

## Reliability Model for subsystems of CNC Machine Tools based on the Repair Degree

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**Keywords:** CNC Machine Tools; Subsystem; Reliability model; repair degree

**Abstract:** The traditional reliability models of CNC machine tools are based on the assumption that the machine after repair is as-good-as-new or as-bad-as-new. But it is not the precise model compared with the Engineering practice. This article focuses on the repair degree to the sub-system of CNC machine tools, and the reliability modeling method was proposed based on incomplete repair circumstances, considering the maintenance degrees. Meanwhile, the model's parameters are calculated using the method integrating maximum likelihood method. Genetic algorithm is used in the solving process. The model is applied to actual data, the results shows that: this model can get the maintenance degrees which reflecting the real states of each sub-system's changes and the intersection of the different failure-periods. It can be the references and basis for the design of reliability and maintainability.

### Introduction

CNC machine tools is a complex and repairable system integrated with technology fields of mechanism, electrics and hydraulic. In addition, CNC machine tool is composed of several subsystems, which can be done independent functions. The reliability is an important index in the evaluation system of the machine tools. According to the classical theories and practical experience, the reliability of subsystems is the basis of evaluating and testing the level of the complete machine's reliability. In the past, the probability statistics theory is used to model the reliability of CNC machine tools frequently, which applies Lognormal distribution, Weibull distribution, Exponential distribution and so forth functions to match the model of the time between failures<sup>[1-3]</sup>. With the development of reliability technology of the whole lifecycle of CNC machine tools, researchers found that only one distribution function to describe the reliability model for a machine would be lead to a certain extent error. In order to reduce the error, Chen. put forward a reliability model with a bathtub curve failure probability function which is composed with twofold Weibull subsectional functions<sup>[4]</sup>. Afterward, Wang Zhiming, Yang Zhaojun and other researchers established the reliability models based on the premise that the machine will be as bad as new after maintenance which considered the influence of maintenance effect. The kind of above model is more suitable for the engineering practice compared with the model of "as good as new"<sup>[5-8]</sup>.

The majority of above reliability models for CNC machine tools are established on the hypothesis of "as good as new" or "as bad as new". The affection of repair maintenance or replace

maintenance is different to the CNC machine tools and the subsystems. For the whole CNC machine tools, the reliability model can be described with an “as bad as new” model. However, such kind of maintenance may generate greater influence to subsystems compared with the whole CNC machine tools, whose repair affection could be between “as good” or “as bad”.

In this case, we put forward the reliability model of CNC’s subsystems considering the degree of maintenance, combined with the reliability rules for the earlier failure period and earlier failure period of CNC subsystem in this paper.

### Reliability model for subsystem of CNC machine tool considering repair degree

“As good as new” and “as bad as new” are regarded as the extreme situation of incomplete repair in Kijima model. While, the incomplete repair model can set up the relationship of effective age and repair effect between these two maintenance strategies. At the same time, the Kijima model can describe the relationship between maintenance and system failure intensity in this method [9]. Therefore, we can put forward the reliability model including the earlier-failure-period and random-period according the relationship between maintenance degree and reliability on the basis of Kijima model. In this new model, it can evaluate the reliability level and the maintenance effect of each subsystem of CNC machine tools.

#### *Kijima I model*

In the Kijima I model of incomplete repair strategy,  $q_i$  represents the age reduction factor of the  $i$ -th maintenance process, which is the quantitative representation of maintenance degree. Virtual age  $V$  is used to describe the "virtual life" of the system after maintenance, with the specific expression:

$$V_i = V_{i-1} + q_i X_i \quad 0 \leq q_i \leq 1 \quad (1)$$

In the Eq.1,  $X_i$  represents the  $i$ -th interval of adjacent failure, namely  $X_i = S_i - S_{i-1}$ .  $S_i$  stands for the failure time of  $i$ -th failure, and  $S_0 \equiv 0$ . Especially in the Kijima I model, the age reduction factor:  $q_i = q$ .

“The age and the failure intensity of the machinery will be changed simultaneously by maintenance activities.” [10]. After the  $(n-1)$ -th failure and maintenance, the system is equivalent to return to the age state when the virtual age  $V_{n-1} = y$ , and the failure intensity function and cumulative distribution function of the  $n$ -th failure duration interval  $X_n$  of system are respectively:

$$w_t = w(t - S_{n-1} + V_{n-1}) \quad (2)$$

$$F_n(X_n | V_{n-1} = y) = \frac{F(X_n + y) - F(y)}{1 - F(y)} \quad (3)$$

#### *Reliability model for subsystem of CNC machine tool considering repair degree*

According to the basic function of Kijima I model, we can set up the reliability model for subsystem of CNC machine tools considering repair degree. In the earlier-failure-period of subsystem, the intensity function presented a gradually increasing trend as a result of high failure rate. But it will rapidly decrease as time goes by, the main reason lies in the defects of design, manufacturing process, the selection of raw materials and so on. Therefore, the maintenance only has the effect of "as bad as new" for the subsystem based on the analysis of the failure reasons, namely, assuming the maintenance strategy in the earlier-failure-period to be the minimal maintenance. When the system reaches the random-failure-period, the defects of earlier-failure-period has been ruled out and as time passed, the failure intensity functions also showed a trend of growth. Therefore, we can assume the phase of maintenance as incomplete maintenance. Then we can set up the reliability

model for subsystem considering repair degree. In the reliability model, the virtual age of subsystem of CNC machine tools is:

$$V_i = \begin{cases} S_i & S_i \leq t_j \\ t_j + q(S_i - t_j) & S_i > t_j \end{cases} \quad (4)$$

In Eq.4,  $t_j$  represents the time of the critical point between earlier-failure-period and random-failure- period.

Because of the difference of reliability model between earlier-failure-period and random-failure-period of CNC machine tool subsystem, we build the model for initial intensity function by sub-sectional Non-Homogeneous Poisson Process (NHPP) model<sup>[7]</sup>. Supposing NHPP fit Weibull process, its initial intensity function is:

$$w(t) = \begin{cases} I_1 b_1 t^{b_1-1} & t \leq t_j \\ I_1 b_1 t_j^{b_1-1} + I_2 b_2 (t - t_j)^{b_2-1} & t > t_j \end{cases} \quad (5)$$

Combined with the Eq.4, Eq.5 and failure intensity function of Kijima I model, we can deduce the failure intensity function for subsystem of CNC machine tool based on the age reduction factor:

$$w_t = \begin{cases} w(t) & S_n \leq t_j \\ w[t - (1-q)(S_n - t_j)] & S_n > t_j \end{cases} = \begin{cases} I_1 b_1 t^{b_1-1} & S_n \leq t_j \\ I_1 b_1 t_j^{b_1-1} + I_2 b_2 [t - S_n + q(S_n - t_j)]^{b_2-1} & S_n > t_j \end{cases} \quad (6)$$

According to Eq.6, when “ $q=1$ ”, we can get “ $\forall i, V_i = S_i$ ”, which represent that all of the stages are minimal maintenance ;when “ $q=0$ ”, we can get “ $\forall i, V_i = t_j, (S_i > t_j)$ ”. In this case, it is supposed that the maintenance process is perfect after the earlier-failure-stage; when “ $0 < q < 1$ ”, the maintenance of random-failure-period and wearing-period is incomplete repair. In this case, the smaller the value of  $q$  is, the better the maintenance effect is. Otherwise, the bigger the value of  $q$  is, the worse the maintenance effect will be. Meanwhile we can draw a conclusion from the calculation that the failure density function is discontinuous when “ $q \neq 0$ ” or “ $q \neq 1$ ”.

And then, we can derive from the function curve of initial cumulative strength:

$$W_t = \begin{cases} W(t) & (S_n \leq t_j) \\ W[t - (1-q)(S_n - t_j)] & (S_n > t_j) \end{cases} = \begin{cases} I_1 t^{b_1} & (S_n \leq t_j) \\ I_1 t_j^{b_1} + I_2 [t - S_n + q(S_n - t_j)]^{b_2} & (S_n > t_j) \end{cases} \quad (7)$$

## The Maximum Likelihood Estimation of the parameters in reliability model of subsystems

### The determination of the likelihood function

In this paper, we estimate the model parameters with the method of Maximum Likelihood Estimation(MLE). Assuming that Time To First Failure(TTFF) of the system fits Weibull distribution, and  $w(t) = I b t^{b-1}$ ,  $(X_1, X_2, \dots, X_n)$  is the interval sequence of failure time. Same as Kijima I model, the cumulative distribution function of  $i$ -th failure interval time for the reliability model of the subsystem of CNC machine tools is:

$$F(s_i | V_{i-1} = v_{i-1}) = \frac{F(x_i + v_{i-1}) - F(v_{i-1})}{1 - F(v_{i-1})} = \frac{\exp[w(v_{i-1})] - \exp[\int_0^{x_i} w(v_{i-1} + u) du]}{\exp[w(v_{i-1})]} \quad (8)$$

So, the conditional probability density of  $S_i$  is:

$$f_i(s_i | s_{i-1}) = w(v_{i-1} + x_i) \exp[\sum_{i=1}^n \int_0^{x_i} w(v_{i-1} + u) du] \quad s_i > s_{i-1} \quad (9)$$

When the test data is type-IIcensored data, the likelihood function is:

$$L(s_1, s_2, \dots, s_n | q) = \prod_{i=1}^n f_i(s_i | s_{i-1}) = \prod_{i=1}^n w(v_{i-1} + x_i) \exp\left[\sum_{i=1}^n \int_0^{x_i} w(v_{i-1} + u) du\right] \quad (10)$$

Where  $q=(I_1, b_1, I_2, b_2, q, t_j)$  represents a collection of model parameters. When the test is type-I censored, the likelihood function is :

$$L(s_1, s_2, \dots, s_n | q) = \prod_{i=1}^n f_i(s_i | s_{i-1}) R(T | s_n) = \prod_{i=1}^n w(v_{i-1} + x_i) \exp\left[\sum_{i=1}^n \int_0^{x_i} w(v_{i-1} + u) du\right] \times R(T | s_n) \quad (11)$$

In Eq.11, T represents censored time of type-I censored test. According to the equations, the censored time  $T=S_n$  when it is type-II censored, and  $R(T/S_n)=1$  apparently.

#### The solving method of likelihood function

To solve the parameters of likelihood functions in section 2.1, it is required to assume the initial value of  $t_j$ , based on the experience and the scatter plot of density function. Generally, we can select the median value as  $t_j$ . Then, we can get the parameters of the likelihood function  $\hat{I}_1, \hat{b}_1, \hat{I}_2, \hat{b}_2, \hat{q}$  according to the parameter estimation methods of minimum maintenance and Kijima I. And the  $i$ -th likelihood parameter is  $\hat{q}_i = (\hat{I}_1, \hat{b}_1, \hat{I}_2, \hat{b}_2, \hat{q}, T_{ij})$  namely:

$$\hat{q} = (\hat{I}_1, \hat{b}_1, \hat{I}_2, \hat{b}_2, \hat{q}, \hat{t}_j) = \arg \max \{L(\hat{q}_i, x_1, x_2, \dots, x_n)\} \quad \text{where} \begin{cases} I_1 > 0, 0 < b_1 \leq 1, \\ I_2 > 0, b_2 \geq 1, \\ 0 \leq q \leq 1, \hat{t}_j > 0 \end{cases} \quad (12)$$

In order to seek the optimum parameters for the model, we assume the likelihood maximum value as the objective function. By preferring the initial value of  $t_j$ , we can calculate the estimation value of the parameters of the reliability model of the subsystems. The flowchart is showed in Figure 1:

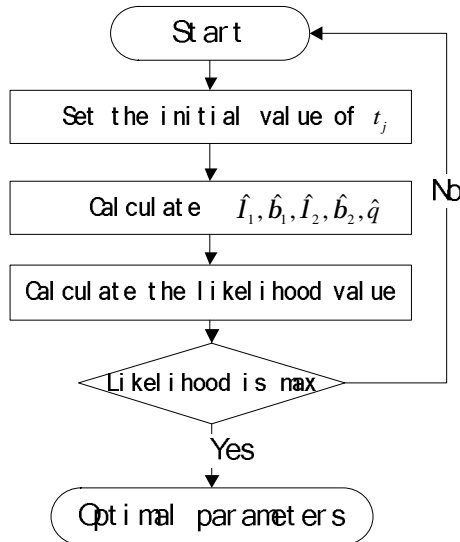


Fig.1. Flowchart for the parameter's estimation of the reliability model of the subsystems

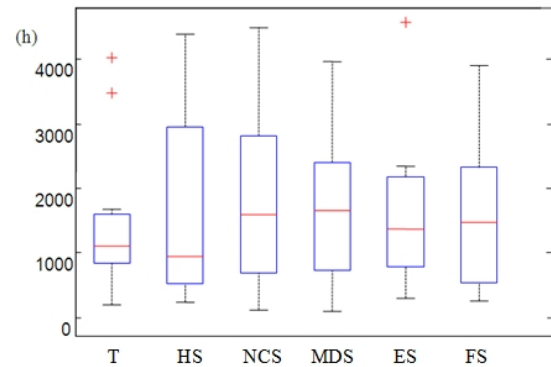


Fig.2. Boxplot of failure time for subsystems a CNC lathe

In Fig.2, T is the abbreviation of Tool holder, HS is the abbreviation of Hydraulic System, NCS is the abbreviation of CNC System, MDS is the abbreviation of Mean Drive System, ES is the abbreviation of Electrical system and FS is the abbreviation of Feed System.

The estimation equation for the parameters in the model of this paper is a likelihood function

with a log derivation, which is a complicated system of implicit functions. In order to solve the maximum likelihood function, we should assume the initial value and repeated iteration. If the initial value is improper, the estimation of the function will be inclined to fall into the local extremum in complex condition. In this case, we need a suitable algorithm which shows good robustness for the model in this paper. So, Genetic Algorithm has been selected for the model.

### Example

In this paper, 23 CNC lathes of the same type are opted for the research subjects. Through random censored failure data collected by means of field tests, we built the reliability model for CNC lathe subsystem<sup>[7,12]</sup>, in which the failure data of main drive are shown in Table 1

Table 1. Failure data for main drive system of a CNC lathe

CNC machine No.	failure data /hour	CNC machine No.	failure data /hour	CNC machine No.	failure data /hour
1	3970.9,(4392.2+)	9	2591.9 (4571.0+)	17	2975.0+
2	1578.5,634.3,(4073.0+)	10	(2322.0+)	18	4021.9+
3	638.4,2094.0,(4481.6+)	11	(1966.3+)	19	3945.3+
4	3817.6+	12	1672.6 (2732.4+)	20	1825.8+
5	1736.5,(3766.6+)	13	2936.7, (4340.0+)	21	102.1, 806.2,(2477.0+)
6	983.1,(3485.7+)	14	1979.0, (3907.0+)	22	820.3,(2847.3+)
7	(2783.4+)	15	1995.4, (2221.6+)	23	229.8,(2311.0+)
8	520.0,1404.5,(1685.4+)	16	293.7, 306.4 , (1430.0+)		

+ means censored data of main drive system of each CNC machine. If no failure happened at the censored time, there is no bracket, such as 3817.6+; otherwise, a bracket is added, such as (2783.4+).

First, we preprocessed the failure data of each subsystem using Total Failure Time Method<sup>[7]</sup>. Secondly, boxplots are drawn according to the data of the various key subsystems, shown in Figure 2. From the boxplot in Fig. 2, we can get quartiles and the median point value of main drive system. Referring these points, we assume the median point to be the initial value of  $t_j$ , i.e.  $T_{ij}=1653.4$ .. On the basis, we seek the optimal solution for objective function Eq.12 using Genetic Algorithm, in which the key parameters are as shown in Table2.:

Table 2. Value of key parameters in GA

Parameters	Value	Parameters	Value	Parameters	Value
Population size	100	Mutation rate	0.1	Elite count	10
Max Generation	1000	Crossover rate	0.8	Error limits	0.001

After taking value for each parameter of GA, by computer simulation program, model parameters of the main drive system are searched out, shown in Table 3:

Table3. Parameters for reliability model of the main drive system

$\hat{I}_1 = 0.0009$	$\hat{b}_1 = 0.8924$	$\hat{t}_j = 1304$
$\hat{I}_2 = 2.80E - 05$	$\hat{b}_2 = 1.2826$	$\hat{q} = 0$

In accordance with the model parameters in Table 3, we can obtain the initial density function of the transmission system and the initial cumulative intensity function curves,

$$I(t) = \begin{cases} 0.0009 \times 0.8924 t^{-0.1076} & t \leq 1304 \\ 0.0009 \times 0.8924 \times 1304^{-0.1076} \\ + 2.80E-5 \times 1.2826 (t - 1304)^{0.2826} & t > 1304 \end{cases} \quad W(t) = \begin{cases} 0.0009 t^{0.8924} & t \leq 1304 \\ 0.0009 \times 1304^{0.8924} + \\ 2.80E-5 \times (t - 1304)^{1.2826} & t > 1304 \end{cases}$$

As can be seen from Figure 3 and Table 3, the age reduction factor “q” of the main drive equals to 0, which means that after the earlier-failure-period, the repair process corresponds to the renew process, namely repair effect is close to "as good as new" in the observation interval. After simulating by means of GA, we can get the critical point of earlier-failure-period and random-period, that is  $t_j = 1304$  hours.

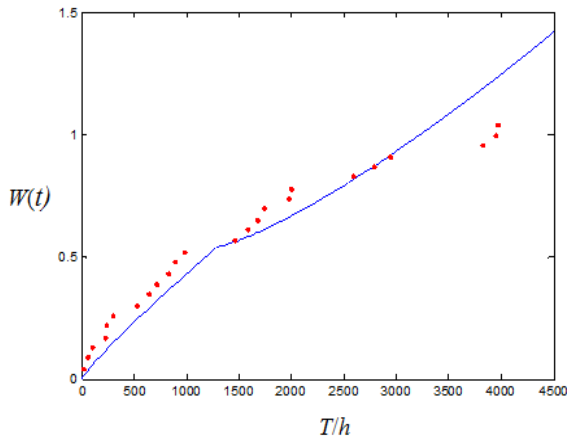


Fig.3. the initial cumulative intensity function curve for the main drive system of a CNC lathe

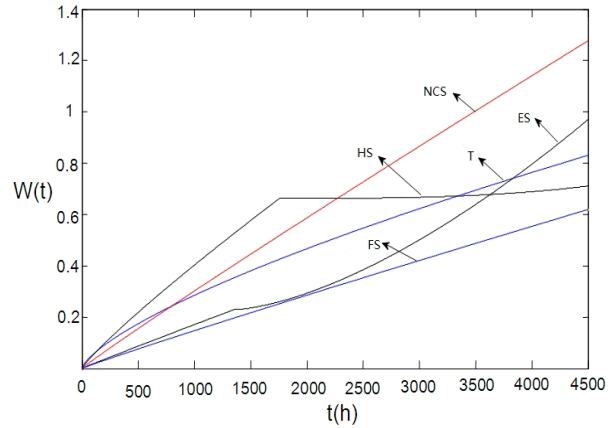


Fig.4. Cumulated failure plots for each subsystem of a CNC lathe

After that, we analysis other subsystems with the above method. The reliability model parameter estimations of the critical subsystems, which have more failure, are shown in Table 4:

Table 4. Parameters for reliability model of other CNC lathe subsystem

Name of the subsystem	Estimations of the model parameters					
	$\hat{t}_1$	$\hat{b}_1$	$\hat{t}_j$	$\hat{t}_2$	$\hat{b}_2$	$\hat{q}$
T-Tool holder	-	-	-	0.0021	0.7109	0.0013
HS-Hydraulic System	0.0009	0.8836	1756	1.75E-13	2.98	0.8683
NCS-CNC System	-	-	-	0.0004	0.9592	1
ES-Electrical System	0.0002	0.9772	1350	2.08E-06	1.55	0
FS-Feed System	0.0002	0.9956	2162	2.35E-07	1.8252	0

Based on the parameter values in Table 4, the failure intensity function curves for each subsystem are plotted, shown in Figure 4. Combined with Table 4 and Figure 4, it can be seen that the age reduction factor q, which represents repair level varies for each subsystem. For example, the age reduction factor of Tool Holder system q equals to 0.0013, it represents a higher maintenance level which approaches the level of "as good as new"; and for hydraulic system,  $q = 0.8683$ , for electrical systems,  $q = 0$ , it means the repair effect of hydraulic system is less as good as the electrical system. It is commonly related to gradual failure of hydraulic system, its influence upon device takes a longer time to be related, because such a failure can not be timely discriminated. At the same time, in Figure 4, the earlier-failure-period of major subsystems are different, which indicates that the earlier-failure-period of subsystem does not necessarily synchronized with the early failure of the machine, therefore, great attention should be paid to such issues during the early troubleshooting test.

During the reliability design process, the reliability model of subsystem established in this paper can provide reference and basis for the growth of reliability and maintainability to designer, depending on the repair degree, failure reason and indexes of related reliability.

## Conclusion

During the reliability design process, the reliability model of subsystem established in this paper can provide reference and basis for the growth of reliability and maintainability to designer, depending on the repair degree, failure reason and indexes of related reliability.

In view of that the reliability model of CNC machine is not improper without repair effect considered, we set up a modified reliability model for subsystems of CNC machine considering repair degrees. Combined with Kijima I model, the repair degree can be calculated, and the critical point of earlier-failure-period and random-period could be estimated. In order to prove the model, we use this model to an application example which collected data in field tests for a type of CNC lathes. According to this method, we can calculate the repair degree and other reliability parameters properly. And the feasibility is proved by the example.

## Acknowledgement

This work is supported by the Specialized Research Fund of the Application engineering of one thousand sets of CNC machines with Dometic CNC systems (Grant No. 2014ZX040015061), the Postdoctoral fund project of Jilin province (Grant No. 801130020414) and the Priority Project of Basic Scientific Research Fund of Jilin University (Grant No. 450060501744).

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