Discussion on Electromagnetic Waves Under the Circumstances of Total Internal Reflection and Optical Tunneling

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Abstract. In the case of total internal reflection, the electromagnetic incident waves will be reflected back to the original medium. Whereas, based on the theory of electromagnetic fields, the incident waves do not wholly go back to the original medium, but generates refracted waves in the second medium, which causes optical tunneling on some electrons. This paper discusses the traveling of electromagnetic waves, distribution of energy and the optical tunneling on some electrons under the circumstance of total internal reflection.

Introduction

The phenomenon of total internal reflection is widely applied to the researches on the electromagnetic waveguide. In the fields of fiber optics and integrated optics, which are branches of contemporary optics, total internal reflection is also used to transmit optical energy. Thus, researches on electromagnetic wave under the circumstance of total internal reflection are significant.

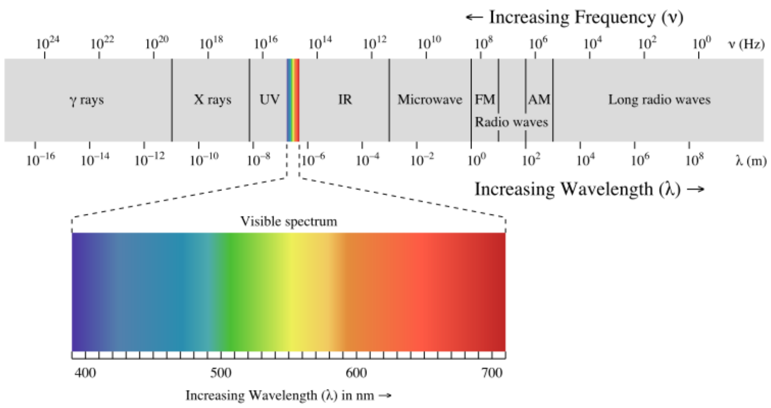
Electromagnetic Wave

In electromagnetism, electromagnetic wave equation is the second-order differential equation to describe the electromagnetic waves transmitted in medium or vacuum. The sources of electromagnetic waves are localized time-dependent density and electric current density. When the wave source is zero, the equation is a second-order homogeneous differential one. With E representing the electric field and B representing the magnetic field, the equation can be expressed as

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E} = 0$$
$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{B} = 0$$

Thereinto, ∇^2 is a laplace operator, c is the speed the electromagnetic waves transmitted in vacuum or medium, and t is the time.

As the optical waves equal the electromagnetic waves, c is the transmission speed of optical waves, which is also called the speed of light. In a vacuum, $c = c_0 = 299,792,458$ (meter/second) represents the transmission speed of electromagnetic waves in free space.



Stated in A Dynamical Theory of the Electromagnetic Field written by James Clerk Maxwell in 1864, displacement current and other established electromagnetic equations were combined, and the wave equation describing electromagnetic waves was obtained. The inspiring thing was that the wavy light speed described in the equation equaled the speed of optical waves. Due to the linear characters of Maxwell's equations in a vacuum, the solution can be decomposed into a set of sinusoidal waves, which can form the original solution when overlapping together. It is considered as the fundamental concept for solving differential equations with Fourier transformation. The sinusoidal wave solution form of the electromagnetic wave equation is:

$$E(r,t) = E_0 \cos(\omega t - k \cdot r + \phi_0) ,$$

$$B(r,t) = B_0 \cos(\omega t - k \cdot r + \phi_0) .$$

The relation between wave vector and angular frequency is:

$$k = |k| = \frac{\omega}{c} = \frac{2\pi}{\lambda} ;$$

Thereinto, λ is the wave length.

According to the wave length, starting from the long waves, electromagnetic waves can be categorized into electric energy, radio wave, microwave, infrared ray, visible light, ultraviolet ray, X-ray, gamma ray and etc. Spectrographs utilized in ordinary experiments are competent to analyze electromagnetic waves with lengths ranging from 2 nanometers to 2500 nanometers. With the apparatus, detailed physical properties of objects, gases or even fixed stars can be acquired. And the apparatus is considered as necessary. For example, hydrogen atoms are able to emit radio waves with the length of 21.12 centimeters.

When the electromagnetic wave frequency is low, the transmission mainly relies on tangible conductors. It is because that in low-frequency electric oscillations, the inter-conversion between magnetism and electricity is relatively slow and nearly all the energy will be reflected to the original circuit rather than being radiated. When the electromagnetic wave frequency is high, transmission within the free space or in tangible conductors can be realized. Transmission within the free space is due to the speedy inter-conversion between magnetism and electricity in the high-frequency electric oscillation, which keeps energy from wholly going back to the original oscillating circuit. Then electric energy and magnetic energy can be transmitted to the space in the form of electromagnetic wave as the periodic variation of electric field and magnetic field. The energy transmission can be realized without medium and it can be called radiation. For instance, the distance between the sun and the earth is quite remote, but we can still feel the light and heat of the sun in the open air. Similarly, "electromagnetic radiation delivers energy through radiation" enjoys the same principle.

Electromagnetic waves belong to the transverse waves. The magnetic field, electric field and advancing direction of electromagnetic waves are mutually perpendicular. The amplitude alternates periodically along the vertical direction of the transmission and its strength is inversely proportional to the squared distance. The waves carry energy and the energy power at any position is directly proportional to the squared amplitude.

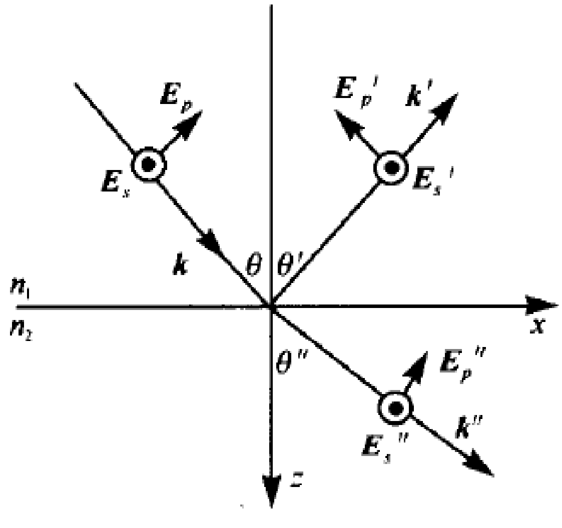
The speed equals the light speed C (3×10^8m/s). Electromagnetic waves transmitted in the space are in the same direction of the electric field (magnetic field) intensity. The distance between the two points with the maximum magnitude is the length of the electromagnetic wave λ. The number of times the electromagnetism changes every second is f. The relation among the three factors can be expressed through the formula c= λ f.

The transmission of electromagnetic waves does not require medium and the transmission speed of electromagnetic waves with the same frequency varies in different media. When electromagnetic waves with different frequency are transmitting in the same medium, the ones with higher frequency and refraction travel at lower speeds. Only in the homogeneous medium can electromagnetic waves be transmitted in straight lines. When the medium of the same category is uneven, the refractive indexes of electromagnetic waves differ and the transmission is curved. Refraction, reflection, diffraction, scattering and absorption will happen when the transmission is in different media. The transmission of electromagnetic waves cover the ground waves transmitting along the ground and air waves as well as sky waves transmitting in the air. The longer waves indicates less attenuation and they are able to transmit through obstacles more easily. Because of the fluctuation of all the waves, refraction, reflection, diffraction and interference can also happen in mechanical waves and electromagnetic waves. Diffraction, refraction, reflection, and interference are the forms of fluctuation.

Total Internal Reflection

As stated in Graph 1, when electromagnetic waves are radiated on the interface of two kinds of isotropic media, reflection and refraction take place. It is hypothesized that the incident waves are monochromatic and plane, then:

E = (E0 s + E0 p) exp j (ωt - k·r) (1)



Incident plane is xz, interface is xy

The forward direction of Es、E′ s and E″ s goes along the forward direction of y axis

Graph 1 Demonstration of electromagnetic wave transmission

Based on the electromagnetic field theory, when the interface is infinite and plane, the reflected waves and refracted waves of the electric field are considered as monochromatic and plane at the same frequency

E′= (E′0 s + E′0 p) exp j (ωt - k′·r) (2)

E″= (E″0 s + E″0 p) exp j (ωt - k″·r) (3)

In the formula (1), (2) and (3), k 、k′ and k″ are the transmission vectors of incident waves, reflected waves and refracted waves respectively.

Laws of reflection and refraction can be acquired based on boundary conditions of electromagnetic field: the angle of reflection meets:

θ′=θ (4)

The angle of refraction θ″ meets:

$$\frac{\sin\ \theta}{\sin\ \theta''} = \frac{k''}{k} = \frac{v_1}{v_2} = \frac{n_2}{n_1} \qquad (5)$$

Then the directions and the magnitude of k′ and k″ can be defined.

When n2 < n1 , make n = n1/ n2 > 1, then formula (5) can be expressed as nsin θ = sin θ ″ , θ ″ > θ , and a critical angle of incidence is necessary. $\theta c = \frac{\arcsin 1}{n}$,

When θ″=π/2 and θ>θc, sin θ ″ =nsin θ > 1 and there is no real solution for θ ″ , then total internal reflection will take place.

Transmission of Electromagnetic Waves Under the Circumstance of Total Internal Reflection

Under the circumstance of total internal reflection, it is can be proved that laws of reflection and refraction are still applicable. [1] Movement directions of reflected and refracted waves are still expressed with k’ and k’’ as stated in Graph 1.

E′ = (E′ 0 s + E′ 0 p) exp j [ω t - k′ (x sin θ - z cos θ)] (6)

E″ = (E″ 0 s + E″ 0 p) exp j [ω t - k″ (x sin θ ″ - z cos θ ″)] (7)

With θ′=θ, reflected and refracted waves can be expressed as

E′= (E′0 s + E′0 p) exp j [ωt - k′(x sin θ- z cosθ)] (6)

E″= (E″0 s + E″0 p) exp j [ωt - k″(x sin θ″- z cosθ″)] (7)

Under the circumstance of total internal reflection, sin θ″> 1,

then cos θ″= ± 1 - sin2θ″= ±j sin2θ″- 1 =±j n2 sin2θ- 1 (8)

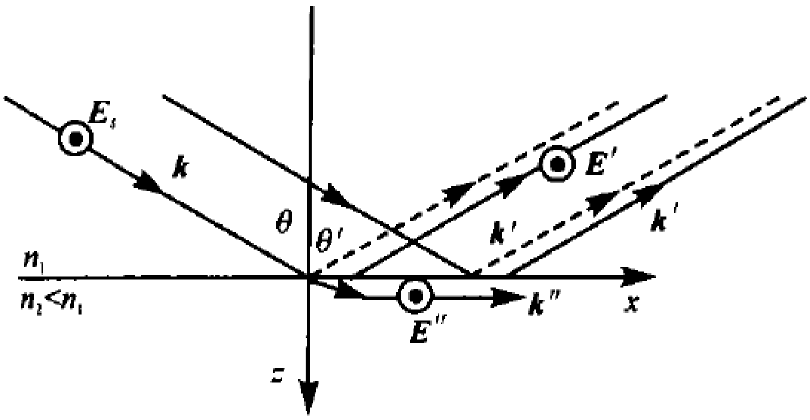
Considering the electric field intensity of z v + ∞ E″ must be finite, then the “+” in front of the square root in the above formula can be omitted, and

cos θ ″ = - j n2 sin2 θ - 1 (9)

Then the refracted wave is:

E″= (E″0 s + E″0 p) exp (- k″z n2 sin2θ- 1) (10)

This is a surface wave radiating along the direction of x with an amplitude in the direction of z and decreased indexes. Effective depth of entrance z is about the magnitude of wave length. Under the circumstance of total internal reflection, the incident wave does not reflect back to medium 1 from the interface as it penetrates into medium 2 and then “retraces” itself to medium 1. The advances of reflected and refracted waves are stated in Graph 2.



Graph 2 Transmission of the reflected and refracted waves of Es

Optical Tunneling

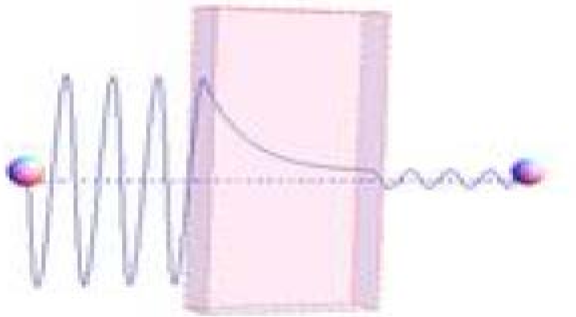
A component called “knot” can be formed through placing an insulating layer with a thickness of 0.1mm between two metal blocks (semiconductor or superconductor). It is hypothesized that the electron was in the left metal block and it was deemed as free with zero potential energy. As electrons cannot easily pass insulating layers, the later is just like barrier and is called potential barrier. The physical image of the right part can be demonstrated as follows: a potential barrier with a height of U0 and width of a, and on the right of the potential barrier is an electron with electron energy E.

The tunneling effect cannot be explained with viewpoints concerning classical mechanics. As the electron energy is less than the potential energy value of Zone II U0, when electrons enter into Zone II, the “negative kinetic energy” is inevitable. Whereas, from the perspective of quantum mechanics, electrons are fluctuant and their movement can be described with wave function, which follows Schrodinger's equation. Based on the solutions of Schrodinger's equation, probability densities of electrons in different zones can be acquired and then the probability of electrons passing through potential barriers are predictable. The probability decreases as the width of potential barrier increases. Therefore, the phenomenon is not observable in macro experiments.

In the quantum mechanics, quantum tunneling effect is a kind of quantum character indicating the phenomenon that electrons or other microscopic particles are capable of passing through the “walls” which they could not traverse originally. This is because that based on quantum mechanics, microscopic particles are fluctuant with non-zero probability of passing through potential barriers.

Quantum tunneling effect belongs to evanescent wave-coupling effect and its quantum behavior follows Schrodinger's equation. Any wave equations will show evanescent wave-coupling effect under appropriate conditions. Wave-coupling effect, mathematically equal to Quantum tunneling effect, also happens under other circumstances. For instance, optical wave or microwave following Maxwell's equations; string wave or acoustic wave following the common dispersive wave equation.

To trigger off the tunneling effect, one thin zone of Type II medium is required to be clamped between two zones of Type I medium. Real valued exponential function solution (rising or decreasing exponential function) is essential to the wave equation of Type II medium, while wave solution must be permitted to solve wave equation of Type I medium . In optics, Type I medium may be glass while Type II medium may be vacuum. In quantum mechanics, from the perspective of particle movement, the zone of Type I medium refers to those whose total particle energy is higher than its potential energy; while in the zone of Type II medium, the total particle energy is less and the zone is called potential barrier.



The diagram of quantum tunneling effect. Energy of tunneling particles remains the same while quantum amplitude decreases, thus the probability of finding particles is decreased.

Under appropriate conditions, when penetrating into the zone of Type II medium from the zone of Type I medium, the amplitude of the traveling wave will pass through the zone of Type II medium and then show up in the zone of Type I medium in the form of traveling wave. In quantum mechanics, the amplitude capable of penetrating can be physically explained as moving particles. Following Schrodinger's equation, the ratio of the squared absolute value of penetrating amplitude and the squared absolute value of incident amplitude tells the transmission coefficient of particle tunneling, which is the transmission probability. For optical waves, microwaves, string waves and acoustic waves following other wave equations, penetrating amplitude can be physically explained as moving energy, and the ratio of the squared absolute value of penetrating amplitude and the squared absolute value of incident amplitude indicates the ratio of the penetrating energy and the incident energy.

Electromagnetic Wave Energy Under the Total Internal Reflection

Under the circumstance of total internal reflection, only the formula (9) needs to be taken into consideration. Fresnel formula is workable and can be compiled as follows:

$$= n\cos\theta + j\left(\sqrt{n\ \sin^2(\ 1)}\right) = \overline{\qquad\qquad} \qquad (11)$$

$$= \cos\theta + j\left(\sqrt{n^2\sin^2(\ 1)}\right) = \overline{\qquad\qquad} \qquad (12)$$

The formula (11) and formula (12) can be simplified as:

E′0 s = E0 sejδ′s (13)

E′0 p = E0 pe jδ′p (14)

Thereinto:

$$\tan\frac{\delta'_s}{2} = \frac{\sin\ \delta'_s}{1+\cos\ \delta'_s} = \frac{\sqrt{n^2\sin^2\ \theta - 1}}{n\ \cos\ \theta} \qquad (15)$$

$$\tan\frac{\delta'_p}{2} = \frac{n\ \sqrt{n^2\sin^2\ \theta - 1}}{\cos\ \theta} \qquad (16)$$

Based on the electromagnetic wave theory, the average energy flux density of electromagnetic wave per unit area radiated on the interface between the two kinds of media:

$$\langle s \rangle = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\, n_1\, E_0^2 \cos\ \theta$$

Taking no account of absorption, scattering and other forms of energy loss, the average energy flux density per unit area reflected on the interface:

$$\langle s' \rangle = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\, n_1\, E_0'^2 \cos\ \theta' =$$

$$\frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\, n_1 (E_{0s}'^2 + E_{0p}'^2) \cos\ \theta' = \langle s \rangle \qquad (17)$$

Then the average energy flow radiated on the interface can be wholly reflected back to media.

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