frequency characteristic derived by the transfer function is consistent with the actual measured curves, and it proves once again that the frequency factor is not enough to make CT come to saturation. CT at high frequencies can still be considered to be a approximate linear system, except that the characteristics worse when in power frequency.

Frequency Characteristics Analysis Based on Transfer Function

Suppose harmonic voltage and harmonic current expression is shown as Eq.22:

$$u(t) = \sum_{h=1}^{\infty} \sqrt{2} U_h \sin(h w_1 + q_{uh}) \quad .$$
(17)

$$i(t) = \sum_{h=1}^{\infty} \sqrt{2} I_h \sin(hw_1 + q_{ih}) \quad .$$
(18)

where, when h=1 U_h , I_h is the RMS of voltage and fundamental component current, When $h \neq 1$, U_h , I_h is the RMS of voltage and current of h-th harmonics. w_1 means fundamental angular frequency. q_{uh} , q_{ih} is voltage and initial phase angle of h-th harmonics. e_h , b_h is the harmonic ratio error and phase displacement of harmonics.

the harmonic power before CT in theory is shown as Eq.24:

$$P_{h} = U_{h}I_{h}\cos(q_{uh} - q_{ih}) = U_{h}I_{h}\cos a_{h} \quad .$$
⁽¹⁹⁾

where, a_h is the power factor angle of h-th harmonic.

This paper mainly studies the impact of energy metering that current transformer brings to, so, we suppose that the error caused by voltage can be ignored. According to the Eq.2, the current after through CT can be shown as:

$$I_h' = \frac{1+e_h}{K} I_h \tag{25}$$

Then the harmonic power after through CT is:

$$P'_{h} = KU_{h}I'_{h}\cos(a_{h} - b_{h}) = KU_{h}\frac{1 + e_{h}}{K}I_{h}\cos(a_{h} - b_{h}) = P_{h}\frac{(1 + e_{h})\cos(a_{h} - b_{h})}{\cos a_{h}}$$
(20)

comparing P_h with P'_h , relative error of the harmonic active power can be shown as:

$$e_{p} = \frac{P_{h} - P_{h}'}{P_{h}} = 1 - \frac{(1 + e_{h})\cos(a_{h} - b_{h})}{\cos a_{h}} = 1 - (1 + e_{h})(\sin b_{h} + \cos b_{h}\tan a_{h})$$
(21)

From the equal above, we can conclude that the power error not only relate to harmonic ratio error and harmonic phase displacement, but also to the power factor angle. And the relationship between power error and power factor angle is a tangent function, so when $a_h = \pm 90^\circ$, the error is considerable. The fundamental power error can be shown as equal(27) while h = 1.



Fig.8.Power error under different power factor angle

According to frequency characteristic at the third quarter and equal (27), power error can be obtained at different power factor angle, which is shown in Fig.8

According to Fig.8, we can conclude :

① The power error multiplied increases with the power factor angle increases. From Figure 3.3, fundamental ratio error is only about three times as much as harmonic ratio error at 2500Hz, but power error increases sharply when harmonics ratio error and power factor angle are considered. At that time, power error is -0.5719% under 50HZ, while it becomes -57.21% under 3000Hz which is increased about 100 times.

② power factor angle can not only change the size of the harmonic power, but also can change the direction of harmonic power. It may cause deviation in the method based on positioning harmonic source via harmonic power, and so does that in harmonic measurement.

Conclusion

Current transformer calibrator can only check CT error in power frequency, but for other bands ,we can indirectly measure the error by using the ratio error in power frequency, rated current and rated load.

Firstly, the frequency characteristics of CT are studied by using the method of single frequency and superposition of single harmonics method. After the study, we can conclude that the results of the two test methods are similar in rated current and load, which indirectly illustrates the change of the frequency can not bring the current transformer from the linear region into the nonlinear region. So, a more efficient method to measure the frequency characteristics of the current transformer - superposition multiple harmonic law, is proposed according the conclusion above.

Then, current transformer transfer function is deduced according to the equivalent circuit model of current transformer. And after analyzing the results compared with the previous measurement of a single frequency, transfer function is verified to be corrected, which lay the foundation for error compensation of current transformer.

References

- Y.M.Li, X.B.Zhang. Research on a New Method of Rational Electric Energy Measurement Under the Distorted Signals Condition [J]. Automation of Electric Power Systems,2008,17:88-91.(In Chinese)
- [2] X.B.Zhang, J.R.Cui, Y.H.Liang, M.K.Wang. Electric Energy Measuring Method Analysis Based on Stochastic Process.[J]. Electrical Measurement & Instrumentation,2006,12:16-19.(In Chinese)
- [3] S.P.Shu,C.C.Wang,W,Y.Jin. Study on Theory and Arithmetic of Electric Energy Measurement for Impact Load [J]..2004,41(10):1-46.(In Chinese)
- [4] L.Y.Zeng, Y.J.Wang, L.K.Zheng. A Method based on fast lifting wavelet transform and modulus maximum for detecting power system singular signals.[J]. ChangSha University of Science & Technology learned journal(Natural Science),2007,04:56-59.(In Chinese)
- [5] G.B.Wang, D.Y.Xiang, W.M.Ma. Improved Algorithm for Non-integer Harmonics Analysis Based on FFT Algorithm and Neural Network.[J]. Proceedings of The Csee. 2008,28(4):102-108.(In Chinese)
- [6] Kondrath, N.; Kazimierczuk, M.K., "Bandwidth of Current Transformers," Instrumentation and Measurement, IEEE Transactions on , vol.58, no.6, pp.2008,2016, June 2009
- [7] Cataliotti, A; Di Cara, D.; Emanuel, AE.; Nuccio, S., "A Novel Approach to Current Transformer Characterization in the Presence of Harmonic Distortion," Instrumentation and Measurement, IEEE Transactions on , vol.58, no.5, pp.1446,1453, May 2009

- [8] GB/T 1208-2006, Current Transformer[S].(In Chinese)
- [9] GB/T 17215.302-2013 (2013). AC measuring equipment Particular requirements Part 2:static harmonic active energy meter,.(In Chinese)
- [10] Y.P.Gao, Z.S.Teng, B.M.Yang, J.X.Wang. Correction method for ratio error and phase error in harmonic energy measurement.[J]. Journal of Scientific Instrument ,2008,07:1376-1381.(In Chinese)
- [11] Emanuel, AE.; Orr, J.A, "Current Harmonics Measurement by Means of Current Transformers," Power Delivery, IEEE Transactions on, vol.22, no.3, pp.1318,1325, July 2007
- [12] Cataliotti, A; Di Cara, D.; Emanuel, AE.; Nuccio, S., "Characterization of Current Transformers in the Presence of Harmonic Distortion," Instrumentation and Measurement Technology Conference Proceedings, 2008. IMTC 2008. IEEE, vol., no., pp.2074,2078, 12-15 May 2008