Single Planetary Mechanism Teeth Matching Conditions

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***Abstract*—Teeth matching conditions for planetary mechanism of single pin, dual pin and multiple pin planetary mechanisms are studied in details in this thesis. Single planetary mechanism teeth matching conditions include: concentric condition, homogeneity distribution condition, neighbor condition and gear ratio condition. With the increase of meshed gear pairs between sun gear and gear ring, the concentric condition will become wider and wider, while neighbor condition become stricter and stricter. Gear ratio condition is applied to check the deviation of actual and theoretical gear ratio of planetary mechanism under the teeth matching condition. If the deviation is large, adjustment could be made by revising structural parameters and number of planetary gear pairs participating in the meshing. Homogeneity distribution condition is most easily ignored, thus enough attention should be paid to it. In this thesis, the angle expression of planetary mechanism when each component is not interfered and homogeneously distributed is inputted its rotation speed equation, and after deduction, the homogeneity distribution condition is gained for various single planetary mechanisms, which will provide reference for engineers.**

Keywords-planetary mechanism; teeth matching condition; meshing; interference

I. Introduction

With the advantages of small size, light weight, compact structure, large gear ratio, smooth transmission, and etc., planetary mechanism is widely used in mechanical transmission field. In recent years, with the development of hybrid power technology, planetary mechanisms can realize the power coupling requirement of hybrid system by two characteristics: 2 degrees of freedom structure and coaxial power input and output [1]. Single planetary mechanism is the most commonly used in hybrid system, during the design process both teeth matching condition and power coupling condition should be satisfied. In the thesis, teeth matching conditions of various single planetary mechanisms will be studied systematically, which will provide reference for engineers when they design these mechanisms.

II. Planetary Mechanism Characteristics

Single planetary mechanism refers to 2K-H structure composed of two sun gears (sun gear and ring gear) and a planet carrier. According to the number of planetary gears participating in meshing transmission on the planet carrier, single planetary mechanism can be divided into three categories: single pin, double pin and multiple pin planet mechanisms.

There are 2 degrees of freedom in planetary mechanism, and the rotation speed of two of the components must be confirmed before the third component's rotation speed is confirmed. The kinematical equation of gear train which utilizes the absolute rotation speed of 3 components and structure characteristic parameter ρ of the planetary mechanism can be expressed as [2]:

$$[1-(-1)^n \cdot \rho] \cdot \omega_H + (-1)^n \cdot \rho \cdot \omega_S - \omega_R = 0 \qquad (1)$$

In the expression: ω_S - Absolute rotation speed of sun gear s^{-1}; ω_R - Absolute rotation speed of ring gear s^{-1}; ω_H - Absolute rotation speed of planet carrier s^{-1}; n -Number of gear pairs meshed externally on the planet carrier; ρ-Characteristic parameters of planetary mechanism, $\rho = Z_S / Z_R$; Z_R - Number of teeth of ring gear; Z_S - Number of teeth of sun gear.

From expression (1), it is known that in 2K-H single planetary mechanism, no matter how many meshed planetary gears there are, the sum of the coefficient of the 3 components' rotation speed is zero. When the rotation speed of one component is equal to another one, the third will also be equal, we call the planetary in "lockout" condition [3].

III. Matching Condition for Single Pin Planetary

Single planetary mechanism with single pin is the basic element to compose all kinds of planetary mechanisms, which is widely used in various mechanical equipments. Its teeth matching condition can be summarized as the following 4 aspects [4]:

A. Concentric Condition

Concentric condition means the center of rotation of the 3 basic components must be on the same axis, as well as each gear must meet right meshing condition, that is, ensure the center distances of each meshed gear be equal to each other.

For single pin planetary mechanism with X-zero gear, or gear with deflection addendum modification,, the concentric condition can be expressed as the expression of the number of teeth of 3 meshed gears.

$$2 \cdot Z_g = (Z_R - Z_S) \qquad (2)$$

In the expression: Z_g -Number of teeth of planetary gear; Z_R -Number of teeth of ring gear; Z_S -Number of teeth of sun gear.

B. Homogeneity Distribution Condition

In order to assemble each planetary gear homogeneously between sun gear and ring gear, number of planetary gears and number of teeth must meet certain relation, or, the assembly will be unable to be done due to the interference between planetary gear and sun gear and (or) ring gear. It is because in planetary mechanism, one gear might be meshed with several other gears, as shown in FIGURE Ⅰ.

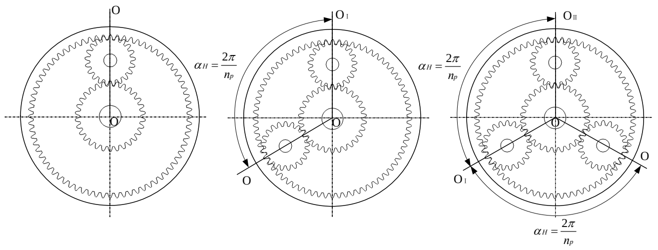

FIGURE I. ASSEMBLY RELATION OF SINGLE PIN PLANETARY MECHANISM

Evenly distribute n_b planetary gears along the periphery, and then the angle between the neighboring gears is $\alpha_H = 2\pi / n_b$. Assume that the first gear is assembled at OO, to assemble the second gear with interval of angle α_H, fix ring gear, rotate planet carrier by angle α_H, OO will be ratated by angle α_H along with planetary gear, OO_I occupies the position of OO, and the sun gear will also rotated by an angle α_S. To assemble the second planetary gear between sun gear and ring gear, the phase relation of sun gear and ring gear at OO_I must to be equal to that of position OO. Fix ring gear, and only sun gear is moved, in order to meet the above requirement, the sun gear needs to be turned by several teeth. The corresponding angle of circumference of each teeth of the sun gear is $2\pi / Z_S$, that means sun gear need to rotates $\alpha_S = N \cdot 2\pi / Z_S$, and here N is integer.

Substitute the above planetary angle to rotation speed expression (1), take n=1 for single planetary mechanism, the following expression can be gained.

$$(1+\rho)\cdot\alpha_H - \rho\cdot\alpha_S - \alpha_R = 0 \tag{3}$$

In the expression: α_H - Angle that planet carrier rotates; α_S - Angle that sun gear rotates; α_R - Angle that ring gear rotates.

Substitute $\rho = Z_S / Z_R$ to expression (3), and after deduction, the homogeneity distribution condition of single planetary mechanism can be gained as follows.

In the expression: n_b - Number of groups of homogenous distributed planetary gears; N - Number of teeth that sun gear rotates, shall be positive integer.

$$(Z_R + Z_S) / n_b = N \tag{4}$$

Expression (4) shows the homogeneity distribution condition of single pin planetary mechanism is: the sum of ring gear and sun gear teeth must be integral multiple of number of planetary gears.

Following the same method, the *k*th planetary gear can be assembled at the position of interval of $\alpha_H = k \cdot 2\pi / n_b$, as shown in FIGURE I.

C. Neighbor Condition

For neighbor condition of single pin planetary gear, ensuring two neighboring planetary gears not interfered with each other means ensuring the center distance of two planet gears longer than their tip diameter. As shown in FIGURE II planetary gear O_1 is not interfered with O_2, $\angle O_1OO_2 = 2\pi / n_p$, the bisecting of $\angle O_1OO_2$ cuts the isosceles ΔO_1OO_3 into two right triangles. In the right triangle, apply sine law to gain the condition of two neighboring planetary gears not interfering with each other. For X-zero gear, the expression is as follows.

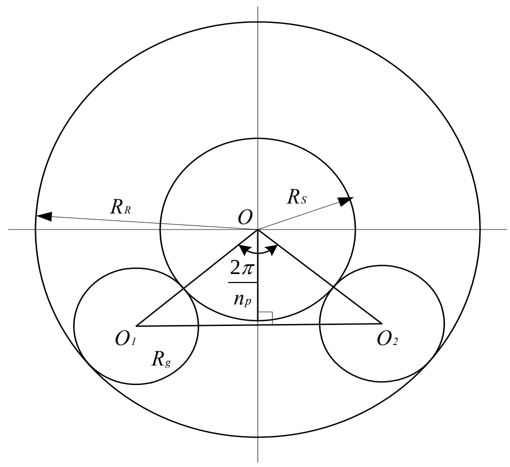

FIGURE II. NEIGHBOR CONDITION OF SINGLE PIN PLANETARY MECHANISM

$$(Z_S + Z_g)\cdot\sin(\frac{2\pi}{n_b}) > Z_g + 2h_a^* + t \tag{5}$$

In the expression: h_a^* - Addendum coefficient.

D. Gear Ratio Condition

To begin designing planetary mechanism, a theoretical gear ratio i_t must be set, and finally due to the restriction of the above 3 teeth matching conditions, the actual gear ratio will be deviated from theoretical gear ratio. During the design of planetary mechanism, the deviation limit of gear ratio must be set according to design requirements. When the deviation exceeds Δi, the design must be redone.

$$\Delta i > \left| i_r - i_t \right| / i_t \tag{6}$$

In the expression: Δi - Deviation limit of gear ratio; i_r - Actual gear ratio; i_t - Theoretical gear ratio.

IV. Matching Condition for Dual Pin Planetary

Due to the special requirements of dual pin planetary mechanism regarding structure and performance, its teeth matching condition is stricter than that of single pin mechanism.

A. Concentric Condition

There are two groups of planetary gears (internal and external) between sun gear and ring gear, thus concentric condition can be achieved by adjusting relative position, which is wider than single pin planetary mechanism. Express the number of teeth of 4 meshed gears as follows:

$$Z_{gmax} + 2h_a^* < \frac{1}{2} \cdot (Z_R - Z_S) \leqslant Z_{gn} + Z_{gw} \tag{7}$$

In the expression: Z_{gmax} - Number of teeth of the biggest planetary gear; Z_{gn} - Number of teeth of internal planetary gear; Z_{gw} - Number of teeth of external planetary gear.

The concentric condition of dual pin planetary mechanism is not strict equation relation, which means there is more freedom to choose number of teeth for each gear. The concentric condition is much wider than single pin planetary mechanism.

B. Homogeneity Distribution Condition

The deduction of homogeneity distribution condition for dual pin planetary mechanism is similar to that of single pin. As shown in FIGURE III, the angle between planetary groups can be gained, it was substituted into equation (1), and take n =2 for dual pin planetary mechanism, the following expression can be gained.

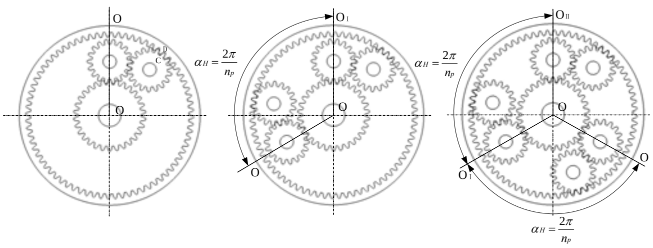

FIGURE III. ASSEMBLY RELATION OF DUAL PIN PLANETARY MECHANISM

$$(1-\rho) \cdot \alpha_H + \rho \cdot \alpha_S - \alpha_R = 0 \tag{8}$$

The homogeneity distribution condition of dual pin planetary mechanism can be gained as follows after deduction:

$$(Z_R - Z_S) / n_b = N \tag{9}$$

Expression (9) shows the homogeneity distribution condition of dual pin planetary mechanism: teeth number of ring gear subtracts teeth number of sun gear should be an integer multiple of number of planetary gear groups.

C. Neighbor Condition

The neighbor condition for dual pin planetary gear is shown in FIGURE IV.

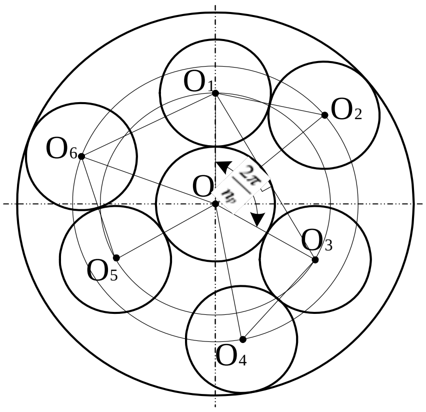

FIGURE IV. NEIGHBOR CONDITION OF DUAL PIN PLANETARY MECHANISM

- Non-interference condition of internal planetary gears and ring gear. Certain interval must be preserved between internal planetary gear tip and ring gear tip, as shown in FIGURE Ⅳ, that is, the tip of internal planetary gear O_1, O_3 and O_5 will not interfere with the tip of ring gear O. For X-zero dual pin planetary mechanism, the relation of teeth number for each gear can be expressed as follows.

$$Z_R - Z_S \geq 2Z_{gn} + 4h_a^* + 2t \tag{10}$$

In the expression: t - interval coefficient.

- Non-interference condition of external planetary gears and sun gear. Certain interval must be preserved between external planetary gear tip and sun gear tip, as shown in FIGURE Ⅳ, that is, the tip of external planetary gear O_2, O_4 and O_6 will not interfere with the tip of sun gear O.It can be expressed as follows.

$$Z_R - Z_S \geq 2Z_{gw} + 4h_a^* + 2t \tag{11}$$

- Non-interference condition for internal planetary gears. Certain interval must be preserved between neighboring internal planetary gear, as shown in FIGUREⅣ, that is, internal planetary gear O_1, O_3 and O_5 will not interfere with each other. It can be expressed as follows.

$$(Z_S + Z_{gn}) \cdot \sin(\frac{\pi}{n_p}) \geq Z_{gn} + 2h_a^* + t \tag{12}$$

- Non-interference condition for internal and external planetary gears not meshed with each other. Certain interval must be preserved between neighboring internal and external planetary gears that are not meshed with each other, as shown in FIGUREⅣ, that is, O_1 and O_6, O_2 and O_3, O_4 and O_5 will not interfere with each other. Take the relation between O_1 and O_6 as an example, that means the distance between O_1 and O_6 is big enough to ensure that O_1 will not interfere with O_6. In triangle ΔO_1OO_6, apply cosine law to calculate the distance from O_1 to O_6. $\angle O_1OO_6=\angle O_2OO_6-\angle O_2OO_1$, $\angle O_2OO_6=2\pi/n_p$, and $\angle O_2OO_1$ can be calculated by applying cosine law again.

$$\angle O_2OO_1 = arccos\frac{(Z_{gn}+Z_S)^2+(Z_R-Z_{gw})^2-(Z_{gn}+Z_{gw})^2}{2\cdot(Z_{gn}+Z_S)\cdot(Z_R-Z_{gw})} \quad (13)$$

Finally the non-interference condition for internal and external planetary gears not meshed with each other can be expressed as follows.

$$\begin{aligned}&(Z_S+Z_{gn})^2+(Z_R-Z_{gw})^2-2\cdot(Z_R-Z_{gw})\cdot(Z_S+Z_{gn})\cdot\\&\cdot\cos(\frac{2\pi}{n_p}-\angle O_2OO_1)\geq(Z_{gn}+Z_{gw}+4h_a^*+2t)^2\end{aligned} \quad (14)$$

The above is the neighbor condition for X-zero gear dual pin planetary mechanism. For neighbor condition of gear with addendum modification, literature [5] and [6] can be referred to.

D. Gear Ratio Condition

The gear ratio verification condition for dual pin planetary mechanism is the same with single pin planetary mechanism. The difference lies in the different sequence of characteristic parameter, which is caused by different teeth matching condition, and the difference will cause the actual gear ratio of dual pin different from single pin mechanism.

V. Matching Condition for Multiple Pin Planetary

A. Concentric Condition

There are more planetary gears between sun gear and ring gear in multiple pin planetary, and there is more space that can be adjusted for putting gears, thus concentric condition can be realized easier.

$$Z_{gmax}+2h_a^* < \frac{1}{2}\cdot(Z_R-Z_S) \leqslant \sum Z_{gi} \quad (15)$$

B. Homogeneity Distribution Condition

The homogeneity distribution condition depends on the times of transmission of the planetary gears between sun gear and ring gear. If it is odd times, the condition will be the same with that of single pin planetary mechanism, while if it is even times, the condition will be the same with dual pin planetary mechanism.

C. Neighbor Condition

Due to the reason that the number of planetary gears in the gear train is increased, it is harsher to ensure non-interference of the gears that are not meshed with each other. However, because the space between sun gear and ring gear is larger, neighbor condition is easier to be satisfied.

D. Gear Ratio Condition

It is the same with other types of planetary mechanism.

VI. Conclusions

To complete assembly of single planetary mechanism, 4 conditions must be satisfied regarding number of gear teeth: concentric condition, homogeneity distribution condition, neighbor condition and gear ratio condition. In the 4 teeth matching conditions, homogeneity distribution condition is more easily ignored. If gears interfere with each other, the assembly will fail. Thus, this condition is critical for teeth matching. With the increasing number of gear pairs that participating in meshing between sun gear and ring gear, the concentric condition will become wider and wider, while neighbor condition become more and more complex. Gear ratio condition is mainly utilized to inspect whether the variance of actual gear ratio compared to theoretical gear ratio lies in reasonable limit.

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