# A Replicator Dynamics Model of Traditional Manufacturing Clusters Low-carbon Evolution

Hongjuan Zhang[1,2]

[1] Management School, Wuhan University of Science and Technology, Wuhan, Hubei, 430081, China

[2]Center for Industrial Policy and Management Research, Wuhan, Hubei, 430081, China

jhz75@126.com

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**Abstract.**The low-carbondevelopmentof traditional manufacturing clustershas becomea hot issuein thecluster field. The cooperative evolution is animportant way of the low-carbon evolutionof traditional manufacturing clusters, and it is also a dynamical, complex and repeatedgame process. This paper builds a replicator dynamics model of traditional manufacturing clusterlow-carbon evolution. Based on evolutionary game theory, the paper analyzes the low-carbon evolutionary paths and evolutionarily stable strategy of traditional manufacturing clusterunder different initial states of cluster system.The analytic results show that, the result of low-carbon evolution depends on the initial conditions of cluster system. The paper studies the mechanism of cluster enterprises changing their low-carbon behavior strategy according to the external environmental changes, and has a good practical value and significance.

## Introduction

The low-carbon developmentof traditional manufacturing clustershas becomea hot issue in theclusterfield,and it is also an important part in the development of low carbon economy[1].

The key influence factor to low-carbon development of traditional manufacturing clusteris R&D strength. Cooperative low-carbon strategy as a major strategy of cluster low-carbon developmentstrategies has many advantages, such as small risk, low cost and advanced technology[2].In order to share resources such as capital, information and technology, cluster enterprises would prefer to choose adopting cooperative strategyto obtain the advantage of low carbon development[2].

The cooperative low-carbon evolution of traditional manufacturing clusters system is a long and complex process. The low-carbon evolutionary path and evolutionarily stable strategy of cluster system are all not uncertain[3].

Replicator dynamics model can describethe behaviorstrategy of the players in the process of game well, and it canpredict the evolutionary path and evolutionarily stable strategy of the gamewell [4].So based onreplicator dynamics model of monomorphic population, the paper can describe and predict the cooperative low-carbon evolution of traditional manufacturing clustervery well.Therefore, thispaper would use replicator dynamics model to study the low-carbon evolutionary mechanism of traditional industrial cluster, and to analyze the changes of cluster enterprise low-carbon behavior strategy according to the external environmental changes.

## Modeling the Replicator Dynamics Model of Cluster Low-carbon Evolution

### Basic Replicator Dynamics Modelof Monomorphic Population

In the monomorphic population symmetric game,assuming that$N$ indicates the total numberof agents of populaiton, $S^K = \{s_1, s_2, \ldots, s_k\}$ indicates the pure strategy sets of game agents. And suppose that every agenthave to adopt a pure strategy at any time.$n_i(t)$ indicates the number of agents adopting pure strategy$i$ at time $t$.$x_i$indicates the proportion of agents adopting pure strategy$i$ at time$t$， then $x_i$ can be expressed as follow[4, 5]:

$$x_i = n_i(t)/N$$

 

Assuming that the expected revenueof agent who adopting pure strategy$i$ can be expressed as $f(s_i, x)$, then the average expected revenueof population can be expressed as follow:

$$\overline{f}(x,x) = \sum x_i \,(s_i, x)$$

So the basic replicator dynamics modelof monomorphic populationcan be expressed as follow:

$$dx_i(t)/dt = [f(s_i, x) - \overline{f}(x,x)]x_i$$

**Modeling the Replicator Dynamics Model of Traditional Manufacturing Cluster Low-carbon Evolution**

(1)Suppose that the traditional manufacturing clusteris composed of a monomorphic population of enterprises, and the totalnumber of cluster enterprises is $N$ ($N \geqslant 2$).

(2)Suppose that when the manufacturing clusterenterprises game with each other, they have the same pure Strategy Sets, that is $S = \{S_1, S_2\}$. $S_1$ is"Cooperative Low-carbonStrategy", $S_2$ is "Independent Low-carbonResearch Strategy". And suppose that every manufacturing cluster enterprise have to choose adopting a pure strategyat any time.

(3)Suppose that the proportion of traditional manufacturing cluster enterprises adopts low-carbon strategy$S_1$ is $x$, and the proportion adopts low-carbonstrategy$S_2$ is $1 - x$($0 \leqslant x \leqslant 1$).

(4)Assuming that all thetraditional manufacturingcluster enterprises in the game distribute low-carbon cooperative benefit evenly, have to pay the same low-carbon cooperative cost, and have the same low-carbon speculative revenue.

Table 1 shows the Game Payoff Matrix of a monomorphic population of traditional manufacturing cluster enterprise.

As shown in Table 1,

$u$is thetraditional manufacturing cluster enterpriseprofit whenit isadoptinglow-carbon strategy$S_2$ in the game;

$c$is thetraditional manufacturing cluster enterprisecooperative cost when it isadoptinglow-carbon strategy$S_1$;

$k$ is thetraditional manufacturing cluster enterprise speculative revenue when it isadoptinglow-carbon strategy$S_2$, while the other side game participantis adopting low-carbon strategy $S_1$;

$h$is the traditional manufacturing cluster enterprise coefficient of profitability;

$h - c$is thetraditional manufacturing cluster enterprisecoefficient of cooperative profitability, when all the game participants are adopting low-carbon strategy $S_1$.

Table1Game Payoff Matrixof manufacturingclusterenterprise 1

| | $S_1$ | $S_2$ |
|---|---|---|
| $S_1$ | $u + h - c$, $u + h - c$ | $u - c$, $u + k$ |
| $S_2$ | $u + k$, $u - c$ | $u$, $u$ |

In order to simplify the calculation, the paper sets $u = 0$. Thisset would not affect the results of the game analysis. Then Table 1 is simplified to Table 2.According to the actual game situations of traditional manufacturing clusterenterprises, the parameters are set as follows: $h > c$, $k > 0$, $c > 0$.

Table 2Game Payoff Matrix of manufacturingclusterenterprise 2

| | $S_1$ | $S_2$ |
|---|---|---|
| $S_1$ | $h-c$，$h-c$ | $-c$，$k$ |
| $S_2$ | $k$，$-c$ | 0，0 |

Whentraditional manufacturing cluster enterprise is adopting low-carbon strategy$S_1$, its average expected revenue$v^1$can be described as Eq. 1:

$v^1 = (h-c)x + (-c)(1-x) = hx - c$ (1)

Whentraditional manufacturing cluster enterprise is adopting low-carbon strategy$S_2$, its average expected revenue$v^2$can be described as Eq. 2:

$v^2 = kx$(2)

Then the Replicator Dynamics Equation of traditional manufacturing cluster enterprise can be described asEq. 3:

$F = \frac{dx}{dt} = x(v^1 - v) = x(1-x)(v^1 - v^2) = x(1-x)(hx - kx - c)$(3)

**Analysis of Evolutionarily Stable Strategy of Traditional Manufacturing Cluster Low-carbon Evolution**

In Eq.3, suppose that$F = \frac{dx}{dt} = 0$，then $x_1^* = 0$，$x_2^* = 1$，$x_3^* = \frac{c}{h-k}$. It means only when$x^* = 0$，1 or $\frac{c}{h-k}$， the proportion of traditional manufacturing cluster enterprises which are adopting strategy $S_1$ is stable.

Based on evolutionary game theory，suppose that$F'(x^*) < 0$, then$x^*$ is evolutionarily stable strategy (ESS)of traditional manufacturing cluster low-carbon evolution. That means the evolutionarily stable strategy of cluster enterprise low-carbon evolution depends entirely on the parameters of the game payoff matrix( $h$，$k$，$c$)[6].

$F'(x_1^*) = F'(0) = -c$

$F'(x_2^*) = F'(1) = k - (h-c)$

$F'(x_3^*) = F'\left(\frac{c}{h-k}\right) = \frac{c(h-c-k)}{h-k}$

Since $h > c$，$k > 0$，$c > 0$， then $F'(0) = -c < 0$. According to the different values of $h$，$k$，$c$，the result of evolutionarily stable strategy of traditional manufacturing cluster can be divided as following situations[6]:

(1)When$h - c > k$and$h < k$，this situation does not exist.

(2) When $h - c > k$and $h > k$，it can be conducted that$F'(x_1^*) < 0$，$F'(x_2^*) < 0$，$F'(x_3^*) > 0$，therefore$x_1^* = 0$ and $x_2^* = 1$ are bothevolutionarily stable strategy (ESS).

Sincethe final result of low-carbon evolution depends on the initial value of$x$, so when $x \in (0，x_3^*)$, through long-term repeated game,traditional manufacturingcluster enterprises finally would adopt independent low-carbon researchstrategy; when $x \in (x_3^*，1)$，through long-term repeated game, cluster enterprises finally would adopt cooperative low-carbon strategy.

(3)When$0 < h - c < k$ and $h < k$，

it can be conducted that $F'(x_1^*) < 0$，$F'(x_2^*) > 0$，$F'(x_3^*) > 0$，therefore, $x_1^* = 0$ is the only evolutionarily stable strategy. It means through long-term repeated game, traditional manufacturing cluster enterprises finally would adopt independent low-carbon research strategy

(4)When$0 < h - c < k$ and $h > k$,

it means$k < h < c + k$，it can be conducted that $F'(x_1^*) < 0$，$F'(x_2^*) > 0$，$F'(x_3^*) < 0$， therefore$x_3^*$is not evolutionarily stablestrategy， $x_1^* = 0$is the only evolutionarily stable strategy. It means through long-term repeated game, traditional manufacturing cluster enterprises finally would adopt independent low-carbon research strategy.

According to the actual game situations of traditional manufacturing clusterenterprises, other values ofparametersof game payoff matrixdon’t exist.

## Conclusions

The low-carbon cooperative evolution oftraditional manufacturing clusters is a dynamical, complex and repeatedgame process. This paper builds a replicator dynamics model oftraditional manufacturing clusterslow-carbon evolution. Based on evolutionary game theory, the paper analyzes the low-carbon evolutionary paths and evolutionarily stable strategy of traditional manufacturing clusterunder different initial states of cluster system.The analysis results show that,the low-carbon evolutionary results of traditional manufacturing clusterdepend on the initial conditions of cluster system.The paper studies the mechanism of traditional manufacturing cluster enterprises changing their low-carbon behavior strategy according to the external environmental changes, and has a good practical value and significance.

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