# Analysis on the Transmission Characteristics and Motion Simulation of Limacon Gear

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*Abstract* —**Limacon gear, which is put forward as a new non-circular gear recent year, is used in the various equipments. In order to analyse the transmission characteristics of limacon gear, three-dimensional models of one-order and high-order limacon gear were established according to its basic meshing theory, and imported the models into ADAMS through the interface, simulation analysis of its motion state in the actual working condition was done, angular velocity curve of conjugate gear and contact forces were obtained. By the comparative analysis on the theoretical and emulational angular velocity curve, relative error of maximum and minimum angular velocity was obtained and the reasons were analyzed. The simulation results further verify the correctness of the modeling method and the theoretical transmission characteristics of limacon gear pair.**

***Keywords-limacon gear; transmission characteristcis; ADAMS; motion simulation***

## I. INTRODUCTION

As one of common mechanical structures which could realize the transmission with variable ratio, non-circular gears have a series of characteristics such as compact structure, steady transmission[1,2]. Limacon gear is widely used in practical production and daily life as a new non-circular gear. The transmission characteristics of non-circular gears are studied by many scholars at home and abroad, Liu Yang[3] designed the three dimensional model and combined it with slider-crank mechanism , and the simulation analysis of its transmission performance was done. Zheng Fang-yan[4] analyzed the transmission ratio, angular velocity and angular acceleration comparatively by studying the kinetics characteristics of an non-circular gear pair with specific transmission ratio and a common elliptical gear pair, the differences on the transmission performance of non-circular gear and cylindrical gear were obtained. Liu Yong-ping[5] studied the kinematics characteristics of elliptical gear and high-order elliptical gear, and the transmission characteristics of non-circular gear were obtained. Marius Vasie[6] studied related meshing characteristics of non-circular gears. Ran Xiao-hu, Lin Chao[7,8] discussed the influence of kinematics parameters on its kinematics characteristics by taking non-circular bevel gear as the research object. This paper analyses the theoretical transmission characteristics of one-order and high-order limacon gear pairs, and its motion simulations are done.

## II. MESHING THEORY OF LIMACON GEAR PAIR

If the driver gear 1 of a conjugate non-circular gear is limacon gear, the polar equation of its pitch curve is

$$r_1 = b\cos\theta_1 + l \tag{1}$$

where $\theta_1$ is the angle between X axis and radius vector of arbitrary point on pitch curve, b is the diameter of pitch curve's generated circle, l is fixed length. When the values of b and l are different, the sharp of pitch curve is aslo different. Figure I is the pitch curve of arbitrary limacon gear, 1 is the pitch curve's generated circle, 2 is the pitch curve of limacon gear.

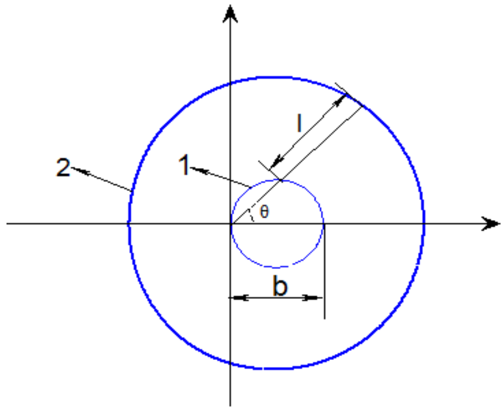



FIGURE I. THE PITCH CURVE OF LIMACON GEAR

According to the differential geometry, curvature radius' formula on each points of pitch curve is

$$\rho_1 = \frac{\left(l^2 + b^2 + 2bl\cos\theta_1\right)^{3/2}}{l^2 + 3bl\cos\theta_1 + 2b^2} \tag{2}$$

Let the order of the conjugated gear 2 is $n_2$, the follow formula could be got by the closed condition of pitch curve

$$\frac{2\pi}{n_2} = \int_0^{2\pi} \frac{r_1}{a - r_1} d\theta_1 = \int_0^{2\pi} \frac{a}{a - (b\cos\theta_1 + l)} d\theta_1 - 2\pi \tag{3}$$

According to the formula (3), the center distance a of the limacon gear pair is

$$a = \frac{(n_2+1)\sqrt{(2n_2+1)b^2 + l^2 n_2^{\,2}} + l(n_2+1)^2}{2n_2+1} \tag{4}$$

Thus, the pitch curve equation of the conjugated gear 2 is

$$\begin{cases} r_2 = a - (b\cos\theta_1 + l) \\ \theta_2 = \int_0^{\theta} \dfrac{(b\cos\theta_1 + l)}{a - (b\cos\theta_1 + l)} d\theta_1 \end{cases} \tag{5}$$

The transmission ratio of external gearing limacon gear pair is

$$i_{12} = \frac{a - r_1}{r_1} = \frac{a - (b\cos\theta_1 + l)}{b\cos\theta_1 + l} \tag{6}$$

According to the closeness of pitch curve and uniform distribution of gear teeth, the arc-length S of driver gear must meet the follow condition

$$S = \int_0^{2\pi} \sqrt{b^2 + l^2 + 2bl\cos\theta_1} d\theta_1 = \pi m Z_1 \tag{7}$$

## III. KINEMATICS SIMULATION ANALYSIS OF LIMACON GEAR PAIR

In order to analyse the transmission characterisitcs and the motion situation of limacon gear pair in actual working conditions, simulation analysis is done to one-order and 2-3 order limacon gears which center distance is a constant(a=135mm), table I are the concrete design parameters of the gears.

TABLE I. THE DESIGN PARAMETERS OF ONE-ORDER AND 2-3 ORDER LIMACON GEARS

| | One-Order | 2-3 Order |
|---|---|---|
| Module ($m$) | 2.5 | 2.5 |
| Tooth Number ($Z$) | $Z_1$=53; $Z_2$=53 | $Z_1$=44; $Z_2$=66 |
| Tooth Width ($b$) | 30 | 30 |
| Addendum Coefficient ( $h_a^*$ ) | 1 | 1 |
| Tip Clearance Coefficient ( $c^*$ ) | 0.25 | 0.25 |

### A. The Design of Pitch Curves and Virtual Prototype Models of Limacon Gears

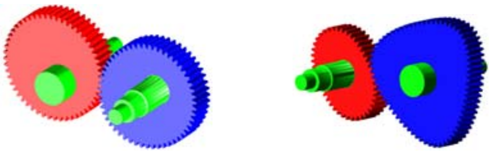
According to the related design parameters in the table, parameters of driver gear's pitch curve equation of one-order limacon gear pair are got by simultaneous equations of formula (4) and formula (7), *b*=19.2181 and *l*=64.8175, thus the pitch curve equations of one-order limacon gear pair are

$$\left.\begin{aligned} & r_1 = 19.2182\cos\theta_1 + 64.8175 \\ & r_2 = 135 - (19.2182\cos\theta_1 + 64.8175) \\ & \theta_2 = \int_0^{\theta_1} \frac{19.2182\cos\theta_1 + 64.8175}{135 - (19.2182\cos\theta_1 + 64.8175)} d\theta_1 \end{aligned}\right\} \tag{8}$$

In the same way, the parameters of driven gear's pitch curve of 2-3 order are got, *b*=9.0236 and *l*=53.4989, thus the pitch curve equations of 2-3 order limacon gear pair are

$$\left.\begin{aligned} & r_1 = 9.0236\cos(2\theta_1) + 53.4989 \\ & r_2 = 135 - [9.0236\cos(2\theta_1) + 53.4989] \\ & \theta_2 = \int_0^{\theta_1} \frac{9.0236\cos(2\theta_1) + 53.4989}{135 - [9.0236\cos(2\theta_1) + 53.4989]} d\theta_1 \end{aligned}\right\} \tag{9}$$

By the above pitch curve equations of the limacon gear pairs, the three-dimensional models of each limacon gear pair are built according to the modeling method which discussed in literature [9], and the models are introduced into ADAMS. Figure II is the virtual prototype models of limacon gear pairs.



(a) one-order (b) 2-3 order

FIGURE II. THE VIRTUAL PROTOTYPE MODELS OF LIMACON GEAR PAIRS

### B. Kinematic Pairs and Driving Adding

According to the actual working condition, if the shaft of driver gear(input shaft) is the driving input terminal, input shaft rotates and pushes conjugated gear shaft(output shaft) to output force and torque. Thus, setting material properties of parts steel 45 in virtual simulation software, the relative motion among every components is determined by adding the constraint relations, rotary motion pairs are added in the input shaft and output shaft. For analyzing easily, the effect of keyway is neglected and fixed constraint is set between driver gear and input shaft, the same constraint is also set between conjugated gear and output shaft, the contact constraint is set between driver gear and conjugated gear. Concrete motion relationship is showed in below table.

TABLE II. THE MOTION RELATIONSHIP BETWEEN THE COMPOENTS

| Component 1 | Component 2 | Motion relationship |
|---|---|---|
| Input shaft | — | Rotation |
| Output shaft | — | Rotation |
| Driven gear | Input shaft | Fixed |
| Conjugated gear | Output shaft | Fixed |
| Driver gear | Conjugated gear | Contact |

When the motion relationships between each components are determined, the driving is set 200r/min on the rotation pair of input shaft, that is 1200 degrees per second(setting 1200d*time), load on the output shaft is set $500N \cdot m$.

### C. Simulation and Result Analysis

The simulation time is set 1 second in simulation control module and its calculation steps is 400, then the virtual simulation situation of limacon gear pairs could be observed. After the simulation, the simulation result could be obtained in the Postprocessor module, the output angular velocity curve figures of one-order and high-order limacon gear are showed in below figures.

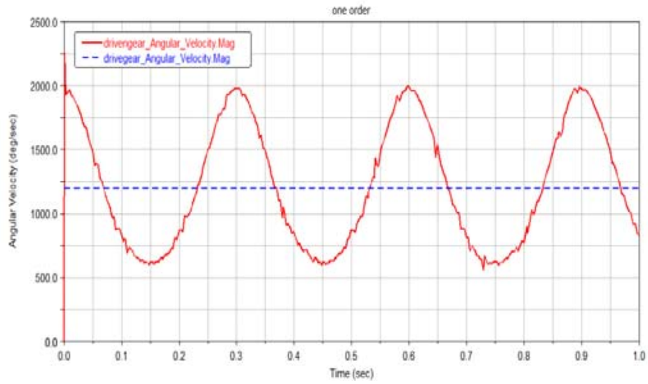



FIGURE III. SIMULATED OUTPUT ANGULAR VELOCITY OF ONE-ORDER LIMACON GEAR PAIR

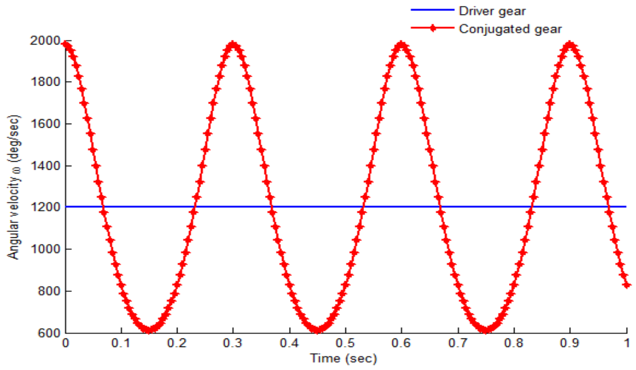



FIGURE IV. THEORETICAL OUTPUT ANGULAR VELOCITY OF ONE-ORDER LIMACON GEAR PAIR

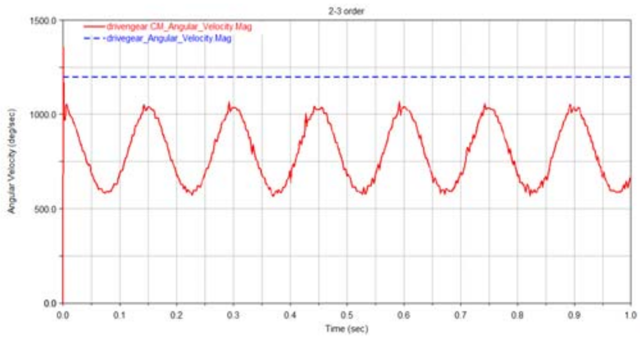



FIGURE V. SIMULATED OUTPUT ANGULAR VELOCITY OF 2-3 ORDER LIMACON GEAR PAIR

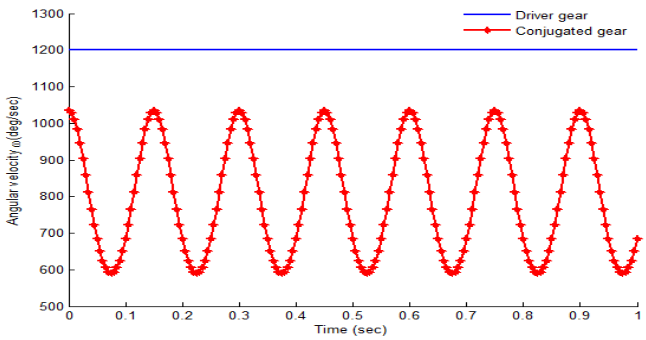



FIGURE VI. THEORETICAL OUTPUT ANGULAR VELOCITY OF 2-3 ORDER LIMACON GEAR PAIR

From figure III and figure IV, the simulation output angular velocity curve's shape is similar to the theoretical one. The maximum value of the simulation output angular velocity is $\omega_{2\max}^{'} = 1992.0115\,\deg/\sec$ and the minimum one is $\omega_{2\min}^{'} = 601.4692\,\deg/\sec$, while the maximum value of the theoretical output angular velocity is $\omega_{2\max} = 1978.6957\,\deg/\sec$ and the minimum one is $\omega_{2\min} = 612.0663\,\deg/\sec$.

From figure V and figure VI, the simulation output angular velocity curve's shape of 2-3 order limacon gear is also similar to the theoretical one. The maximum value of the simulation output angular velocity is $\omega_{2\max}^{'} = 1067.1304\,\deg/\sec$ and the minimum one is $\omega_{2\min}^{'} = 570.0141\,\deg/\sec$, while the maximum value of the theoretical output angular velocity is $\omega_{2\max} = 1035.1764\,\deg/\sec$ and the minimum one is $\omega_{2\min} = 589.5668\,\deg/\sec$ .The relative errors of angular velocity's extreme value of one-order and high-order limacon gear pair's conjugated gears are showed in below table.

TABLE III. THE RELATIVE REEORS OF ANGULAR VELOCITY'S EXTREME VALUE OF CONJUGATED GEARS

| Order | Simulation value(deg/sec) | | Theoretical value(deg/sec) | | Relative errors( $\lambda$ ) | |
|---|---|---|---|---|---|---|
| | $\omega_{2\max}^{'}$ | $\omega_{2\min}^{'}$ | $\omega_{2\max}$ | $\omega_{2\min}$ | $\lambda_{\max}$ | $\lambda_{\min}$ |
| One-order | 1992.0115 | 601.4692 | 1978.6957 | 612.0663 | 0.673% | 1.731% |
| 2-3 order | 1067.1304 | 570.0141 | 1035.1764 | 589.5668 | 3.087% | 3.316% |

Comparing the relative errors of the extreme value of one-order and high-order limacon gear, the simulation precision of one-order limacon gear is higher than high-order one, because the pitch curve of high-order gear is more complex than that of one-order, and its number of teeth is more, so its accumulative error is larger. In the figures of simulation output angular velocity curve, the velocity has an obvious changing at initial time, and the value of velocity is more than its maximun value, it is caused by large pulse input excitation. When the motion of gear pair becomes stationary, this phenomenon disappears quickly, and it could be avoided by setting the STEP function as its driving function. There are some cusps in the figures and it is mainly caused by the vibration during the motion as well as its design and simulation precision, but it has no effect on the analysis of transmission characteristics of limacon gear.

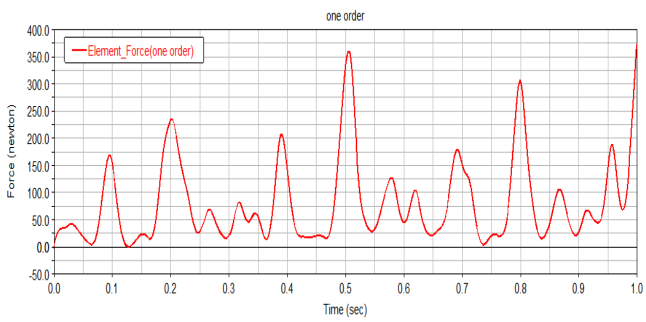



FIGURE VII. THE CONTACT FORCE OF ONE-ORDER LIMACON GEAR

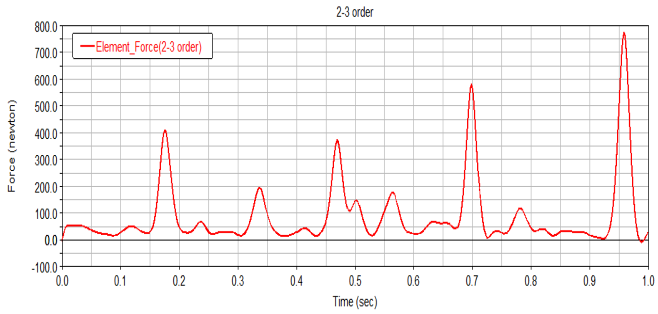



FIGURE VIII. THE CONTACT FORCE OF 2-3 ORDER LIMACON GEAR

The above figures are the contact force curves of one-order and 2-3 order limacon gear. From the figures, the contact force is changeable during the motion process, and the value of contact force of one-order gear mainly changes from 0N to 100N, and the value of contact force of 2-3 order gear mainly change from 0N to 200N, and it is caused by the difference of teeth profile of limacon gear. It is obvious that the contact force has a changing at some time nodes, and at this nodes the angular velocity of the conjugated gear also has an obvious change by comparing the figure of contact force to that of angular velocity of the conjugated gear, it is mainly also caused by the vibration during the motion as well as its design and simulation precision, and it can be improved by enhancing the machining accuracy of gears in actual manufacturing.

## IV. CONCLUSION

This Paper virtual simulated the motion process of one-order and high-order limacon gear in the actual working condition, and the figures of output angular velocity of conjugated gear and the contact force of gear pairs. The correctness of its theoretical transmission characteristics of limacon gear pair is verified, the simulation reflects the accuracy and feasibility of the modeling method laterally.

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