The Evolution Solutions for Complex Ginzburg–Landau equation

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Abstract. The expanded F-expansion method was used to construct the evolution wave solutions involving parameters for the Complex Ginzburg–Landau equation. The evolution wave solution in several forms are shown and the numerical simulation figures are given.

Introduction

In recent years, due to the wide applications of soliton theory in natural science, it is important to seek more exact solutions of nonlinear partial differential equations, which become more attractive topic in physical science and nonlinear science.

Ravoux et al. [1] studied the discrete analog of the complex cubic Ginzburg–Landau equation having pattern formation phenomena in mind. Abdullaev et al. [2] studied the discrete analogue of the complex cubic–quintic Ginzburg–Landau equation with a more general form for the nonlinear terms. Using a perturbation technique, they found a soliton solution which is valid at small values of the dissipative terms for this equation. Efremidis and Christodoulides also studied a different complex cubic–quintic Ginzburg–Landau equation [3].

The expanded F-expansion method [4,5] was used to construct the evolution wave solutions involving parameters for the Complex Ginzburg–Landau equation. The evolution wave solution in several forms are shown and the numerical simulation figures are given.

The description for expanded F-expansion method

For the nonlinear equation with two independent variables *x* and *t* and a dependent variable *u* is given by

$$G(u, u_x, u_t, u_{xt}, u_{xx}, u_{tt}, \ldots) = 0 \tag{1}$$

u is the evolution wave function to be determined, which can be given as the form

$$u = u(x,t) = u(x)e^{i(kx-wt)}, \quad x = x - gt \tag{2}$$

where *k, w, g* are constant parameters. $x = x - gt$ is a arbitrary function with the variables *x* and *t*. Then, the Eq. (2) is changed into an ordinary differential equation

$$G(u, u_x, u_{xx}, u_{xxx}, \ldots) = 0 \tag{3}$$

We can obtain the solutions from the abover formation. The main steps of the expanded F-expansion method are shown. Suppose that the solution of equation (3) can be expressed by a polynomial as follows:

$$u(x) = \sum_{i=0}^{n} K_i f^{i} + \sum_{i=1}^{n} S_i f^{-i}, \tag{4}$$

where f satisfies the second order linear ordinary differential equation in the form

$$f'' = pf + qf^3, \quad (f')^2 = r + pf^2 + \frac{qf^4}{2}. \tag{5}$$

The parameter n can be determined by balancing the highest order derivative terms with the nonlinear terms in Eq.(3). K_i ,G_i are constants parameter to be determined later.

Substituting Eq. (4) and (5) into Eq.(3), and collecting all terms with the same power of f together. Equating each coefficient of f to zero yields a set of algebraic equations for K_i ,G_i. The parameters K_i ,G_i are obtained from the algebraic equations. Substituting these parameters into Eq. (4) and (2), the evolution wave solutions for nonlinear Eq. (1) are obtained.

Solutions for comples Ginzburg-Landau equations

We begin with the Complex Ginzburg–Landau equation[6-8] as follow:

$$j_t-(1+ib)\nabla^2 j+(1+ia)j\left|j\right|^2=0 \tag{7}$$

Here, we suppose that

$$j(x,t)=u(x)e^{i(kx-wt)}, \qquad x=x-gt. \tag{8}$$

Substituting Eq. (8) into Eq. (7) and by using $2ik=2bk-g$, Eq.(7) is changed into

$$Pu_{xx}+Ru+Tu^3=0 \tag{9}$$

where $P=1+ib; \quad R=iw-ik^2b-k^2; \quad T=-1-ia.$

By considering the homogeneous balance, we obtain n=1 for $u(x)$. Then, equation (4) is written as

$$u(x)=K_0+K_1 f+S_1 f^{-1}; \tag{10}$$

Employing Eq. (5), the first and the second derivative for Eq. (10) are given

$$u_x=K_1 f'-S_1 f'f^{-2};$$

$$u_{xx}=K_1 f''-S_1(f''f-2(f')^2)f^{-3}; \tag{11}$$

Substituting equations (10), (11) into equation (9), collecting all terms with the same power of f together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$P(K_1(pf+qf^3)f^3)-S_1((pf+qf^3)f-2(r+pf^2+qf^4/2)))+RK_0f^3+RK_1f^4+RS_1f^2+T(K_0^3f^3+$$
$$3f^4K_1K_0^2+3f^5K_0K_1^2+3K_0^2S_1f^2+6K_0K_1S_1f^3+3f^4K_1^2S_1+f^6K_1^3+3K_0S_1^2f+3K_1S_1^2f^2+S_1^3)=0$$

Let the coefficient of f^i (i=6,5,….1,0) equal to zero, we have

$$K_1Pq+K_1^3T=0;$$
$$3K_0K_1^2T=0;$$
$$K_1Pp+K_1R+3K_1TK_0^2+3S_1TK_1^2=0;$$
$$K_0R+K_0^3T+6K_0K_1S_1T=0;$$
$$pPS_1+RS_1+3S_1TK_0^2+3K_1TS_1^2=0$$
$$3K_0S_1^2T=0$$
$$2rS_1P+S_1^3T=0$$

Solving the above equations, we obtain the two cases:

Case(I) $K_0=0; \quad K_1=\pm\sqrt{\frac{(1+ib)q}{1+ia}}; \quad S_1=\pm\sqrt{\frac{2r(1+ib)}{1+ia}}$ with the condition $18rqP^2=P^2p^2+R^2+2RPp$;

Case (II) $K_0=0; \quad K_1=\pm\sqrt{\frac{(1+ib)q}{1+ia}}; \quad S_1=0$ with the condition $R=-Pp$.

The evolution solutions for Complex Ginzburg–Landau Equation are obtained by substituting the abover cases into Eq. (10), the parameter p, q, r are given [9-11] as the following

$$p=-(1+m^2),\ \ q=2m^2,\ \ r=1;\ \ f=sn(x);\ \ p=-1+2m^2,\ \ q=-2m^2,\ \ r=1-m^2;\ \ f=cn(x);$$

$p=2-m^2,\quad q=-2,\quad r=m^2-1;\quad f=dn(x);\qquad p=(m^2-2)/2,\ q=m^2/2,\ r=m^2/4;\ f=msn(x)/(1+dn(x));$

Substituting the case (I) and Eq. (10) to Eq.(8), which is written as:

$j_{11}=(\pm\sqrt{\dfrac{2(1+ib)m^2}{1+ia}}sn(m,x-gt)\pm\sqrt{\dfrac{2(1+ib)}{1+ia}}\dfrac{1}{sn(m,x-gt)})e^{i(kx-wt)}$ with $18rqP^2=P^2p^2+R^2+2RPp$;

$j_{12}=(\pm\sqrt{\dfrac{-2(1+ib)m^2}{1+ia}}cn(m,x-gt)\pm\sqrt{\dfrac{2(1-m^2)(1+ib)}{1+ia}}\dfrac{1}{cn(m,x-gt)})e^{i(kx-wt)}$ with $18rqP^2=P^2p^2+R^2+2RPp$;

$j_{13}=(\pm\sqrt{\dfrac{-2(1+ib)}{1+ia}}dn(m,x-gt)\pm\sqrt{\dfrac{2(m^2-1)(1+ib)}{1+ia}}\dfrac{1}{dn(m,x-gt)})e^{i(kx-wt)}$ with $18rqP^2=P^2p^2+R^2+2RPp$;

$j_{14}=(\pm\sqrt{\dfrac{m^2(1+ib)}{2(1+ia)}}\dfrac{msn(m,x-gt)}{1+dn(m,x-gt)}\pm\sqrt{\dfrac{m^2(1+ib)}{2(1+ia)}}\dfrac{1+dn(m,x-gt)}{msn(m,x-gt)})e^{i(kx-wt)}$ with $18rqP^2=P^2p^2+R^2+2RPp$.

Substituting the case (II) and Eq. (10) to Eq.(8), which is written as:

$j_{11}=\pm\sqrt{\dfrac{2(1+ib)m^2}{1+ia}}sn(m,x-gt)e^{i(kx-wt)}$ with $R=-Pp$;

$j_{12}=\pm\sqrt{\dfrac{-2(1+ib)m^2}{1+ia}}cn(m,x-gt)e^{i(kx-wt)}$ with $R=-Pp$;

$j_{13}=\pm\sqrt{\dfrac{-2(1+ib)}{1+ia}}dn(m,x-gt)e^{i(kx-wt)}$ with $R=-Pp$;

$j_{14}=\pm\sqrt{\dfrac{m^2(1+ib)}{2(1+ia)}}\dfrac{msn(m,x-gt)}{1+dn(m,x-gt)}e^{i(kx-wt)}$ with $R=-Pp$.

The real part of the solutions for Case (I) are given as the following:

$j_{11}=(\sqrt{2m^2}sn(m,x-gt)+\sqrt{2}\dfrac{1}{sn(m,x-gt)})\cos(kx-wt)$ with $a=0$; $b=0$ and $18rqP^2=P^2p^2+R^2+2RPp$. The simulation of $j(x,t)_{11}$ is shown in Fig. 1 with $m=0.3$, $w=0.5$, $k=1$, $x\in[-10,10]$ $t\in[-10,10]$, respectively.

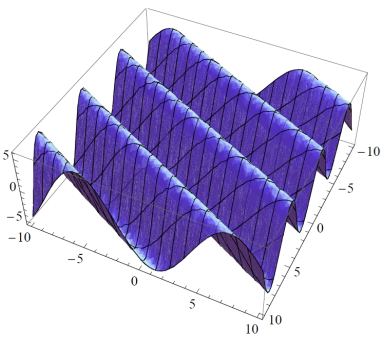

Fig.1 Simulation for real part of $j(x,t)_{11}$, $m=0.3$, $w=0.5$, $k=1$, $x\in[-10,10]$ $t\in[-10,10]$, respectively.

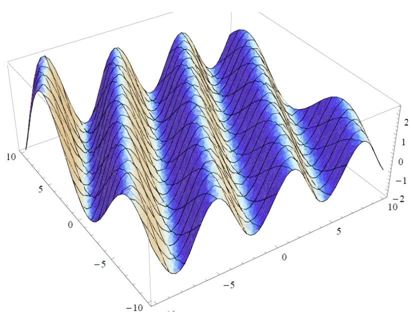

Fig.2 The simulation for real part of $j(x,t)_{12}$ is shown with $m=0.6$, $w=0.5$, $k=1$, $x\in[-10,10]$ $t\in[-10,10]$, respectively.

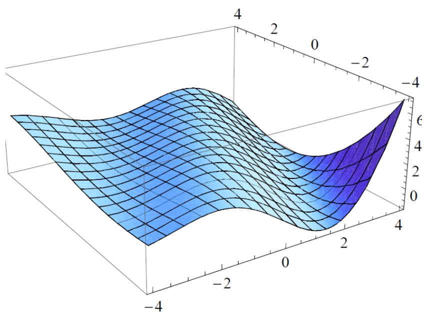

Fig. 3 The simulation for real part of $j(x,t)_{13}$ is shown with $m=0.6$, $w=0.5$, $k=1$, $x\in[-4,4]$ $t\in[-4,4]$, respectively.

The simulation for real part of $j(x,t)_{12}$ is shown in Fig. 2 with $m=0.6$, $\mathrm{w}=0.5$, $k=1$, $x\in[-10,10]$ $t\in[-10,10]$, respectively.

$$j_{13}=(\sqrt{2}dn(m,x-gt)+\sqrt{2(1-m^2)}\frac{1}{dn(m,x-gt)})\cos(kx-wt)$$

with $a=0$, $b=2i$ and $18rqP^2=P^2p^2+R^2+2RPp$; The numerical simulation for $j(x,t)_{13}$ is shown in Fig. 3 with the parameters *m, w, k, t, x* are given. The fugure is more smooth than the above figures.

Conclusions

In this paper, we presented the evolution wave solutions in terms of expanded F-expansion method for complex Ginzburg–Landau equation. These equations are very difficult to be solved by traditional methods. This present work afirms that the expansion method is an easy straight forward method to solve nonlinear partial differential equations.

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References

[1] J.F. Ravoux, S. Le Dizes, P. Le Gal, Stability analysis of plane wave solutions of the discrete Ginzburg-Landau equation, Phys. Rev. E 61 (2000) 390-393.

[2] F.Kh. Abdullaev, A.A. Abdumalikov, B.A. Umarov, Discrete solutions for the complex cubic–quintic Ginzburg–Landau equation Phys. Lett. A 305 (2002) 371-375.

[3] N.K. Efremidis, D.N. Christdoulides, Discrete Ginzburg-Landau solitons, Phys. Rev. E, 67 (2003) 26606-4.

[4] Mingliang Wang, Lingzheng Xiang, Extended F-expansion method and periodic wave solutions for the generalized ZaKharov equations, Phys Lett A. 343: (2005), 48-54.

[5] M.A.Abdou, The extended F-expansion method and its application for a class of nonlinear evaluation equations. Chaos, Solitons and Fractals.31: (2007), 95-104.

[6] Ken-ichi Marunoa, Adrian Ankiewiczb, Nail Akhmediev, Exact localized and periodic solutions of the discrete complex Ginzburg–Landau equation, Optics Communications, 221, (2003) 199–209.

[7] Lutz Bruscha, Alessandro Torcinib, Martin van Heckea, Martín G Zimmermannf, Markus Bär, Modulated amplitude waves and defect formation in the one-dimensional complex Ginzburg–Landau equation, Physica D: Nonlinear Phenomena, 160, (2001), 127–148.

[8] Jiahong Wu, The inviscid Limit of the Complex Ginzburg–Landau Equation, Journal of Differential Equations, 142, (1998), 413–433.

[9] Huamei Li, New exact solutions of nonlinear Gross—Pitaevskii equation with weak bias magnetic and time-dependent laser fields, 14 (2005) Chinese Physics B 251-256.

[10] Huamei Li, Mapping deformation method and its application to nonlinear equations, 11, (2002) Chinese Physics B 1111-1114.

[11] Lunxun Gong, Some new exact solutions of the Jacobi elliptic functions of NLS equation, Acta Physics Sinica, 55(9), 2006, 4414-4419.