

A Combination Predicting Method Based on Innovation GM(1,1) and RBF

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Abstract—Proposed a combination predicting method based on innovation grey theory and neural network to improve the precision of spacecraft power predicting. Established innovation GM(1,1)+RBF combination predicting model with optimal predicting precision according to the spacecraft power seasonal fluctuating. Given embedded combination predicting model and compensated combination predicting model. Contrastive analyzed the performance of those different combination predicting methods. The results show that innovation GM(1,1)+RBF combination predicting model with optimal predicting precision gains the best prediction performance. The method is of application space because it is suitable to other parameters predicting.

Keywords-GM(1,1); RBF; spacecraft power predicting

I. INTRODUCTION

In the gray system theory, GM(1,1) is the major forecasting model, but this model is not ideal to predict the effect of non-monotonic number sequence [1]. However, satellite solar array output power changes with the seasons and increasing its power will appear alternately decreasing, showing non-monotonic characteristic, so GM(1,1) model does not apply. To solve this problem, [2] proposed a method for real-time online application, create a dynamic dimension new information GM(1,1) prediction model and an example to verify the validity of the simulation model. But the single innovation GM(1,1) model accuracy is limited, if combined gray neural network forecasting system to predict the stability can be further improved and the reliability of the system [3-4]. RBF network has a good nonlinear function approximation and convergence speed, avoiding local minima, robustness [5]. Therefore, this article as a new message Select RBF GM(1,1) combination forecasting another data processing method.

Based on neural network and gray combination system can be divided into tandem, parallel-type, embedded and mixed type. This article will be a combination of several different methods for detailed analysis and comparison. Firstly, given the GM(1,1) and its improved forecasting model innovation; and the establishment of neural network combination forecasting model of gray system; Finally, simulation experiments and comparative analysis of the single innovation GM(1,1) and this merits of several models in a spacecraft power of prediction.

II. GM(1,1) AND IMPROVED EQUAL-DIMENSION AND NEW INFORMATION GRAY MODEL

A. GM(1,1) Model

Gray prediction establishes differential equations based on the original data column, which can make random noise component of the original series reduced or eliminated, while allowing the certainty information enhanced by accumulating. GM (1,1) modeling is as follows [6]:

Let observational data value as $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n))$, To generate one-time accumulated generating operator(1-AGO) sequence of $X^{(0)}$

$$X^{(1)}(k) = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)) \quad (1)$$

Where

$$x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i) \quad (2)$$

To generate the albino differential equations for $x^{(1)}(k)$ as following forms

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = u \quad (3)$$

$$x^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{u}{a} \right) e^{-ak} + \frac{u}{a} \quad (4)$$

The estimation value of parameter a is expressed as

$$\hat{a} = (B^T B)^{-1} Y \quad (5)$$

Where

$$B = \begin{bmatrix} -\frac{1}{2}[x^{(1)}(1) + x^{(1)}(2)] & 1 \\ -\frac{1}{2}[x^{(1)}(1) + x^{(1)}(2)] & 1 \\ \dots & \dots \\ -\frac{1}{2}[x^{(1)}(n-1) + x^{(1)}(n)] & 1 \end{bmatrix} \quad (6)$$

$$Y = [x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(n)]^T \quad (7)$$

The restore data is obtained by differencing

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1), \quad k = (1, 2, \dots, n) \quad (8)$$

B. Equal-Dimension and New Information GM(1,1) Model

Equal-Dimension and New Information GM (1,1) is essentially to update parameters with the latest measurement data instead of the oldest measurement data. The steps are as follows: by coding measurement data with equal interval to form the time sequence $\{x^{(0)}(k)\}$, $(k=1, 2, \dots, n)$. Let forecast time be t , the number of predicted original data be N . First read N data from the time series at $t-N+1$ to t of the original input for prediction when modeling, then to establish methods and predict output at time $t+1, t+2, \dots$ according to GM (1,1) model of 2.1. When the value at time $t+1$ is obtained, update the model using $t-N$ to $t+1$ as the new of columns in the original input sequence and predict the output value at time $t+2, t+3, \dots$.

III. RBF NETWORK MODEL

Radial basis function neural network(RBFNN) is a three-layer forward network [7], the structure of which is shown in Figure I.

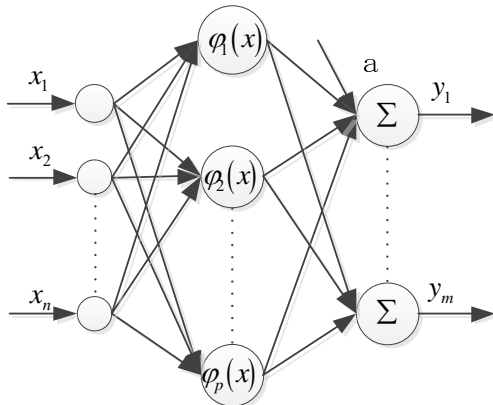


FIGURE I. RBF NEURAL NETWORK STRUCTURE

Input space to the hidden layer space conversion is non-linear, the conversion from the hidden layer space to the output space is linear. The input and the output of the network can be considered to achieve the y_i : Mapping of $R_n \rightarrow R$ [8].

$$y_i = \sum_{j=1}^{n_c} w_{ij} * \Phi(\|X - c_j\|/\beta) + b_j + \xi_j, \quad i = 1, 2, \dots, N \quad (9)$$

Where, $X = (x_1, x_2, \dots, x_n)^T \in R^n$ is the input vector, y_i is the i -th-output value of the output unit; w_{ij} is the output unit weights of the i -th to the j -th hidden neuron; $\|\bullet\|$ is the euclidean norm; $\Phi(\bullet)$ is the nonlinear transfer function; c_j is the center of RBF; β is the distributed constant; n_c is the number of centers;

The sample data set is required to meet the following characteristics:

1. The association between the input and output, which requires certain function relation between output and input.
2. No redundancy in the sample data, namely the input vectors are relatively independent.
3. The forecast data is comparable to the prior data, namely a priori data and forecast data are consistency.
4. The data sample has a certain scale, the number of samples is not too little.

IV. COMBINATION FORECAST

A. Optimal Model Based on the Prediction Accuracy

The so-called prediction accuracy refers to the prediction error in all periods are relatively small; the overall average maintains a high precision level. The optimal gray neural network system structure based on the prediction accuracy is shown in Figure II.

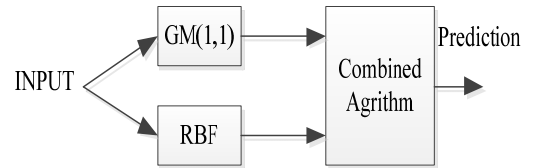


FIGURE II. THE OPTIMAL GRAY NEURAL NETWORK SYSTEM STRUCTURE BASED ON THE PREDICTION ACCURACY

Let the fitting error of the i -th method at time t be e_{it} , that

$$e_{it} = \begin{cases} -1 & (x_t - x_{it})/x_t < -1 \\ (x_t - x_{it})/x_t & -1 \leq (x_t - x_{it})/x_t \leq 1 \\ 1 & (x_t - x_{it})/x_t > 1 \end{cases} \quad (10)$$

$$A_{it} = \begin{cases} 1 - |e_{it}| & 0 \leq |e_{it}| \leq 1 \\ 0 & |e_{it}| \geq 1 \end{cases} \quad (11)$$

Where, A_{it} is called the prediction accuracy of the i -th method at time t .

Define combined forecasting model on the sample interval:

let $\hat{x}_t = \sum_{i=1}^m w_i x_{it}$ be a combination of time-series forecasting value at time t , then this time fitting accuracy of the combined forecasting model is:

$$A_t = 1 - \left| (x_t - \hat{x}_t) / x_t \right| = 1 - \left| \sum_{i=1}^m w_i e_{it} \right|, t = 1, 2, \dots, N$$

Then the forecasting accuracy of sample interval combination at time t is $m_t = \sum_{i=1}^N Q_i A_t$, where Q_i is the weight coefficient of A_t , which meets non-negative and normalization condition. Then the optimization model can be expressed as: $\max m_1 = \sum_{t=1}^N Q_t A_t$

$$s.t. \begin{cases} A_t = 1 - \left| \sum_{i=1}^m w_i e_{it} \right| & t = 1, 2, \dots, N \\ \sum_{i=1}^m w_i = 1 \end{cases} \quad (12)$$

The prediction accuracy of the forecast range is $m_2 = \sum_{t=N+1}^{N+T} Q_t A_t$, the optimization model can be expressed as:

$$\max m_2 = \sum_{t=N+1}^{N+T} Q_t A_t \quad (13)$$

$$s.t. \begin{cases} A_t = 1 - \left| \sum_{i=1}^m w_i e_{it} \right| & t = N+1, N+2, \dots, N+T \\ \sum_{i=1}^m w_i = 1 \end{cases} \quad (14)$$

The total range of optimization models is:

$$\max m_3 = a \sum_{t=1}^N Q_t A_t + (1-a) \sum_{t=N+1}^{N+T} Q_t A_t \quad (15)$$

$$s.t. \begin{cases} A_t = 1 - \left| \sum_{i=1}^m w_i e_{it} \right| & t = 1, 2, \dots, N+T \\ \sum_{i=1}^m w_i = 1 \end{cases} \quad (16)$$

As can be seen that the combination forecasting model based on optimization objective function is linear, and the use of the weighted average of the absolute value of the relative error can eliminate the influence of dimension.

B. Embedded Gray Neural Network

Embedded gray neural network is constituted by respectively adding a white layer at the input and the output of the neural network. The system structure is shown in Figure III.

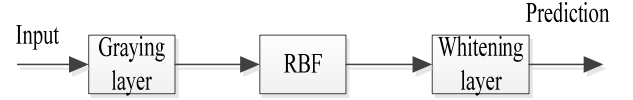


FIGURE III. THE STRUCTURE OF EMBED GRAY NEURAL NETWORK

Gray layer can play a role in weakening the randomness of the original data, so it is easier to approach. Ashing layer generate new data by accumulating raw data one or more times to be neural network training samples. And because the accumulated data has monotonically increasing trend, the nonlinear activation function is relatively easy to approach, thereby greatly reducing the learning time and improve forecast accuracy convergence speed.

C. Mixed Gray Neural Network

For a given data sequence, forecast the data of the sequence columns by constructing the constant dimensional dynamic gray model. Compare the predicted data and the sequence to generate the residual error. Then, the neural network approximation model is established by using the neural network using these residual errors and the corresponding data. In this way, the recurrent neural network is a mapping relationship between the residual error and the selected gray model data. The output of the neural network is used to compensate the output of the neural network. The system structure is shown in Figure IV

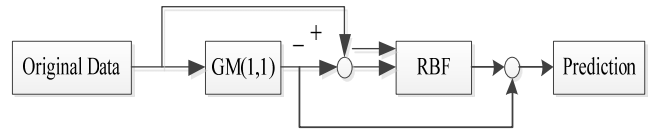


FIGURE IV. THE STRUCTURE OF COMPENSATED GRAY NEURAL NETWORK

V. ACCURACY SPECIFICATIONS OF THE PREDICTION MODEL

As can be seen that the combination forecasting model based on optimization objective function is linear, and the use of the weighted average of the absolute value of the relative error can eliminate the influence of dimension.

Only after comprehensive accuracy inspection and evaluation, the forecasting model can be established to meet the actual conditions. Referring to the GM (1,1) prediction

model, the common methods include three kinds of methods: residual error test, relevance test and posterior test. Relevance test is to examine the similarity between the model value curve and the curve of modeling sequence, it is generally believed that the prediction that is credible when the relevance degree $r > 0.68$. Posterior test is to test the distribution's statistical characteristics of residual error, which is composed of a posteriori and small error ratio C and the minimum error P gives model accuracy grade (Table I) according to C, P value.

TABLE I. THE DATA RANGE OF MODEL ACCURACY CLASS

Prediction accuracy Classification standard	Test Specifications	
	P	C
Class 1 (Good)	>0.95	<0.35
Class 2 (Qualified)	>0.80	<0.50
Class 3 (Just)	>0.70	<0.65
Class 4 (disqualified)	≤ 0.70	≥ 0.65

The specific calculations are as follows:

Let the original data be $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n))$, the prediction data be $\hat{X}^{(0)} = (\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \dots, \hat{x}^{(0)}(n))$. Then the prediction residual error of the k-th datum is $\varepsilon(k) = x^{(0)}(k) - \hat{x}^{(0)}(k) (k=1, 2, \dots, n)$. The relative prediction residual error of the k-th datum is $\Delta k = |\varepsilon(k) / x^{(0)}(k)|$; the average of the original data is $\bar{x} = \frac{1}{n} \sum_{k=1}^n x^{(0)}(k)$, the average of the residual error is $\bar{\varepsilon} = \frac{1}{n} \sum_{k=1}^n \varepsilon(k)$; The variance of the original data is $S_1^2 = \frac{1}{n} \sum_{k=1}^n [x^{(0)}(k) - \bar{x}]^2$; The variance of the residual error is $S_2^2 = \frac{1}{n} \sum_{k=1}^n [\varepsilon(k) - \bar{\varepsilon}]^2$; Thus, every coefficient of the parameters is obtained:

Relative error test parameters: $a = \frac{1}{n} \sum_{k=1}^n \Delta k$;

Posterior variance ratio test parameters: $C = S_2 / S_1$;

Small error probability test parameters: $p = p\{|\varepsilon(k) - \bar{\varepsilon}| < 0.6745S_1\}$.

VI. SIMULATIONS

In this paper, a set of data of the output power of a satellite battery array given in Table II is analyzed. From table II, the power value is slowly decreasing at time $t=1\sim5$ and slowly increasing at time $t=6\sim11$ time. The traditional GM (1,1) models using the first 6 data, including the new information system with the former 2-7 data to get the forecast value as shown in TABLE III.

TABLE II. MEASURED POWER OF SOME SATELLITE

t	$x^{(0)}$	t	$x^{(0)}$
1	718.6	7	696.8
2	710.2	8	704.7
3	690.5	9	712.5
4	687.8	10	720.9
5	683.5	11	731.5
6	688.7	12	742.8

TABLE III. FORECASTING RESULTS OF VARIOUS METHODS

Power/ t	Traditio nal GM(1,1)	New Informa tion GM(1,1)	Optimal model prediction accuracy	Embe d model	Compensa tion model
8	677.0	693.7	692.7	682.8	695.2
9	667.2	695.1	699.2	691.1	696.6
10	662.4	686.4	702.1	678.0	687.9
11	657.5	697.9	707.6	662.3	698.4
12	652.9	699.3	718.8	658.4	700.8

TABLE IV. THE ACCURACY OF THE RESULTS OF THE VARIOUS METHODS OF PREDICTION

Predictio n method Test indicators	Traditio nal GM(1,1)	New informa tion GM(1,1)	Optimal model prediction accuracy	Embed model	Compensa tion model
$a / \%$	8.95	4.03	2.54	7.18	3.84
C	0.111	0.379	0.112	0.838	0.423
ε	8.18	3.88	3.21	6.64	3.70

From TABLE IV methods can be listed from good to poor performance: the prediction accuracy of the optimal compensation model > new information GM (1,1) > traditional embedded model. Embedded model method is limited for performance improvement, because this method only make data graying and whitening, the amount of data is still valid so the RBF fitting is unable to reach the optimal. Compensation model compensates the error based on the original new information GM (1,1), the more close to the real value after compensation, so the performance has improved, but is not obvious, this is because the relative power of the compensation value is very small. While prediction accuracy optimal model obtains weighted average using the absolute value of the relative error, eliminating the impact of the dimension, and reasonably allocates the prediction weight of the RBF and GM (1,1). This takes the advantages of the two methods, so that the performance has been greatly improved.

VII. CONCLUSION

Fault prediction and health management of spacecraft is a hot research topic at home and abroad, which can play a role in fault prediction and alarm. In this paper, the power of a spacecraft is predicted by the analysis of several different GM (1,1) and RBF combined forecasting methods in the application of the spacecraft power prediction. In this application, the best combination method is the optimal prediction model, which is more accurate than the new information GM(1,1). The method can also be applied to some

other parameters prediction of the spacecraft, so it has a certain application prospect.

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