

# Evaluation of Particle Swarm Optimization Factors Using Gray Situation Decision-Making Model

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**Abstract**—This study investigates three factors of the acceleration equation, i.e., acceleration constants  $c_1$  and  $c_2$  and inertia weight  $w$ , which are then used as events in particle swarm optimization for parameter optimization in the para-tank model (PTM) during rainfall–runoff simulation. The values of 0.2, 0.5, and 0.8 are respectively used to create 27 groups of situation sets using the indices of the two decision-making objectives, root mean squared error and coefficient of efficiency, in order to analyze the systematic effectiveness. After comparing the comprehensive effect measures, an optimal decision is reached when the combined effectiveness was at the highest when  $c_1 = 0.2$ ,  $c_2 = 0.8$ , and  $w = 0.2$  and becomes the optimal parameter value for the PTM.

**Keywords**—Para-tank model; particle swarm optimization; gray system theory; gray situation decision making

## I. INTRODUCTION

Both the above and below surface runoff mechanisms were used to simulate the regional rainfall–runoff relationship, and subsequently, a new hydrology model known as the para-tank model (PTM) was developed using particle swarm optimization (PSO) for optimizing variables in the model. PSO will be used to optimize parameters within the flood PTM [1]. However, when searching for model parameters using PSO, the acceleration constants  $c_1$  and  $c_2$  and inertia weight  $w$  are inherent in the acceleration equation. The existence of these factors indirectly affects the accuracy and simulation speed of the PTM. Therefore, gray decision making from gray system theory will be used to analyze the three factors in order to find the optimal combination.

PSO was developed in 1995 by Kennedy et al. [2] when studying the hunting behavior of bird. In 1998, Shi et al. [3] introduced inertia weight to improve convergence, which has thereafter become the standard version of PSO. Gray system theory was proposed by Deng Julong [4] in 1982 and has gradually refined over the years. Most studies of related theories and applications of grey prediction and grey relational analysis have found few applications in gray situation decision making. Xue (2000) used gray system theory on explosion design and analysis. As there were multiple factors and objectives in his study, a multi-objective gray situation decision-making model was established from the numerical relationships among the data to show the rules of connections and restrictions. This model helps to find the optimal solution in finite explosion testing in a scientific manner [5]. Cao et al. (2006) used gray situation decision-making theory on the

selection of sites for waste and sanitary landfill [6]. Zhang et al. (2009) used data from 1998 to 2007 to establish a gray situation decision-making model to help investors make the best decisions in real estate and the stock market [7]. Wong (2015) applied gray situation decision making to select industrial factory design proposals using the calculated comprehensive effect measure in size to choose the optimal design. His study presents a relatively scientific approach for choice evaluations in civil engineering [8].

This study will investigate the change in the three factors in the acceleration equation, i.e., acceleration constants  $c_1$  and  $c_2$  and inertia weight  $w$ , in PSO when finding optimizing the parameters in the PTM during rainfall–runoff simulation and their impact on accuracy and simulation speed. Two objective decision-making indices, root mean squared error (RMSE) and coefficient of efficiency (CE) are used in the analysis to find the combined effectiveness when  $c_1$ ,  $c_2$ , and  $w$  are optimal.

## II. GRAY SITUATION DECISION-MAKING ANALYSIS

Gray decision making is a process of elimination that chooses strategies for different events based on the effectiveness of the objectives.

1) *Case (A)*: cases that need to be handled. Let case set  $A = \{a_i, i = 1, 2, \dots, n\}$ .

2) *Strategy (B)*: plans for handling a certain case. Let strategy set  $B = \{b_j, j = 1, 2, \dots, m\}$ .

3) *Situation(S) and results (U)*: the effects of certain strategies on specific cases.

a) Let the Cartesian product of  $A$  and  $B$ ,  $S = A \times B = \{S_{ij} = (a_i \times b_j) \mid a_i \in A, b_j \in B\}$  be the situation set.

b) Construct an objective decision index set  $P = \{p_k, k = 1, 2, \dots, p\}$  to clearly set the index for consideration of decision making strategy.

c) Confirm that the resulting sample value  $u_{ij}^p$  is from situation  $S_{ij}$  under the effects of objective  $p$ .

d) Unify effect measure: define function  $\theta: u_{ij}^p \rightarrow r_{ij}^p \in [0, 1]$ , where  $r_{ij}^p$  is the effect measure value after unification.

4) *Objective (P)*: Evaluate the index of strategy effectiveness.

a) *Upper effect measure*: Reflects the degree of divergence between the result sample value and the maximum result value, defined as (1).

$$r_{ij}^p = \frac{u_{ij}^p}{i_{\max, j_{\max}, u_{ij}^p}} \quad (1)$$

where  $i_{\max}$ ,  $j_{\max}$ , and  $u_{ij}^p$  are the maximum values of all the sample values related to objective p.

b) *Lower effect measure*: Reflects the degree of divergence between the result sample value and the minimum result value, defined as (2).

$$r_{ij}^p = \frac{i_{\min, j_{\min}, u_{ij}^p}}{u_{ij}^p} \quad (2)$$

where  $i_{\min}$ ,  $j_{\min}$ , and  $u_{ij}^p$  are the minimum values of all the sample values related to objective p.

c) *Medium effect measure*: Ideally, this result should be within a certain range of a designated objective, defined as (3).

$$r_{ij}^p = \frac{\min(u_{ij}^p, u_0)}{\max(u_{ij}^p, u_0)} \quad (3)$$

where  $u_0$  is the designated moderate value.

5) Find the comprehensive effect measure  $r_{ij}^\Sigma$  of the situation  $S_{ij}$ .

$$r_{ij}^\Sigma = \frac{1}{l} \sum_{p=1}^l r_{ij}^p \quad (4)$$

6) Evaluate the comprehensive effect measure according to the  $r_{ij}^\Sigma$  value found, and then sort and confirm the optimal or satisfactory situation and strategy.

### III. CASE STUDY

#### A. Summary of the Research Region

Kaohsiung metropolitan region was chosen as the research area. According to the report of "Kaohsiung City flood prevention and drainage plan" [9], on July 11, 2001 at 5pm, a violent south west airflow due to Typhoon Trami caused continuous torrential rain in the Kaohsiung region for 10 h. According to records from Cianjhen and Zuoying weather stations of the weather bureau, the total accumulated rainfall was 525 and 493 mm, respectively. The highest recorded rainfall within 1 h at Zuoying weather station was 126.5 mm, which was close to the 100-year frequency storm rainfall of 130 mm; the rainfall within 3 h peaked at 329 mm, which exceeded the 200-year frequency storm rainfall of 300 mm. Cianjhen weather station recorded the highest rainfall of 119.5 mm in 1 h and 239 mm over 3 h. Both of the above-mentioned rainfall amounts recorded greatly exceeded Kaohsiung City's flood drainage design standards (instant water drainage: 5 years, flood prevention: 20 years). In addition, the tide level at the mouth of the Love River increased dramatically, obstructing the flood. Even though the city's sewage system was at over 90% capacity, the system was seriously overloaded, causing floods

of over 1 foot high in low-lying regions of Yancheng District, Benguan and Benhe Village, Baozhugao, Canal No. 2, and Cianjhen District spreading across an area of 300 hectares. In order to compare the results of the PTM, rainfall data of 711 from the Kaohsiung City flood prevention and drainage plan was used, and the average rainfall value of Kaohsiung, Zuoying, and Fengshan was used as the rainfall  $r(t)$  in the simulation.

In this study, the PTM was primarily used to investigate the effects of torrential rain on urban areas that were relatively more impermeable. The Love River Basin (LR Basin) in Kaohsiung was the main area of focus in this research. As the result the 711 flood distribution, the LR Basin was divided into nine catchment sections. A certain section was selected as the target section by its geographical properties.

#### B. Analysis Results

Variations in  $c_1$ ,  $c_2$ , and  $w$  in the acceleration equation used in PSO are the focus of this study. The gray situation decision-making model is used to analyze the factors,  $c_1$ ,  $c_2$  and  $w$ , in finding the optimal combination. The related information used in the gray situation decision-making model is listed.

1) *Event (A)*: This study focuses on analysis of  $c_1$ ,  $c_2$  and  $w$ ; hence, we define  $a_1 = c_1$ ,  $a_2 = c_2$ , and  $a_3 = w$ .

2) *Strategy (B)*: Events  $A_1$ ,  $A_2$ , and  $A_3$  have values between 0 and 1. To simplify the analysis in terms of the values used, we selected values of 0.2, 0.5, and 0.8 based on the rule of thirds to represent the median value of the three regions of front, middle, and back. Hence,  $b_1 = 0.2$ ,  $b_2 = 0.5$ , and  $b_3 = 0.8$ .

#### 3) Objective (P):

a) Lower effect measure using RMSE to find the minimum value.

b) Upper effect measure using CE to find the maximum value.

c) Medium effect measure do not use to find the moderate value in the study.

Based on a recently published paper in 2015 on finding parameters in the rainfall-runoff PTM [1], the rainfall amount in the water catchment areas and the flow rate were entered into the PTM analysis. Global area acceleration constants  $c_1$ ,  $c_2$  and inertia weight  $w$  in the region were substituted into the situation set  $S_{ij}$ . The following parameters were set: number of particles  $m = 30$ , dimensions  $d = 4$ , residue height before rain  $h_0 = 3$  mm, accumulated rainfall  $r_0 = 1$  mm, infiltration capacity  $i(t) = 1$  mm, and the capacity of the sewage designed  $q_c$  was set as the standard design specifications for that region in Taiwan. The section design standard of water drainage was set at 70.9 mm/h, the rainfall intensity of the 5-year storm in Kaohsiung City rainwater sewage system.  $X_{i1}$  represents the residue height of the infiltration and depression storage and  $H_1$  was set between 1 and 100,  $X_{i2}$  represents the outflow rate of the geographical flood characteristic  $\lambda_1$  set between 0 and 1,  $X_{i3}$  represents the sewage system's residue height  $H_2$  set between 50 and 100,  $X_{i4}$  represents the sewage system's outflow rate  $\lambda_2$  at load capacity set between 0 and 1, and frequency count  $t_{\max}$  was set at 1000. After calculations, we obtained 27 groups of

$H_1$ ,  $\lambda_1$ ,  $H_2$ , and  $\lambda_2$  as PTM parameters and the objective decision index values of RMSE and CE. Gray decision making as shown in Table I.

TABLE I. SITUATION SET RESULTS.

Case	PSO factor			PTM parameters				Objective decision index	
	$c_1$	$c_2$	$w$	$H_1$ (mm)	$\lambda_1$ (%)	$H_2$ (mm)	$\lambda_2$ (%)	RMSE	CE
1	0.2	0.2	0.2	37.3486	59.04	52.1690	24.27	3.6585	0.9996
2	0.2	0.2	0.5	37.4537	59.20	52.1626	24.10	3.6581	0.9996
3	0.2	0.2	0.8	36.5143	58.52	53.8165	23.96	3.7644	0.9996
4	0.2	0.5	0.2	37.4536	59.20	52.1626	24.10	<b>3.6581</b>	0.9996
5	0.2	0.5	0.5	37.4432	59.18	52.1681	24.12	3.6581	0.9996
6	0.2	0.5	0.8	38.1944	57.20	49.4884	24.79	4.2088	0.9995
7	0.2	0.8	0.2	37.4536	59.20	52.1626	24.10	3.6581	0.9996
8	0.2	0.8	0.5	37.9559	59.81	52.8354	22.97	3.7171	0.9996
9	0.2	0.8	0.8	35.1007	57.52	57.7463	26.87	4.1050	0.9995
10	0.5	0.2	0.2	37.2949	58.94	52.1412	24.36	3.6592	0.9996
11	0.5	0.2	0.5	37.4451	59.19	52.1813	24.11	3.6581	0.9996
12	0.5	0.2	0.8	35.4207	53.67	50.5187	31.28	4.4024	0.9994
13	0.5	0.5	0.2	37.4534	59.20	52.1628	24.10	3.6581	0.9996
14	0.5	0.5	0.5	37.8917	60.17	52.9285	23.12	3.6754	0.9996
15	0.5	0.5	0.8	41.4631	60.60	44.4441	23.21	4.3734	0.9994
16	0.5	0.8	0.2	37.4566	59.20	52.1635	24.10	3.6581	0.9996
17	0.5	0.8	0.5	36.3037	58.05	50.8781	23.92	3.8747	0.9996
18	0.5	0.8	0.8	35.2222	53.30	53.6074	29.58	4.8889	0.9993
19	0.8	0.2	0.2	37.4683	59.22	52.1592	24.08	3.6581	<b>0.9996</b>
20	0.8	0.2	0.5	36.9238	58.41	52.1357	24.83	3.6690	0.9996
21	0.8	0.2	0.8	40.9988	59.94	38.2418	20.89	5.1020	0.9992
22	0.8	0.5	0.2	37.4196	59.13	52.1440	24.18	3.6582	0.9996
23	0.8	0.5	0.5	36.6787	58.95	51.6804	23.73	3.7633	0.9996
24	0.8	0.5	0.8	38.6492	62.02	60.9766	24.58	4.9530	0.9993
25	0.8	0.8	0.2	37.1967	58.78	52.1407	24.59	3.6615	0.9996
26	0.8	0.8	0.5	37.3942	59.52	54.4564	23.98	3.7245	0.9996
27	0.8	0.8	0.8	39.4834	61.45	54.3769	24.67	4.4578	0.9994

Note: Numbers in bold and highlighted in gray are the extreme (max and min) values.

#### IV. DISCUSSION OF ANALYSIS RESULTS

##### A. Comprehensive effect measure

After using PSO to find the optimal  $H_1$ ,  $\lambda_1$ ,  $H_2$ , and  $\lambda_2$  in each case, the four parameter values were entered into the PTM calculation using time step to obtain the flow rate  $Q_c(t)$  and objective flow rate  $Q_o(t)$ . The objective minimum RMSE value and the maximum CE value were used for evaluation. Results are shown in Table II. Case 7 (when  $c_1 = 0.2$ ,  $c_2 = 0.8$ , and  $w = 0.2$ ) shows the highest comprehensive effect measure.

##### B. Verification of the statistical method

Table III was generated after the comprehensive effect measure were sorted and grouped into front, middle, and back sections using the frequency count statistics. In the first nine entries (front section) of the results,  $c_1 = 0.2$  occurred four times,  $c_2 = 0.5$  occurred four times, and  $w = 0.2$  occurred six times. From the viewpoint of statistics, when  $c_1 = 0.2$ ,  $c_2 = 0.5$  and  $w = 0.2$ , most of the first nine entries after sorting of the comprehensive effect measures were a little different with the results from gray situation decision-making comprehensive

effect measures (objective decision indices: RMSE, CE). Only the value of  $c_2$  is different. In Table II, the value of comprehensive effect measure is the 2nd place of Case 4 ( $c_1 = 0.2$ ,  $c_2 = 0.5$ , and  $w = 0.2$ ), which is a high representation.

TABLE II. UNIFIED EFFECT MEASURE ANALYSIS TABLE OF GRAY SITUATION DECISION MAKING ON PSO FACTORS.

Case	PSO factor			Objective decision index		Comprehensive effect measure
	$C_1$	$C_2$	$W$	RMSE	CE	
7	0.2	0.8	0.2	1.0000	1.0000	<b>1.0000</b>
4	0.2	0.5	0.2	<b>1.0000</b>	1.0000	1.0000
2	0.2	0.2	0.5	1.0000	1.0000	1.0000
13	0.5	0.5	0.2	1.0000	1.0000	1.0000
16	0.5	0.8	0.2	1.0000	1.0000	1.0000
5	0.2	0.5	0.5	1.0000	1.0000	1.0000
11	0.5	0.2	0.5	1.0000	1.0000	1.0000
19	0.8	0.2	0.2	1.0000	<b>1.0000</b>	1.0000
22	0.8	0.5	0.2	1.0000	1.0000	1.0000
1	0.2	0.2	0.2	0.9999	1.0000	0.9999
10	0.5	0.2	0.2	0.9997	1.0000	0.9999
25	0.8	0.8	0.2	0.9991	1.0000	0.9995
20	0.8	0.2	0.5	0.9970	1.0000	0.9985
14	0.5	0.5	0.5	0.9953	1.0000	0.9977
8	0.2	0.8	0.5	0.9841	1.0000	0.9921
26	0.8	0.8	0.5	0.9822	1.0000	0.9911
23	0.8	0.5	0.5	0.9721	1.0000	0.9860
3	0.2	0.2	0.8	0.9718	1.0000	0.9859
17	0.5	0.8	0.5	0.9441	1.0000	0.9720
9	0.2	0.8	0.8	0.8911	0.9999	0.9455
6	0.2	0.5	0.8	0.8691	0.9999	0.9345
15	0.5	0.5	0.8	0.8364	0.9998	0.9181
12	0.5	0.2	0.8	0.8309	0.9998	0.9154
27	0.8	0.8	0.8	0.8206	0.9998	0.9102
18	0.5	0.8	0.8	0.7482	0.9997	0.8740
24	0.8	0.5	0.8	0.7386	0.9997	0.8691
21	0.8	0.2	0.8	0.7170	0.9996	0.8583

Note: Unified effect measures are rounded to four decimal places, and values in bold and highlighted in gray are the extreme values (min and max) of the objective decision index.

TABLE III. FACTOR'S FREQUENCY OF OCCURRENCE IN ORDERED COMPREHENSIVE EFFECT MEASURE.

Section Factor	Front		Middle		Back	
	Value	Frequency of occurrence	Value	Frequency of occurrence	Value	Frequency of occurrence
$c_1$	<b>0.2</b>	<b>4</b>	0.2	3	0.2	2
	0.5	3	0.5	2	0.5	4
	0.8	2	0.8	4	0.8	3
$c_2$	0.2	3	0.2	4	0.2	2
	<b>0.5</b>	<b>4</b>	0.5	2	0.5	3
	0.8	2	0.8	3	0.8	4
$w$	<b>0.2</b>	<b>6</b>	0.2	3	0.2	0
	0.5	3	0.5	5	0.5	1
	0.8	0	0.8	1	0.8	8

Note: Numbers in bold and highlighted in gray are the most times of occurrences.

### C. Discussion

When using PSO to optimize parameters in the PTM, the acceleration constants  $c_1$  and  $c_2$  and inertia weight  $w$  exist in the acceleration equation, which indirectly impact the accuracy of parameter optimization and simulation speed. But after adding the gray situation decision making, an optimal comprehensive effect can be achieved by choosing different strategies in different cases through the elimination process according to the objective results.

In our example, there were in total 27 cases and the objective decision indices, RMSE (smaller the better) and CE (larger the better), were used for evaluation. For the objective decision index RMSE, the minimum unified effect measure value occurred in case 4 ( $c_1 = 0.2$ ,  $c_2 = 0.5$ ,  $w = 0.2$ ). For the objective decision index CE, the maximum unified effect measure value occurred in case 19 ( $c_1 = 0.8$ ,  $c_2 = 0.2$ ,  $w = 0.2$ ). After evaluation of the comprehensive effect measures, the optimal case was found to be case 7 ( $c_1 = 0.2$ ,  $c_2 = 0.8$ ,  $w = 0.2$ ) with a value of 1.0000, the decision of which could not be made using a single objective decision index, implying that multiple objectives are desirable for the best optimization.

Using statistical methods to verify the results, we found that in the sorted front section,  $c_1 = 0.2$  occurred four times,  $c_2 = 0.5$  occurred four times, and  $w = 0.2$  occurred six times. We can see from the results that the combination of  $c_1 = 0.2$ ,  $c_2 = 0.5$ , and  $w = 0.2$  appears most frequently, which is the 2nd optimal selection of the gray situation decision making. This confirms that gray situation decision making can indeed select a case with the optimal comprehensive effect. This is a simple and easy-to-use application of multi-objective optimization.

### V. CONCLUSIONS

This paper proposes a new para-tank model with particle swarm optimization. Grey theory is applied for the decision making of the multi-objective function of PSO factors in the acceleration equation. After the objective decision index evaluation to the effect measures of RMSE and CE were evaluated in our results; case 7 ( $c_1 = 0.2$ ,  $c_2 = 0.8$ ,  $w = 0.2$ ) with an comprehensive effect measure of 1.0000 was the best out of the 27 cases. The statistics of frequency occurrence of the nine cases in the front section of the ordered data results verified the effectiveness of gray situation decision making. It also confirmed that PSO could generate a set of optimized factors during parameter simulation of the new hydrology model (PTM) that would help to a certain extent in enhancing speed and accuracy in future model simulations.

In the future, the number of Objective Decision Index as well as the Medium Effect Measure will be introduced in this research to make a contribution to the increase of decision factors. Then, we are looking forward to acquire a best selection of the gray situation decision-making while the results have been tested by statistical methods. Using gray situation decision making to solve multi-objective optimization is a good reference with regard to decision making. However, finding out how to establish a more objective and quantitative index factor in the right weight ratios will be the topic for future development.

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