Characterizing Instantaneous Difference between Received Powers at Two Ends of a Bidirectional Optical Wireless Communication Link through Atmospheric Turbulence

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Abstract. Numerical simulations are used to investigate the statistical properties of the instantaneous difference in the received powers between the two ends of a bidirectional optical wireless communication (OWC) link through atmospheric turbulence. The probability density estimates of the instantaneous difference between the received powers at the two link ends are obtained with various receiver aperture sizes being considered. It is found that asymmetry of turbulence profiles with respect to the path midpoint may incur obvious difference in the received powers between the two link ends. When atmospheric turbulence is distributed uniformly over the propagation path, the probability of the received powers at the two link ends being the same is nontrivial; however, it gets lower as the turbulence becomes stronger. A large enough receiver aperture may result in negligible difference between the received powers at the two link ends.

Introduction

In recent years, long-distance optical wireless communications (OWC) have been under intensive study in the communications community. The reason for this phenomenon is that OWC has certain advantages over the conventional radio frequency communications, e.g., no frequency regulatory restrictions, small antenna, high security, and so on [1]. However, the performance of OWC links is susceptible to atmospheric turbulence, which is ubiquitous in the earth's atmosphere. In fact, when a light wave propagates in atmospheric turbulence, both the amplitude and phase of the wave will fluctuate randomly [2]. This nature further gives rise to fluctuations in the received light power of OWC systems and hence degrades their performance. So far, researchers have proposed various types of methods to mitigate the deleterious effects of atmospheric turbulence on OWC systems, for instance, spatial diversity, time delayed diversity, adaptive optics, code-rate-adaptive transmission, among others [2-4]. However, up to now, the turbulence-induced performance degradation of OWC systems is still a problem that is required to be further addressed. To use code-rate-adaptive transmission to reduce the turbulence-induced negative effects on OWC systems, the transmitter needs to obtain the channel state information [4]; traditionally, a low data-rate feedback channel is employed to send the channel state information detected by the receiver to the transmitter [4,5]. Nevertheless, time delay caused by the feedback operation may make the channel state information arriving at the transmitter stale, and consequently weaken the performance improvement that the use of rate-adaptive transmission may result in. Very recently, several researchers found that fluctuations in the received light powers at the two ends of a bidirectional OWC link through atmospheric turbulence may manifest significant correlation [5,6]. This property has been referred to as the channel intensity reciprocity in the literature [6], and can be employed to implement code-rate-adaptive transmission in bidirectional OWC links of which the adaptive transceivers directly obtain the instantaneous channel state information according to their received light powers. On the other hand, it should be noted that the instantaneous difference between the received light powers at the two ends of a bidirectional OWC link through atmospheric turbulence plays a critical role in code-rate-adaptive OWC systems whose transceivers get the instantaneous channel state information based on channel intensity reciprocity. As a result, it is worth characterizing the instantaneous difference between turbulence-induced fluctuations in the light powers received by finite-size apertures at the two ends of a bidirectional OWC link.

In this paper, by considering a bidirectional OWC link through atmospheric turbulence, we use numerical simulations to investigate the statistical properties of the instantaneous difference between fluctuations in the received light powers at the two ends. For a bidirectional OWC link, there are two light waves that propagate in opposite directions along a common path in atmospheric turbulence. This kind of propagation geometry is referred to as counter propagation in the literature [7]. Here, the optical-wave propagation in atmospheric turbulence is numerically simulated by making use of the split-step beam propagation techniques based on multiple phase screens (MPS) [8]. According to the simulation results, the probability density estimates of the instantaneous difference between the received light powers at the two link ends will be calculated and the effects of characteristic turbulence profiles on the probability density will be analyzed in detail.

Aperture y

Formulations for Instantaneous Received-Power Difference between Two Link Ends

Fig. 1. Schematic diagram of beam-wave counter propagation in atmospheric turbulence

Figure 1 illustrates the geometry of beam-wave counter propagation under consideration in this paper. Two Gaussian beams propagate in opposite directions from the aperture of one transceiver along a common path to the aperture of the other transceiver. We calculate the statistical properties of the instantaneous difference between the light power collected by the left aperture and that collected by the right aperture. The basic idea used to attain this goal is as follows: first by employing the MPS-based split-step beam propagation method, we numerically generate numerous realizations of both the forward- and inverse-propagation beam-wave fields at the two receiver planes; second, we calculate the difference between the powers collected by the two apertures for each of these realizations, and then carry out statistical analysis of the difference. In this work, we primarily concentrate our attention on the effects of turbulence profiles and aperture size on statistical properties of the instantaneous difference in the received powers at the two link ends. As a result, for simplicity, two monochromatic counter-propagating beam waves with the same wavelength will be considered in the simulations.

At this point, we elucidate how to calculate the instantaneous difference between the received powers at the two link ends. To begin, we first assume that the two counter-propagating Gaussian beams have unit on-axis amplitude at their respective transmitter planes. This assumption actually does not incur any loss of generality. In accordance with the formulations given by Andrews and Phillips [2], for the forward-propagation beam, the field in the receiver plane at z = L can be formally expressed as follows: $U_F(\mathbf{r}, z = L) = U^{(0)}(\mathbf{r})\exp[\psi_F(\mathbf{r}, z = L)]$, where \mathbf{r} denotes a point in the plane at z = L, L is the separation distance between the two transceivers, $\psi_F(\cdot)$ is the turbulence-induced complex phase perturbation of the beam field, $U^{(0)}(\cdot)$ represents the field of a Gaussian beam after propagating a distance L in a vacuum. The expression for $U^{(0)}(\cdot)$ is given by [2]

$$U^{(0)}(\mathbf{r}) = \frac{1}{1 + i\alpha_0 L} \exp\left[ikL + \frac{ik}{2} \left(\frac{i\alpha_0}{1 + i\alpha_0 L}\right)r^2\right],\tag{1}$$

where $k = 2\pi/\lambda$ is the optical wavenumber with λ being the wavelength, $\alpha_0 = 2/(kw_0^2) + i/R_0$ with w_0 representing the initial beam radius and R_0 the initial phase front radius of curvature. Similarly, the field of the inverse-propagation beam in the receiver plane at z = 0 takes the form: $U_I(\mathbf{s}, z = 0) = U^{(0)}(\mathbf{s})\exp[\psi_I(\mathbf{s}, z = 0)]$, where **s** denotes a point in the plane at z = 0, $\psi_I(\cdot)$ is the turbulence-induced complex phase perturbation of the beam field. The realizations of both $U_F(\mathbf{r}, z = L)$ and $U_I(\mathbf{s}, z = 0)$ are calculated based on the MPS-based split-step beam propagation method, which in essence belongs to Monte Carlo simulations. The light power collected by the receiver aperture at z = L can be formulated by

$$P_{F} = \iint_{\infty}^{\infty} U_{F} \left(\mathbf{r}, z = L \right) U_{F}^{*} \left(\mathbf{r}, z = L \right) H \left(\mathbf{r} \right) d^{2} \mathbf{r} , \qquad (2)$$

where the asterisk denotes complex conjugate, $H(\mathbf{r})$ is equal to 1 if $|\mathbf{r}| \le D/2$ and 0 otherwise, *D* is the diameter of the receiver aperture. Similarly, the expression for the light power collected by the receiver aperture at z = 0 can be written by

$$P_{I} = \int_{-\infty}^{\infty} U_{I}(\mathbf{s}, z = 0) U_{I}^{*}(\mathbf{s}, z = 0) H(\mathbf{s}) d^{2}\mathbf{s}.$$
(3)

To characterize the instantaneous difference in the received light powers between the two link ends, we define the relative deviation of P_F from P_I as $\delta_F = (P_F - P_I)/P_F$, and the relative deviation of P_I from P_F as $\delta_I = (P_I - P_F)/P_I$. In the next section, the two quantities δ_F and δ_I will be used to describe the relative difference between the received light powers, computed according to the numerical simulation results, at the two link ends.

Simulation Results and Analysis

The phase screens used in numerical simulations are generated based on the sparse spectrum (SS) model [9]. In all of the simulations, the outer and inner scales of turbulence are specified as 10 m and 2.1 mm, respectively; the parameters of both the forward- and inverse-propagation Gaussian beams at their respective transmitter planes are given as follows: $\lambda = 800$ nm, $w_0 = 2$ cm, and $R_0 = \infty$; the propagation distance *L* is a fixed value of 5 km.





To begin, we consider the case of simplified asymmetrical turbulence profiles with respective to the path midpoint, in which it is assumed that turbulent cells are concentrated in a thin layer located somewhere between the two link ends. A single phase screen is used to model the thin layer of atmospheric turbulence. The coherence radius for the band-limited power-law spectral density [9] based on which the phase screen is generated is 5.38 cm. Figure 2 shows the probability density estimates of δ_F and δ_I associated with different receiver aperture sizes, where the phase screen is positioned at z = 0.5 km for Fig. 2(a) and at z = 1.5 km for Fig. 2(b). The quantity $w_L = w_0[(1-L/R_0)^2+4L^2/(k^2w_0^4)]^{1/2}$ denotes the beam radius at the receiver plane in the absence of

turbulence [2]. It can be found from Fig. 2 that there are great probabilities that the instantaneous difference in the received light powers between the two link ends is nontrivial, implying that the intensity reciprocity of the channel is poor. The underlying physical reason for this can be elucidated as follows: the impact of turbulent cells on a propagated beam depends on their location on the path; turbulent cells closer to the transmitter are more likely to make contribution to beam wander, and however those closer to the receiver are more likely to make the cross-section spot of the beam break up into small patches. When a phase screen is positioned at z = 0.5 km, realizations of the irradiance distribution of the two counter-propagating Gaussian beams in the receiver planes are shown by Fig. 3. It is noted that the phase screen used to produce Fig. 3 is closer to the transmitter for the forward-propagation beam and is closer to the receiver for the inverse-propagation beam. It is apparent that the dominate effect that atmospheric turbulence has on the forward-propagation beam is beam wander, and that on the inverse-propagation beam is breakup of the cross-section spot. Moreover, it is seen from Fig. 2 that a larger receiver aperture leads to a greater peak value of the probability density when other parameters are fixed. By comparing Figs. 2(a) and 2(b), one finds that the intensity reciprocity of the channel becomes better when the phase screen moves to a position closer to the midpoint of the path.



Fig. 3. Instantaneous irradiance distribution in receiver planes of two counter-propagating Gaussian beams in the case of a single phase screen being positioned at z = 0.5 km. (a) the forward-propagation beam; (b) the inverse-propagation beam.

Now we investigate the probability density of δ_F and δ_I in the case that atmospheric turbulence is distributed uniformly over the propagation path, in which the turbulence profile is symmetric with respect to the midpoint of the path. Figure 4 shows the probability density estimates of δ_F and δ_I associated with various receiver aperture sizes, where the plane-wave Rytov variance σ_l^2 [2] is used to describe the strength of atmospheric turbulence. Compared with the results shown by Fig. 2, it is observed from Fig. 4 that although both δ_F and δ_I are likely to be nonzero, there are also large probabilities that these two quantities are equal to zero, meaning that the instantaneous state of received power fluctuations at one transceiver can be used to characterize that of received power fluctuations at the other transceiver. One can also find from Fig. 4 that the probability of $\delta_F = \delta_I = 0$ decreases as the turbulence strength increases. Moreover, as far as the simulation examples presented here are concerned, when the receiver aperture size becomes relatively large, the probability of $\delta_F = \delta_I = 0$ grows relatively great. Indeed, if the receiver aperture size is large enough, most of the energy carried by the beams can be collected, and the turbulence-induced fluctuations in the received powers become trivial, leading to the increase in the probability of $\delta_F = \delta_I = 0$. This can be exemplified by Fig. 5, which demonstrates the instantaneous irradiance distribution in receiver planes of two counter-propagating Gaussian beams in the case that atmospheric turbulence is distributed uniformly over the path. From Fig. 5, one can see that there are multiple separated

irradiance patches in the beam cross section. If the receiver aperture is so large that it can collect all these patches, the aperture averaging of wave scintillations is very great and hence reduces the received power fluctuations significantly. In the limiting case that the received powers no longer fluctuate, it is apparent that $\delta_F = \delta_I = 0$.



Fig. 4. The probability density estimates of δ_F and δ_I with different receiver aperture sizes in the case that atmospheric turbulence is distributed uniformly over the path; w_L denotes the beam radius at the receiver plane in the absence of turbulence. (a) $\sigma_l^2 = 0.398$; (b) $\sigma_l^2 = 2.182$.



Fig. 5. Instantaneous irradiance distribution in receiver planes of two counter-propagating Gaussian beams in the case that atmospheric turbulence is distributed uniformly over the path; $\sigma_l^2 = 2.182$. (a) the forward-propagation beam; (b) the inverse-propagation beam.

Conclusion

In this work, we have investigated the instantaneous difference in the received powers at the two ends of a bidirectional OWC link. The MPS-based split-step beam propagation method has been used to simulate the realizations of the fields of two counter-propagating Gaussian beams at their respective receiver planes. Based on the simulation results, the probability density estimates of the instantaneous relative difference between the received powers at the two link ends have been obtained by considering different receiver aperture sizes. It has been found that asymmetry of turbulence profiles with respect to the path midpoint can incur obvious instantaneous difference between the received powers at the two link ends. On the other hand, in the case that atmospheric turbulence is distributed uniformly over the path, the probability that the instantaneous received powers at the two link ends are the same is nontrivial; however, stronger turbulence leads to a lower value of this probability. When the receiver aperture is large enough, the instantaneous difference between the received powers at the two link ends will become negligible.

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