Using of Kriging Surrogate Model in the Multi-Objective Optimization of Complicated Structure

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***Abstract*—To solve the problem of too large calculating amount in the multi-objective optimization of complicated structure, a method based on Kriging surrogate model is proposed and being used in an aerospace assembly to verify its effectiveness. There are two objectives in the structural optimization of the assembly, min mass and max frequency. The method is based on the following steps, first get the sample point by the design of the experiment, and then create the Kriging surrogate model based on the sample points, and at last get the Pareto set by using multi-objective genetic algorithm method working on the established surrogate model to solve the multi-objective problem. The designer can choose the suitable Pareto solution to modify the structure. The method can save the design time and cost at the same time and can provide a reference for similar products.**

Keywords—Kriging surrogate model; multi-objective optimization; complicated structure; design of experiment; sample points; Pareto set

I. INTRODUCTION

Spacecraft will experience complex mechanics environment in the process of launching, and vibration is one of most important factors to be considered in the development phase of space products. For an aerospace assembly, the first integral vibration frequency is very important because it should be large than the rocket to avoid resonance. Also spacecraft is strict to quality characteristics because the cost can decrease 10000 dollars as the weight reduce 1 kilogram [1], and one of the most important factors that limit human to explore space is the weight. For these many reasons, we choose to use structural optimization technology to find the minimum mass and the maximum frequency of the first integral vibration at the same time [2].

Weighted variables method is often used to solve the multi-objective optimization problem, which give a coefficient for each objective by a human. But it has its disadvantages that different coefficient will get different optimization results. In order to get the Pareto set of relation value between min mass, and max the first integral vibration frequency ('frequency' for short), multi-objective genetic algorithm (MOGA) method [3] is used to solve the multi-objective problem. But the calculating amount is too large using MOGA, especially for complicated structure. Thus, we choose to use Kriging surrogate model to replace the original finite element model to reduce calculated amount. There are three steps to create an approximate model (including Kriging model). Firstly, get sample points from the design of experiment. Secondly, choose one approximate model to replace the above data. Thirdly, fit the model with the sample points [4]. The multi-objective optimization flow chart can be described as Fig. 1.

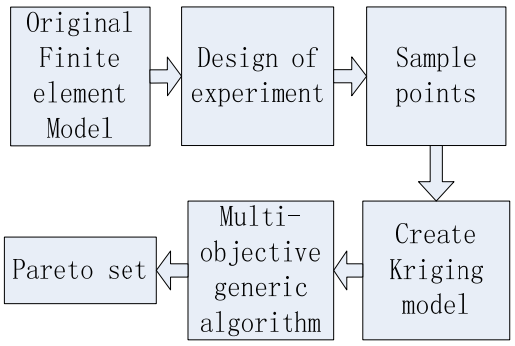

Fig. 1. Flow chart of multi-objective structural optimization based on Kriging model.

In this paper, we choose the experimental design to realize multi-objective structural optimization of an aerospace assembly, and two objectives are mass and the first integral vibration frequency. The experimental design is a good method in experimental design which can get the influence regularization between the objective and the variable just from fewer the experiment times. Based on the data, a Kriging surrogate model was created to equivalent the solver of the optimum problem. In the final, the Pareto optimal solutions can be found by multi-objective genetic algorithm on the response surface. The designer can choose suitable Pareto solution to modify the structure.

II. PROBLEM DESCRIPTIONS

The aerospace assembly is including external framework and internal function products. As the internal function

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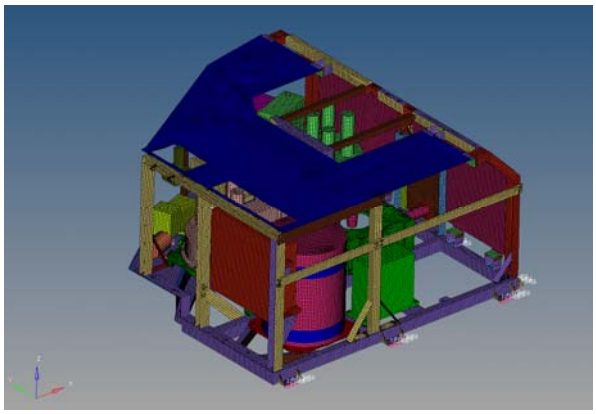
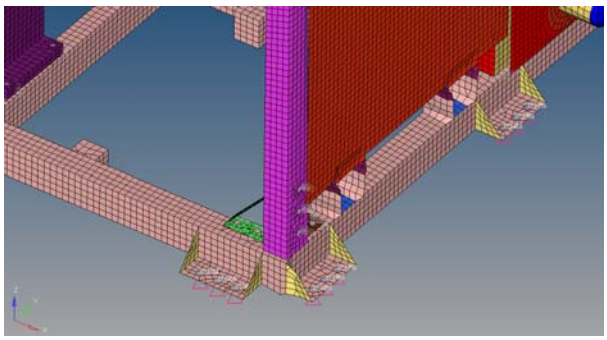
products are hard to modify, then the optimization concerns on the external framework which is consist of thin-walled beams. So the finite model of the frame is divided by two dimension grids while the internal function products are divided by three dimension grids. The finite element model of the aerospace assembly is Fig. 2.



(a) Mesh generation



(b) Setting of boundary

Fig. 2. Finite element model of the aerospace assembly.

To get a better look at the objective before the optimization, we can get the first integral vibration frequency based on modal analysis [5], the results showed that the first integral vibration frequency of the aerospace assembly is 58.2Hz, and the first integral vibration feature is Fig. 2.

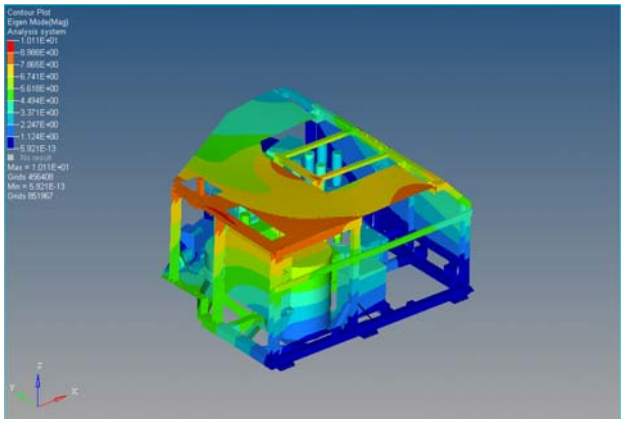


Fig. 3. First integral vibration feature of the aerospace assembly (58.2Hz).

For this issue, the variable for the optimum problem is size variable of the frame. By analysis of all probable size variables, finally we choose 10 variables which can be changed of the aerospace assembly. The objective for the optimum problem is two: mass and the first integral vibration frequency. The constraints for the optimum problem are the maximum and minimum limit of size variable [6]. The problem can be described as (1) [7].

$$\begin{cases} \text{find} & \{x_i\} \\ \text{Min} & (\text{Mass}) \\ \text{Max} & (\text{Frequency}) \\ \text{s.t.} & x_i^{min} \le x_i \le x_i^{max} \\ & i=1,2,...10 \end{cases} \tag{1}$$

In considering of the requirement of manufacture and other factors, the probable size variable value can be described as Table I. Table I give the initial value, the maximum and minimum limit of size variable.

TABLE I. INITIAL VALUE, THE MAXIMUM AND MINIMUM LIMIT OF SIZE VARIABLE

Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Initial value/mm	8	4.4	4.3	9	10	3	6	4	5	7
Max limit/mm	10	5	5	12	15	4	8	5	6	5
Min limit /mm	6	3	3	6	5	2	4	3	4	9

III. DESIGN OF EXPERIMENT

The design of experiment (DOE) is a statistics method which is concentrating on researching the influence of input variables on the output response [8]. There is two objective of DOE in this paper, first to get which variable influence the response most and second to get the sample points to create the Kriging surrogate model in the next section. In this section, we first do the design of experimental to get the sample points, in other words, the data of responses (mass and frequency) as the size variable selects a different value.

If one variable has three levels, then 10 variable have experimental design times, which is two larger to calculate for the common computer. So we choose Fractional factorial design method to finish the optimum problem. Finally, we choose L27, which mean the experimental design times is 27. In every experimental design, one group data of variable will import to the finite element model and by computing of FEA software the response of mass and frequency will be achieved. By analysis of final results, the interpolation sensitivity of response versus variable can be achieved as Table II and can be described as Fig. 4.

TABLE II. INTERPOLATION SENSITIVITY OF RESPONSE VERSUS VARIABLE

Project	dv_1	dv_2	dv_3	dv_4	dv_5	dv_6	dv_7	dv_8	dv_9	dv_10
mass	0.19	0.03	0.02	0.09	0.20	0.24	0.24	0.42	1.39	0.05
frequency	0.51	-0.01	0.01	-0.02	0.21	0.58	0.17	1.83	5.23	-0.01

From the data of Table II, we can see the mass response will increase as the variable increase, but the frequency is not always changing in this way. Because the stiffness it may not increase as much as the mass change. So the interpolation sensitivity of frequency may be negative. Besides, the data shows that dv_9 influence most both mass and frequency, in the second place is dv_8, dv_6, dv_1, dv_5 and dv_7, while the variable dv_2, dv_3, dv_4 and dv_10 value of influence on the response is relatively small.

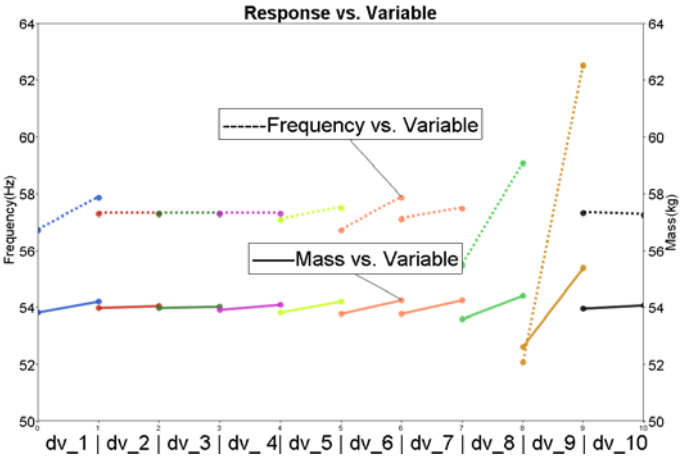


Fig. 4. Effect of each design variable on the response.

IV. Kriging Surrogate Model

In this section, we first do the orthogonal experimental design to get the data of responses (mass and frequency) as the size variable selects different value. Based on the data, we can create a Kriging surrogate to equivalent the solver of the optimum problem to reduce calculated quantity [9, 10]. Based on the response surface, we can solve the optimum problem, and finally get the Pareto front and the Pareto set.

The calculation amount of complicated problem is too large and sometimes is not necessary. Thus, people choose to use the approximate model instead of the initial model to solve the optimization problem. The common used approximate model includes Kriging method and response surface method. Unlike other methods smoothing the data, Kriging model goes through response value of the sample points.

The kriging method is an equivalent method using Kriging surrogate model replacing the initial solver. From the data of experimental design, the equivalent Kriging surrogate model can be created, and the optimization of the assembly can be carried out based on the Kriging surrogate model.

Besides, the reliability of the surrogate model should be evaluated based on extra sample points, one of the parameter is R^2 described as (2). R^2 is by definition less than 1.0 and closer to 1.0, the larger the R^2, the better the approximation is. Fig. 5 is the frequency response between Kriging model and original model. Table II is the error parameter of Kriging surrogate model.

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{2}$$

TABLE III. Error parameter of Kriging surrogate model

Parameter	Criterion	Input Matrix	validation Matrix	Merged Matrix
1	Mass R-Square	1.0000000	1.0000000	1.0000000
2	Frequency R-Square	1.0000000	0.9990806	1.0000000
3	Number of samples	27	16	43

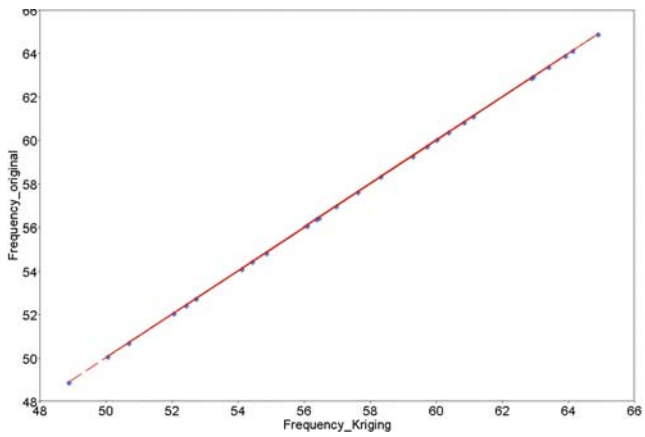


Fig. 5. Frequency response between Kriging model and original model.

V. Multi-Objective Structural Optimization of Complicated Structure

In the previous section, we can get the Kriging surrogate model, and in this section, we can use the surrogate model to do multi-objective structural optimization of the aerospace assembly. MOGA (multi-objective genetic algorithm) is used to solve the problem. The iteration is set as 100, and finally we can get the Pareto set of the multi-objective optimization problem. The initial value of the assembly is frequency= 58.2Hz and mass=54.05kg as point A. From the Pareto set, we can see if frequency keeps invariant, the mass can drop 0.64kg (from 54.05kg to 53.41kg) as point B, and if mass keep invariant, the frequency can increase 2.1Hz (from 58.2Hz to 60.3Hz). And the designer can choose suitable Pareto solution as finial solution.

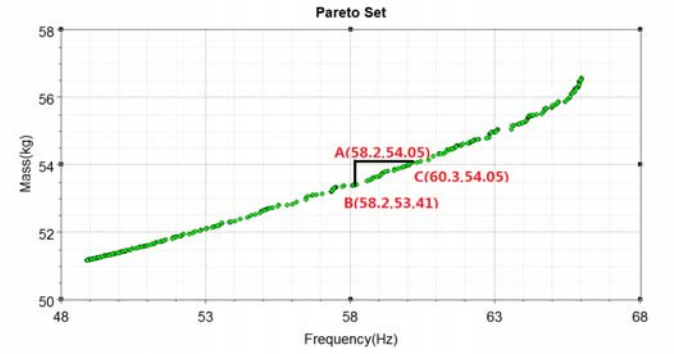


Fig. 6. Pareto set of multi-objective optimization problem.

VI. CONCLUSIONS

In this paper, we first get the finite model of the aerospace complicated structure and by modal analysis we get the first integral vibration frequency and feature. Then we do the experimental design to get the data of responses (mass and frequency) as the size variable changes. Based on the data, we create a Kriging surrogate model to equivalent the solver of the optimum problem. Finally, we get the Pareto set by using MOGA on the surrogate model. The case proves the effectiveness and practical applicability of the given method.

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