# Study on Humanoid Robot's Dual-arm Collision Avoidance Motion Planning Algorithm 

Huazhong LI', a ${ }^{\text { }}$, Zhuo LIANG ${ }^{2, b}$<br>${ }^{1}$ Dept. of Software Engineering Shenzhen Institute of Information Technology Shenzhen, China<br>${ }^{2}$ Shenzhen Middle School Shenzhen, China<br>${ }^{\text {alihz }}$ @sziit.edu.cn, ${ }^{\text {b } 2862001893 @ q q . c o m ~}$


#### Abstract

Keywords: humanoid robot, dual-arm collision avoidance, bi-directional rapidly-exploring random tree, DLS-SVD method. Abstract. Aiming at humanoid robot's dual-arm collision avoidance motion planning, this paper firstly establishes kinematics model for humanoid robot and mathematic model for Workspace Target Areas (WTAs). Secondly, it has deduced inverse kinematics formula based on DLS-SVD method and proposed a bi-directional rapidly-exploring random tree collision avoidance motion planning algorithm. Finally, the effectiveness of the algorithm proposed in this paper has been verified via computer 3D simulation.


## Introduction

Humanoid robot is a hotspot [1-2] in current robot technical research field, and its dual-arm control collision avoidance planning is a key technology [3-4] in humanoid robot path planning research. With the help of virtual humanoid robot with virtual reality technology, visualization online simulation research on key technical problems relating to humanoid robot path planning can be conducted. It is widely applied in virtual assembly, computer animation, virtual prototype and anthropomorphic robot simulation, etc.

Humanoid robot (HR) dual-arm collision control planning can be described as the following: to meet the constraint conditions of control object target pose, dual-arm kinematics and dynamic mechanism, environment and obstacle, a collision-free smooth path connecting original structure and target structure is searched in HR structure space. This is a problem about typical high DoFs and nonlinear redundancy constraint path planning. As DoFs increases, its calculated amount will increase exponentially, and programming solver efficiency will reduce significantly. To solve this problem, there are two typical random motion planning methods based on sampling: Probabilistic Road Map (PRM) [4-5] and Rapidly-exploring Random Tree (RRT) [6-12].

Based on further research on humanoid robot dual-arm direct and inverse kinematics modeling, this paper has proposed a dual-arm collision avoidance control algorithm based on bi-directional rapidly-exploring random tree through introducing the concept model of WTAs. Then, the effectiveness of the algorithm proposed in this paper has been verified via computer 3D simulation.

## HR Dual-Arm Kinematics Modeling

## HR Dual-Arm Structure

The dual-arm motion mechanism of humanoid robot (HR) is similar to upper limbs and dual-arm structure of human being. It is composed of torso and left (right) arm. The torso has 3 DoFs. Define joint motion $\boldsymbol{\theta}^{t}=\left[\varphi^{t}, \theta^{t}, \psi^{t}\right]^{T}$ by referring to euler angle RPY (the upper right mark t is for torso). They are Roll (rotate left and right side sways around $R$, abbreviated as $R_{R}$ ), itch (bending rotate back and forth around axis $P$, abbreviated as $R_{p}$ ) and Yaw (rotate leftwards and rightwards around axis $Y$, abbreviated as $\mathrm{R}_{\mathrm{Y}}$ ). Left (right) arms have 7 DoFs respectively: $\boldsymbol{\theta}^{k}=\left[\theta_{1}^{k}, \theta_{2}^{k}, \Lambda, \theta_{7}^{k}\right]^{T}$ (the upper right mark $\mathrm{k}=1, \mathrm{r}$ is for left arm and right arm. That is, $\boldsymbol{\theta}^{l}$ and $\boldsymbol{\theta}^{r}$ are for left and right arm structures. The following text is similar.). They are 3 rotations $\left(R_{p}, R_{R}, R_{Y}\right)$ of shoulder joint, 1 rotation ( $R_{p}$ ) for pivot
joint to rotate around axis $P$ and 3 rotations of wrist joint $\left(R_{p}, R_{R}, R_{Y}\right)$. 3 joints in wrist are intersected at one point. The vector has totally 17 dimensions, describing the pose structures at the end of HR dual arms. It can be defined as $\left.\boldsymbol{\Theta}=\left[\begin{array}{lll}\boldsymbol{\theta}^{t}\end{array}\right]^{T}, \quad\left[\boldsymbol{\theta}^{l}\right]^{T}, \quad\left[\boldsymbol{\theta}^{r}\right]^{T}\right]^{T}$. Describing the pose structures at the end of HR left (right) arms, the vector can be defined as $\left.\boldsymbol{\Theta}^{k}=\left[\begin{array}{ll}{\left[\boldsymbol{\theta}^{t}\right.}\end{array}\right]^{T}, \quad\left[\boldsymbol{\theta}^{k}\right]^{T}\right]^{T}$. The original point $o_{w}$ of the world coordinate system (world) $\sum_{w}$ is located at the center of two feet, the original point of torso coordinate system (torso) $\sum_{t}$ is located at the center of two buttocks, and the original point of two shoulders center coordinate system $\sum_{C o A}$ is located at the center of two shoulders. The distance between the two shoulders is 2D, the torso height is $H_{t}$, and the height of lower limb is $H_{f}$. Each coordinate system of left (right) arms is defined as $\sum_{i}^{k}\left(o_{i}^{k}, x_{i}^{k}, y_{i}^{k}, z_{i}^{k}\right)(i=0,1, \ldots, 7)$.

## HR Dual-Arm Direct Kinematics

The research focus of HR kinematics is on determining the relation between the pose of HR dual-arm end-effector and joint structure and providing means and methods for dual-arm collision avoidance control. HR dual-arm direct kinematics (DK): given geometric parameters of HR's connecting rods and structure variables which connect connecting rods, solve the pose of HR dual-arm end-effector relative to specific coordinate system.

Denavit-Hartenberg (D-H) method is adopted in this paper to confirm the pose of HR dual arms. A kinematic diagram is established, as shown in Fig.1, and then a family tree structure diagram from HR dual arms is formed. D-H parameter is $\left(\theta_{i}, \alpha_{i}, a_{i}, d_{i}\right)$. D-H method is adopted to establish $4 \times 4$ homogeneous transformation matrix ${ }^{i-1} \mathbf{T}_{i}$ for the coordinate system $\sum_{i}$ for connecting rod at every joint, describing its spatial relation relative to the previous coordinate system $\sum_{i-1}$ for connecting rod. The structure principle of D-H method is as follows:
(1) The original point $o_{i}$ of $\sum_{i}$ is at the axis intersection of connecting rod length $\mathbf{a}_{i}$ and the joint $\mathbf{j}_{i+1}$;
(2) $\mathbf{z}_{i+1}$ axis coincides with the joint $\mathbf{j}_{i+1}$ axis, pointing at any direction;
(3) $\mathbf{x}_{i}$ axis coincides with $\mathbf{a}_{i}$, and $\mathbf{j}_{i}$ axis points at $\mathbf{j}_{i+1}$ axis along $\mathbf{a}_{i}$;
(4) $\mathbf{y}_{i}$ is confirmed according to the right-hand rule.

All joints of HR dual arms are rotational joints ( 3 rotational joints in torso and 7 rotational joints in left or right arm), so the general formula for ${ }^{i-1} \mathbf{T}_{i}$ is:

$$
\begin{equation*}
{ }^{i-1} \mathbf{T}_{i}=\mathbf{R}_{\text {rot }}\left(z_{i-1}, \theta_{i}\right) \cdot \mathbf{T}_{\text {trans }}\left(z_{i-1}, d_{i}\right) \cdot \mathbf{T}_{\text {trans }}\left(x_{i}, a_{i}\right) \cdot \mathbf{R}_{\text {rot }}\left(x_{i}, \alpha_{i}\right) \tag{1}
\end{equation*}
$$

In Formula (1), $\mathbf{R}_{\text {rot }}(\bullet, \bullet)$ and $\mathbf{T}_{\text {trans }}(\bullet, \bullet)$ are rotation and translation transformation matrix. $c_{i} \equiv \cos \theta_{i}, s_{i} \equiv \sin \theta_{i}, c_{\alpha i} \equiv \cos \alpha_{i}, s_{\alpha i} \equiv \sin \alpha_{i}$, and other marks are similar.

According to classical chained multiplying principle in HR kinematics (combining family tree structure and homogeneous coordinate transformation), the kinematics equation for direct homogeneous coordinate transformation of HR dual arms from $\sum_{w}$ to the end of left (right) $\operatorname{arms} \sum_{7}^{l}\left(\sum_{7}^{r}\right): \quad \quad{ }^{w} \mathbf{T}_{7}^{k}\left(\boldsymbol{\Theta}^{k}\right)={ }^{w} \mathbf{T}_{t}{ }^{t} \mathbf{T}_{C o A}\left(\boldsymbol{\theta}^{t}\right) \cdot{ }^{C o A} \mathbf{T}_{0}^{k} .{ }^{0} \mathbf{T}_{7}^{k}\left(\boldsymbol{\theta}^{k}\right)$

In Formula (2), ${ }^{w} \mathbf{T}_{t}=\left[\begin{array}{cc}\mathbf{1}_{3 \times 3} & { }^{w} \mathbf{p}_{t} \\ \mathbf{0}_{1 \times 3} & 0\end{array}\right]$ is reversible homogeneous transformation matrix for $\sum_{t}$ relative to $\sum_{w}, \mathbf{1}_{3 \times 3}$ is rotation matrix for unit opposite angles, $\mathbf{0}_{1 \times 3}$ is null matrix, and ${ }^{w} \mathbf{p}_{t}=\left[\begin{array}{lll}0 & 0 & H_{f}\end{array}\right]^{T}$ is position vector. ${ }^{t} \mathbf{T}_{C O A}\left(\boldsymbol{\theta}^{t}\right)$ is reversible torso transformation matrix for $\sum_{C O A}$ relative to $\sum_{t}$ :

$$
{ }^{t} \mathbf{T}_{C o A}\left(\boldsymbol{\theta}^{t}\right)=\left[\begin{array}{cc}
{ }^{t} \mathbf{R}_{C O A}\left(\boldsymbol{\theta}^{t}\right) & { }^{t} \mathbf{p}_{C O A}  \tag{3}\\
\mathbf{0}_{1 \times 3} & 0
\end{array}\right]
$$

In Formula (3), position translation vector ${ }^{t} \mathbf{p}_{C O A}$ is determined by $H_{t}$ (constant). Rotation matrix ${ }^{t} \mathbf{R}_{C o A}\left(\boldsymbol{\theta}^{t}\right)$ is obtained by multiplying the three rotation matrixes, namely torso roll, pitch and yaw $\boldsymbol{\theta}^{t}=\left[\varphi^{t}, \boldsymbol{\theta}^{t}, \boldsymbol{\psi}^{t}\right]^{T}$ :

$$
\begin{equation*}
{ }^{t} \mathbf{R}_{C o A}\left(\boldsymbol{\theta}^{t}\right)=\mathbf{R}_{z}\left(\varphi^{t}\right) \mathbf{R}_{y}\left(\boldsymbol{\theta}^{t}\right) \mathbf{R}_{x}\left(\boldsymbol{\psi}^{t}\right) \tag{4}
\end{equation*}
$$

In Formula (2), ${ }^{C o A} \mathbf{T}_{0}^{k}$ refers to reversible homogeneous transformation from basic coordinate system for left and right arms to dual-arm center coordinate system $\sum_{C o A}$ :

$$
{ }^{C O A} \mathbf{T}_{0}^{k}=\left[\begin{array}{cc}
\mathbf{1}_{3 \times 3} & { }^{C O A} \mathbf{p}_{0}^{k}  \tag{5}\\
\mathbf{0}_{1 \times 3} & 1
\end{array}\right]
$$

In Formula (5), the position vector for left arm is ${ }^{C o A} \mathbf{p}_{0}^{l}=[0,0,-D / 2]^{T}$, and the position vector for right arm is ${ }^{C o A} \mathbf{p}_{0}^{r}=[0,0, D / 2]^{T}$.In Formula (2), ${ }^{0} \mathbf{T}_{7}^{k}\left(\boldsymbol{\theta}^{k}\right)$ is transformation matrix for coordinate system $\sum_{7}^{k}$ for left (right) arms end-effector relative to coordinate system $\sum_{0}^{k}$ for left (right) shoulders:

$$
{ }^{C o A} \mathbf{T}_{0}^{k}=\left[\begin{array}{cc}
\mathbf{1}_{3 \times 3} & { }^{C o A} \mathbf{p}_{0}^{k}  \tag{6}\\
\mathbf{0}_{1 \times 3} & 1
\end{array}\right]
$$

In Formula (6), ${ }^{i-1} \mathbf{T}_{i}^{k}\left(\theta_{i}^{k}\right)$ is homogeneous transformation matrix for left (right) arms from $\sum_{i-1}^{k}$ to $\sum_{i}^{k}$ ( $\mathrm{i}=1,2, \ldots, 7, \mathrm{k}=\mathrm{l}, \mathrm{r}$ are for left and right arms). It can be obtained by substituting D-H four parameters ( $\theta_{i}^{k}, \alpha_{i}^{k}, a_{i}^{k}, d_{i}^{k}$ ) for 7 connecting rods of left and right arms into Formula (1):

In Formula (2), ${ }^{w} \mathbf{T}_{7}^{k}\left(\boldsymbol{\Theta}^{k}\right)=\left[\begin{array}{cc}{ }^{w} \mathbf{R}_{7}^{k} & { }^{w} \mathbf{p}_{7}^{k} \\ \mathbf{0}_{1 \times 3} & 1\end{array}\right]$ is DK $\left(\boldsymbol{\Theta}^{k}\right)$ equation for HR dual-arm coordinate transformation. After expansion, it can be expressed as a vector $\mathrm{DK}\left(\boldsymbol{\Theta}^{k}\right)$ equation:

$$
\begin{equation*}
\mathbf{x}_{e}^{k}=\mathbf{f}^{k}\left(\boldsymbol{\Theta}^{k}\right) \tag{7}
\end{equation*}
$$

In Formula (7), $\mathbf{x}_{e}^{k}$ is an expression for pose vector $\left[x_{e}^{k}, y_{e}^{k}, z_{e}^{k}, \phi_{e}^{k}, \theta_{e}^{k}, \Psi_{e}^{k}\right]^{T}$ of pose matrix $\left[\begin{array}{cc}{ }^{w} \mathbf{R}_{7}^{k} & { }^{w} \mathbf{p}_{7}^{k} \\ \mathbf{0}_{1 \times 3} & 1\end{array}\right]$ at the end of HR dual arms. $\left.\boldsymbol{\Theta}^{k}=\left[\begin{array}{lll}{\left[\boldsymbol{\theta}^{t}\right.}\end{array}\right]^{T}, \quad\left[\boldsymbol{\theta}^{k}\right]^{T}\right]^{T}$ indicates that $6 \mathrm{D} \mathbf{x}_{e}^{k}$ vector for the pose at the end of HR left (right) arms is the function or mapping relation of torso 3D structure $\boldsymbol{\theta}^{t}$ and left (right) arm 7 D structure $\boldsymbol{\theta}^{l}\left(\boldsymbol{\theta}^{r}\right)$.

## HR Dual-Arm Inverse Kinematics

HR dual-arm inverse kinematics (IK) is described as follows: knowing geometric parameters for the connecting rods of HR dual arms and torso and the pose of dual-arm end-effector relative to specific reference coordinate system, judge whether HR dual-arm end-effector reaches the pose, and solve the corners of joint structures while reaching the expected pose.

Take the derivative of the time $t$ by Formula (7) to obtain the differential relation between HR dual arm configuration $\boldsymbol{\Theta}^{k}$ and the pose $\mathbf{x}_{e}^{k}$ :

$$
\begin{equation*}
\mathbf{x}_{e}=\mathbf{J}_{k}\left(\mathbf{(}^{k}\right) \boldsymbol{\Theta}^{k} \tag{8}
\end{equation*}
$$

In Formula (8), is generalized velocity for HR dual-arm ends in work space, $\boldsymbol{\Theta}^{k}$ is joint velocity, $\mathbf{J}_{k}\left(\boldsymbol{\Theta}^{k}\right)$ is partial derivative matrix of $6 \times 10$ which is jacobian matrix for HR dual arms. Its No. $i$ row and No. $j$ column are ( $\mathrm{k}=\mathrm{l}, \mathrm{r}$ for left and right arms):

$$
\begin{equation*}
J_{i j}^{k}\left(\boldsymbol{\Theta}^{k}\right)=\frac{\partial f_{i}^{k}\left(\boldsymbol{\Theta}^{k}\right)}{\partial \Theta_{j}^{k}}(\mathrm{i}=1, \ldots, 6, \mathrm{j}=1, \ldots, 10) \tag{9}
\end{equation*}
$$

Apparently, there is redundancy in HR dual-arm kinematics. While solving IK, singularity problem will be met. If the generalized inverse of $\mathbf{J}_{k}\left(\boldsymbol{\Theta}^{k}\right)$ exists, IK's differential form for HR dual arms can be obtained from Formula (8):

$$
\begin{equation*}
\boldsymbol{\Theta}^{k}=\mathbf{J}_{k}^{+}\left(\boldsymbol{\Theta}^{k}\right) \boldsymbol{\delta}_{e}^{k} \tag{10}
\end{equation*}
$$

To facilitate numerical calculation, the increment form of Formula (10) is:

$$
\begin{equation*}
\Delta \boldsymbol{\Theta}^{k}=\mathbf{J}_{k}^{+}\left(\boldsymbol{\Theta}^{k}\right) \Delta \mathbf{x}_{e}^{k} \tag{11}
\end{equation*}
$$

To this end, this paper has researched the following solution method for IK of HR dual arms based on DLS-SVD.

## DLS-SVD Method

To improve operability and stability of HR dual-arm control, this paper proposes DLS-SVD method combining Damped Leased Square (DLS) and Singular Value Decomposition (SVD) to solve IK numerical solution of HR dual arms. By using DLS method, seeking the solution $\Delta \Theta^{k}$ of IK which meets $\Delta \mathbf{x}_{e}^{k}=\mathbf{J}_{k}\left(\boldsymbol{\Theta}^{k}\right) \Delta \boldsymbol{\Theta}^{k}$ can be transformed to seek the minimum $\Delta \boldsymbol{\Theta}^{k}$ in the following formula:

$$
\begin{equation*}
\left\|\mathbf{J}_{k}\left(\boldsymbol{\Theta}^{k}\right) \Delta \boldsymbol{\Theta}^{k}-\Delta \mathbf{x}_{e}^{k}\right\|+\lambda_{k}^{2}\left\|\Delta \boldsymbol{\Theta}^{k}\right\| \tag{12}
\end{equation*}
$$

In Formula (12), $\lambda_{k} \in R$ is a non-zero damping coefficient, and it is equivalent to the following matrix form:

$$
\left\|\left[\begin{array}{c}
\mathbf{J}_{k}\left(\boldsymbol{\Theta}^{k}\right)  \tag{13}\\
\lambda_{k} \mathbf{I}
\end{array}\right] \Delta \boldsymbol{\Theta}^{k}-\left[\begin{array}{c}
\Delta \mathbf{x}_{e}^{k} \\
\mathbf{0}
\end{array}\right]\right\|
$$

The regular equation of Formula (14) can be written as:

$$
\left[\begin{array}{c}
\mathbf{J}_{k}  \tag{14}\\
\lambda_{k} \mathbf{I}
\end{array}\right]^{T}\left[\begin{array}{c}
\mathbf{J}_{k} \\
\lambda_{k} \mathbf{I}
\end{array}\right] \Delta \boldsymbol{\Theta}^{k}=\left[\begin{array}{c}
\mathbf{J}_{k} \\
\lambda_{k} \mathbf{I}
\end{array}\right]^{T}\left[\begin{array}{c}
\Delta \mathbf{x}_{e}^{k} \\
\mathbf{0}
\end{array}\right]
$$

Formula (14) is equivalent to the following formula: $\left(\mathbf{J}_{k}^{T} \mathbf{J}_{k}+\lambda_{k}^{2}\right) \Delta \boldsymbol{\Theta}^{k}=\mathbf{J}_{k}^{T} \Delta \mathbf{x}_{e}^{k}$
According to Formula (12), the solution for DLS of IK of HR dual arms is:

$$
\Delta \Theta^{k}=\mathbf{J}_{k}^{T}\left(\mathbf{J}_{k}^{T} \mathbf{J}_{k}+\lambda_{k}^{2} \mathbf{I}\right)^{-1} \Delta \mathbf{x}_{e}^{k}
$$

SVD decomposition for $\mathbf{J}_{k}^{T} \mathbf{J}^{k}+\lambda_{k}^{2} \mathbf{I}$ is conducted, and it can be expressed as:

$$
\begin{equation*}
\mathbf{J}_{k} \mathbf{J}_{k}^{T}+\lambda_{k}^{2} \mathbf{I}=\left(\mathbf{U}_{k} \mathbf{D}_{k} \mathbf{V}_{k}^{T}\right)\left(\mathbf{V}_{k} \mathbf{D}_{k}^{T} \mathbf{U}_{k}^{T}\right)+\lambda_{k}^{2} \mathbf{I}=\mathbf{U}_{k}\left(\mathbf{D}_{k} \mathbf{D}_{k}^{T}+\lambda_{k}^{2} \mathbf{I}\right) \mathbf{U}_{k}^{T} \tag{17}
\end{equation*}
$$

In Formula (17), $\mathbf{D}_{k} \mathbf{D}_{k}^{T}+\lambda_{k}^{2} \mathbf{I}$ is diagonal matrix, and its diagonal element is $\sigma_{k, i}^{2}+\lambda_{k}^{2}$, then:

$$
\begin{equation*}
\mathbf{J}_{k}^{T}\left(\mathbf{J}_{k} \mathbf{J}_{k}^{T}+\lambda_{k}^{2} \mathbf{I}\right)^{-1}=\mathbf{V}_{k} \mathbf{D}_{k}^{T}\left(\mathbf{D}_{k} \mathbf{D}_{k}^{T}+\lambda_{k}^{2^{2}} \mathbf{I}\right)^{-1} \mathbf{U}_{k}^{T}=\mathbf{V}_{k} \mathbf{E}_{k} \mathbf{U}_{k}^{T} \tag{18}
\end{equation*}
$$

Where, $\mathbf{E}_{k}$ is the diagonal matrix of $10 x 6 \mathrm{~m}$ and its diagonal element is:

$$
\begin{equation*}
e_{i, i}^{k}=\frac{\sigma_{k, i}}{\sigma_{k, i}^{2}+\lambda_{k}^{2}} \tag{19}
\end{equation*}
$$

Therefore, the solution for DLS in Formula (16) can be expressed as:

$$
\begin{equation*}
\Delta \Theta^{k}=\sum_{i=1}^{r} \frac{\sigma_{k, i}}{\sigma_{k, i}^{2}+\lambda_{k}^{2}} \mathbf{v}_{k, \mathbf{u}} \mathbf{u}_{k, i}^{T} \Delta \mathbf{x}_{e}^{k} \tag{20}
\end{equation*}
$$

Where, $\boldsymbol{\lambda}_{k}$ is damping coefficient for left (right) arms, which is directly related to operability and stability of HR dual arm control: When left (right) arms are close to singularity configuration, the selected $\lambda_{k}$ is smaller than $\sqrt{\operatorname{det}\left(\mathbf{J}_{k} \mathbf{J}_{k}^{T}\right)}$; when left (right) arms are far away from singularity configuration, the selected $\lambda_{k}$ is greater than $\sqrt{\operatorname{det}\left(\mathbf{J}_{\mathbf{k}} \mathbf{J}_{k}^{T}\right)}$.

## WTAs 's Physical Modeling

In this paper, the concept of Workspace Target Areas (WTAs) is introduced. When HR carries out the tasks which are similar to human being, set the continuous area of end-effector in 6D work space as the target pose of dual-arm control planning tool. In this paper, the pose for dual arms is set as $\left[x_{e}^{k}, y_{e}^{k}, z_{e}^{k}, \varphi_{e}^{k}, \theta_{e}^{k}, \psi_{e}^{k}\right]^{T}$.

## Mathematic Model of WTAs

Mathematic model of WTAs is described in three aspects:
(1) ${ }^{w} \mathbf{T}_{W T A}^{k}$ : Reference homogeneous transformation matrix of WTA of left and right arms in relative world coordinate system $\sum_{w}$.
(2) ${ }^{W T A} \mathbf{T}_{e}^{k}$ : The specific deviated homogeneous transformation matrix of end-effector of left and right arms in WTA coordinate system $\sum_{\text {WTA }} . \sum_{\text {WTA }}$ is defined in the object center.
(3) ${ }^{W T A} \mathbf{B}_{e}^{k}$ : Set $6 \times 2$ bordered matrix of WTA in $\sum_{W T A}(\mathrm{~L}$ and U in the lower right table are for minimum lower bound and upper bound): $\quad{ }^{W T A} \mathbf{B}_{e}^{k}=\left[\begin{array}{cccccc}x_{L}^{k} & y_{L}^{k} & z_{L}^{k} & \varphi_{L}^{k} & \theta_{L}^{k} & \psi_{L}^{k} \\ x_{U}^{k} & y_{U}^{k} & z_{U}^{k} & \varphi_{U}^{k} & \theta_{U}^{k} & \psi_{U}^{k}\end{array}\right]^{T}$

The front three rows of ${ }^{W T A} \mathbf{B}_{e}^{k}$ are for the allowable displacement along axis x, y and z described in $\sum_{\text {WTA }}$. The next three rows are for the allowable displacement of Roll-Pitch-Yaw (RPY) described in $\sum_{\text {WTA }}$.

## Distance Measure of WTA

To use WTAs in random planning for HR dual-arm control collision avoidance, the distance the structure $\left.\boldsymbol{\Theta}_{s}^{k}=\left[\begin{array}{ll}\boldsymbol{\theta}_{s}^{k}\end{array}\right]^{T}, \quad\left[\boldsymbol{\theta}_{s}^{k}\right]^{T}\right]^{T}$ given by left (right) arms to WTA must be calculated.

As for the given $\Theta_{s}^{k}$, the homogeneous transformation matrix for the pose of end-effector in this structure an be obtained by Formula (2) direct kinematics: ${ }^{w} \mathbf{T}_{W T A}^{k}\left(\Theta_{s}^{k}\right)={ }^{w} \mathbf{T}_{e}^{k}\left(\boldsymbol{\Theta}_{s}^{k}\right)\left({ }^{W T A} \mathbf{T}_{e}^{k}\right)^{-1}$

In $\sum_{w}$, the pose of the object grabbed by end-effector is:

$$
\begin{equation*}
{ }^{w} \mathbf{T}_{g}^{k}\left(\boldsymbol{\Theta}_{s}^{k}\right)={ }^{w} \mathbf{T}_{\text {WTA }}^{k} \cdot\left[{ }^{w} \mathbf{T}_{\text {WTA }}^{k}\left(\boldsymbol{\Theta}_{s}^{k}\right)\right]^{-1} \tag{23}
\end{equation*}
$$

The following formula can be obtained by transforming the object pose from $\sum_{w}$ to $\sum_{\text {WTA }}$ :

$$
\begin{equation*}
{ }^{W T A} \mathbf{T}_{g}^{k}\left(\boldsymbol{\Theta}_{s}^{k}\right)=\left({ }^{w} \mathbf{T}_{W T A}^{k}\right)^{-1} \cdot{ }^{w} \mathbf{T}_{g}^{k}\left(\boldsymbol{\Theta}_{s}^{k}\right) \tag{24}
\end{equation*}
$$

In the formula, ${ }^{\text {WTA }} \mathbf{T}_{g}^{k}\left(\boldsymbol{\Theta}_{s}^{k}\right)=\left[\begin{array}{cc}{ }^{W T A} \\ \mathbf{R}_{g}^{k}\left(\boldsymbol{\Theta}_{s}^{k}\right) & { }^{\text {WTA }} \mathbf{p}_{g}^{k} \\ 0 & 1\end{array}\right]$.
Now, transform this pose matrix into 6D pose vector described in $\sum_{\text {WTA }}$, including 3 translations ( x , $\mathrm{y}, \mathrm{z})$ and 3 RPY rotations ( $\varphi, \theta, \psi$ ). It is consistent with what is defined by ${ }^{W T A} \mathbf{B}_{e}^{k}$ :

$$
{ }^{W T A} \mathbf{x}_{g}^{k}=\left[\begin{array}{c}
{ }^{W T A} \mathbf{p}_{g}^{k}  \tag{25}\\
a \tan 2\left({ }^{W T A} \mathbf{R}_{g[3,2]}^{k},{ }^{W T A} \mathbf{R}_{g[3,3]}^{k}\right) \\
\left.-\arcsin { }^{\left({ }^{W T A}\right.} \mathbf{R}_{g[3,1]}^{k}\right) \\
a \tan 2\left({ }^{W T A} \mathbf{R}_{g[2,1]}^{k},{ }^{W T A} \mathbf{R}_{g[1,1]}^{k}\right)
\end{array}\right]
$$

Considering the boundary constraint ${ }^{\text {WTA }} \mathbf{B}_{e}^{k}, 6 \mathrm{D}$ displacement vector $\Delta^{W T A} \mathbf{x}_{g}^{k}$ of WTA can be obtained:

Where, $i$ is the subscript for traversing ${ }^{\text {WTA }} \mathbf{B}_{e}^{k}$ and ${ }^{\text {WTA }} \mathbf{x}_{g}^{k} \cdot\left\|\Delta^{\text {WTA }} \mathbf{x}_{g}^{k}\right\|$ is the distance measure from the structure $\boldsymbol{\Theta}_{s}^{k}$ given by HR left (right) arms to WTA area.

## Sampling Strategy of WTAs

To use WTAs in random planning for HR dual-arm control collision avoidance, the random sampling needs to be conducted for WTAs so that they can be applied IK method for HR. The sampling strategy for single WTA is to use uniform probability to obtain the pose vector ${ }^{W T A} \mathbf{x}_{\text {rand }}^{k}$ which is randomly sampled from the object controlled by left and right arms within the boundary defined by ${ }^{\text {WTA }} \mathbf{B}_{e}^{k}$ and then transform it into the pose matrix ${ }^{\text {WTA }} \mathbf{T}_{\text {rand }}^{k}$ :

$$
\begin{gather*}
{ }^{W T A} \mathbf{x}_{\text {rand }}^{k}={ }^{\text {WTA }} \mathbf{B}_{e, L}^{k}+\operatorname{rand}(0,1)\left({ }^{W T A} \mathbf{B}_{e, U}^{k}{ }^{-}{ }^{\text {WTA }} \mathbf{B}_{e, L}^{k}\right)  \tag{27}\\
{ }^{\text {WTA }} \mathbf{T}_{\text {rand }}^{k}=\left[\begin{array}{cc}
{ }^{W T A} & \mathbf{R}_{\text {rand }}^{k} \\
{ }^{\text {WTA }} & \mathbf{p}_{\text {rand }}^{k} \\
1
\end{array}\right] \tag{28}
\end{gather*}
$$

The expression of the object which is randomly sampled from the object in $\sum_{w}$ can be obtained by applying the transformation matrix of end-effector of HR left (right) arms:

$$
\begin{equation*}
{ }^{w} \mathbf{T}_{\text {rand }, g}^{k}={ }^{w} \mathbf{T}_{\text {WTA }}^{k}{ }^{W T A} \mathbf{T}_{\text {rand }}^{k}{ }^{W T A} \mathbf{T}_{e}^{k} \tag{29}
\end{equation*}
$$

The random structure $\Theta_{\text {rand }}^{k}$ of HR dual arms can be obtained by the above DLS-SVD method solving IK of HR dual arms.

Similarly, set ${ }^{W T A} \mathbf{B}_{e}^{k_{s}}=\left\{{ }^{W T A} \mathbf{B}_{e}^{k_{1}}, \mathrm{~L},{ }^{W T A} \mathbf{B}_{e}^{k_{m}}\right\}$, sample multiple WTAs and adopt the method based on weight.

$$
\begin{equation*}
{ }^{W T A} \mathbf{x}_{\text {rand }}^{k}=\sum_{i=1}^{m} \zeta_{i}^{k_{i} W T A} \mathbf{x}_{\text {rand }}^{k_{i}} \tag{30}
\end{equation*}
$$

Where, the weight factor $\zeta_{i}^{k_{i}}\left(\sum_{i=1}^{m} \zeta_{i}^{k_{i}}=1\right)$ is: $\quad \zeta_{i}^{k_{i}}=\sum_{j=1}^{6}\left({ }^{W T A} \mathbf{B}_{e[j, U]}^{k_{i}}-{ }^{W T A} \mathbf{B}_{e[j, L]}^{k_{i}}\right)$
According to Formula (31), ${ }^{\text {WTA }} \mathbf{x}_{\text {rand }}^{k}$ is obtained. By using the above similar method, the random structure $\boldsymbol{\Theta}_{\text {rand }}^{k}$ of HR dual arms can be obtained.

## Dual-Arm Collision Avoidance Motion Planning Algorithm

By combining the solution algorithm for DK and IK of HR dual arms and classical bi-directional RRT (BiRRTs) algorithm ${ }^{[1-11]}$, this paper proposes a dual-arm collision avoidance control algorithm with complete probability whose core algorithm is $\operatorname{MCIKRRT}\left(\boldsymbol{\Theta}_{s}^{k},{ }^{W T A} \mathbf{B}_{e}^{k_{s}}\right)$. This algorithm starts from initial structure $\boldsymbol{\Theta}_{s}^{k}$ of HR left (right) arms and takes the target pose area ${ }^{W T A} \mathbf{B}_{e}^{k_{s}}$ as a heuristic search target structure $\boldsymbol{\Theta}_{g, b}^{k}$ to bi-directionally generate random trees $\mathbf{T}_{a}^{k}$ and $\mathbf{T}_{b}^{k}$. In every iterative process, MCIKRRT selects one of the following two search modes:
(1) Expand the structure space of HR dual arms by classical BiRRTs.
(2) Sample from the target area ${ }^{W T A} \mathbf{B}_{e}^{k_{s}}$ by WTAs sampling strategy, obtain ${ }^{W T A} \mathbf{x}_{\text {rand }}^{k}$ according to Formula (27) or (30) and conduct expansion search by taking it as the heuristic information for target pose.

The selection probability of every model is controlled by the parameter $f_{r}$. The pseudo-code of MCIKRRT core algorithm is described as follows:

```
Algorithm :MCIKRRT ( \(\left.\Theta_{s}^{k},{ }^{\text {WTA }} \mathbf{B}_{e}^{k_{s}}\right)\) \{
\(01 \quad \mathbf{T}_{a}^{k} \cdot \mathbf{I n i t}\left(\Theta_{s}^{k}\right) ; \mathbf{T}_{b}^{k} \cdot \mathbf{I n i t}(\mathrm{NULL}) ;\)
02 do \{
\(03 \quad \operatorname{if}\left(\mathbf{T}_{b}^{k} . \operatorname{size}()=0 \| \operatorname{rand}(0,1)<f_{r}\right)\left\{\mathbf{T}_{b}^{k} \cdot \mathbf{H R} \_\mathbf{I K}\left(\Theta_{\text {gool, }, b}^{k}{ }^{\text {WTA }} \mathbf{B}_{e}^{k_{s}}\right) ;\right\}\)
\(04 \quad\) else \(\left\{\quad \boldsymbol{\Theta}_{\text {rand }, a}^{k}=\mathbf{T}_{a}^{k} \cdot \operatorname{RandomConfig}() ; \quad \boldsymbol{\Theta}_{\text {near }, a}^{k}=\mathbf{T}_{a}^{k} \cdot \operatorname{NearNeighbor}\left(\boldsymbol{\Theta}_{\text {rand }, a}^{k}\right)\right.\);
\(05 \quad \Theta_{\text {reach }, a}^{k}=\mathbf{T}_{a}^{k} \cdot \operatorname{Connect}\left(\boldsymbol{\Theta}_{\text {near }, a}^{k}, \boldsymbol{\Theta}_{\text {rand }, a}^{k}\right) ; \quad \boldsymbol{\Theta}_{\text {near }, b}^{k}=\mathbf{T}_{b}^{k} \cdot \operatorname{NearNeighbor}\left(\boldsymbol{\Theta}_{\text {reach }, a}^{k}\right)\);
\(06 \quad \Theta_{\text {reach }, b}^{k}=\mathbf{T}_{b}^{k} . \operatorname{Connect}\left(\boldsymbol{\Theta}_{\text {near }, b}^{k}, \boldsymbol{\Theta}_{\text {reach }, a}^{k}\right)\);
\(07 \quad \operatorname{if}\left(\left\|\boldsymbol{\Theta}_{\text {reach }, b}^{k}-\boldsymbol{\Theta}_{\text {reach }, a}^{k}\right\| \leq \delta \boldsymbol{\Theta}_{\min }^{k}\right)\left\{\quad \mathbf{S}^{k}=\operatorname{BuildSolu}\left(\mathbf{T}_{a}^{k}, \boldsymbol{\Theta}_{\text {reach }, a}^{k}, \mathbf{T}_{b}^{k}, \boldsymbol{\Theta}_{\text {reach }, b}^{k}\right)\right.\);
08
                                    return SmoothSolution ( \(\mathbf{S}^{k}\) ); \}
            else \(\left.\left\{\operatorname{Swap}\left(\mathbf{T}_{a}^{k}, \mathbf{T}_{b}^{k}\right) ;\right\}\right\}\)
    \}while(!TimeOut());
    return NULL; \(\}\)
```


## Computer 3D Simulation

In the computer whose CPU is I7-3770 (CORE I7 3.4G 8M cache) and memory is 8G DDRIII1600, HR models are established by Open Inventor. Virtual prototype and environment model ${ }^{[13-14]}$ can be constructed through VC++ programming to verify mechanism modeling method and control algorithm for HR dual arms proposed in this paper. Set the initial pose $\mathbf{x}_{e}^{l}=[-464,-269,1036,2.4,-0.2,-2.3]^{T}$ of HR left arm and the pose at the end of right $\operatorname{arm} \mathbf{x}_{e}^{r}=[531,134,1290,-0.9,1.2,0.5]^{T}$. The pose of water
cup is: ${ }^{W T A} \mathbf{B}_{e}^{k}=\left[\begin{array}{cc}-0.15 & 0.15 \\ 505.85 & 506.15 \\ 1116.95 & 1117.05 \\ 0.575 & 0.625 \\ -0.308 & -0.309 \\ 0.102 & 0.104\end{array}\right]$.
In MCIKRRT algorithm, the planning result for solving IK by DLS-SVD method is shown in Fig.1. The joint configuration track of HR dual arms is shown in Fig.2. The average planning time is $4,210.88$ ms.


Fig. 1 Result of collision avoidance planning when HR dual arms grab the bowl


Fig. 2 Joint configuration track when HR dual arms grab the water cup

## Summary

WTAs algorithm for HR dual arm collision avoidance control proposed in this paper can set the target pose area (WTAs) of the object in work space visually and effectively. Thus, it can effectively cover the whole target pose set covered by machine vision and easily measure the distance from the end-effector. Direct kinematics model of HR dual arms established based on homogeneous transformational matrix and chained multiplying rule can easily realize fast recursion numerical calculation via computer. IK solution for HR dual arms based on DLS-SVD method is of universality.

## Acknowledgment

This work was sponsored by Shenzhen Science and Technology Program (JC201006020820A and JCYJ20120615101640639), Team of Scientific and Technological Innovation of Shenzhen Institute of Information Technology (CXTD2-002, LG2014005), Natural Science Foundation of Guangdong Province, China (S2013010013779), Guangdong Vocational Education Information Technology Fund (XXJS-2013-1019), School-enterprise Cooperation Projects(HX-054,HX-077).

## References

[1] Shin-Yu Liu,Wen-June Wang,Rong-Jyue Wang,Cheng Wang,I-Ping Chang. Image recongnition and force measurement application in the humanoid robot imitation[J]. IEEE Transactions on Instrumentation and Measurement ,2012, 61(1):149-161.
[2] Ke Wende,Hong Bingrong,Cui Gang,et al. Study on similarity imitation constraints of biped walking for humanoid robot[J]. International Journal of Computing Science and Mathemaics,2013,4(1):51-61.
[3] BERTRAM D, KUFFNER J, DILLMANN R, et al. An integrated approach to inverse kinematics and path planning for redundant manipulators[C] //Proc IEEE International Conference on Robotics and Automation (ICRA).Orlando, USA, 2006: 1874-1879
[4] Z. Sun, D. Hsu, T. Jiang, H. Kurniawati, and J. H. Reif. Narrow passage sampling for probabilistic roadmap planning[J]. IEEE Transactions on Robotics and Automation, 2005, 21(6):1105-1115.
[5] K.I. Tsianos, I.A. Şucan, and L.E. Kavraki. Sampling-based robot motion planning: Towards realistic applications[J]. Computer Science Review, 2007,1(1): 2-11.
[6] S La Valle, J Kuffner. Rapidly-exploring random trees:Progress and Prospects[A]. Proceedings of Algo-rithmic and Computational Robotics:New Direc-tions,2001[C].2001:293-308.
[7] LI Huazhong, LIANG Yongsheng, WANG Meini, Dan Tangren. Design and Implementation of Improved RRT Algorithm for Collision Free Motion Planning of High-Dimensional Robot in Complex Environment[C]. ICCSNT 2012, 2nd Int. Conference on Computer Science and Network Technology: 1391-1397.
[8] D. Bertram, J.J. Kuffner, R. Dillmann and T. Asfour. An integrated approach to inverse kinematics and path planning for redundant manipulators. In Proceedings of IEEE International Conference on Robotics and Automation, 2006[C].2006:1874-1879.
[9] M.K. Weghe, D. Ferguson, and S.S. Srinivasa. Randomized path planning for redundant manipulators without inverse kinematics.In Proceedings of IEEE International Conference on Humanoid Robots, 2007[C]. 2007:477-482.
[10] D. Berenson, S.S. Srinivasa, D. Ferguson, and J.J. Kuffner. Manipulation planning on constraint manifolds.In Proceedings of IEEE International Conference on Robotics and Automation, 2009[C]. 2009:625-632.
[11]E. Drumwright and V. Ng-Thow-Hing.Toward interactive reaching in static environments for humanoid robots[C]. Intelligent Robots and Systems, 2006 IEEE/RSJ International Conference on, Oct. 2006, pp.846-851.
[12]GRAD A, MILITAOF, NADEN K. Improving RRT with context sensitivity[C]// Proceedings of the IEEE International Conference on Robotics and Automation,April 3,2010,Detroit MI.
IEEE,2010:1-5.
[13] LI Huazhong, LIANG Yongsheng, WANG Meini, Dan Tangren. Design and Implementation of Improved RRT Algorithm for Collision Free Motion Planning of High-Dimensional Robot in Complex Environment[C]. //ICCSNT 2012, 2nd Int. Conference on Computer Science and Network Technology, CHANGCHUN, CHINA, 2012:1391-1397.
[14]N. Vahrenkamp, C. Scheurer, T. Asfour, R. Dillmann, and J. Kuffner. Adaptive motion planning for humanoid robots[C].// Intelligent Robots and Systems, 2008. IROS 2008. IEEE/RSJ International Conference on, 2008, Nice, 22-26 Sept. 2008: 2127 - 2132.

