Analysis of Cargo Sway Characteristics of Tower Cranes

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Abstract. To study the cargo motion and sway characteristics of tower cranes to find effective methods to suppress the cargo sway and to position the cargo, we modeled the cargo sway, and then analyzed the equilibrium states of the cargo sway. The research results show that the center line of the cargo pendulum is declining, just as the cargo oscillates in the inclined gravity field because of there are inertia forces and the centrifugal forces.

Introduction

Tower cranes have 3 kinds of basic working condition that are the transition, the rotation and the hoisting movement. Rotary motion causes centrifugal force and inertia force in the varying direction, so that the analysis of the dynamic characteristics is more complex. In this paper, the dynamic characteristics of the tower cranes at the working condition of the transition, the rotation, and the hoisting movement at the same time. The nonlinear models of the cargo sway were set in the non-inertia reference system and the equilibrium states of the cargo are calculated. The models are linear near the equilibrium states and the cargo sway characteristics are simulated [1-3].

Non-Linear Models of Cargo Sway

Tower cranes transport cargoes by rotation of crane jib and trolley motion along the jib. According to the motion characteristics, we set up a polar coordinate system $\{e_\rho, e_\psi\}$ whose coordinate origin is located at the intersection of the rotary center line of the tower body and the rotary surface of the crane jib. The inertia force, centrifugal force, and external disturbance causes the cargoes to oscillate in space pendulum and hoisting motion changes the length of the pendulum. Based on this, we set up spherical coordinate system $\{e_l, e_\theta, e_\phi\}$ whose coordinate origin located at the suspension point of the hoisting cable and the suspension point moves with the trolley and rotates synchronously with the jib[4,5]. The coordinate system is shown in Fig. 1.

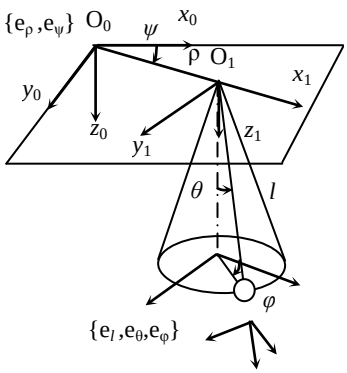


Fig.1 Tower crane coordinate system

The payload suspension point is located at (ρ, ψ) in the polar coordinates, where ρ and ψ respectively denote the displacement of the trolley and the rotation angle of the jib. The position of the payload in the non-inertia spherical coordinate system is described by three generalized coordinates (l, θ, φ).

Analysis of the External Forces Acting on the Cargo. The external forces acting on the cargo include the gravity *G* and hoisting cable tension force *T*, where the gravity *G* is decomposed as $F_{g\theta}$ and $F_{g\phi}$ along e_θ direction and e_ϕ direction and tension force *T* along the direction of hoisting cable not affects the movement of the cargo in e_θ direction and e_φ direction. $F_{g\theta}$ and $F_{g\phi}$ are following.

$$F_{g\theta} = -mg\sin\theta . \tag{1}$$

$$F_{g\phi} = 0 . \tag{2}$$

Analysis of the Inertia Forces. When the trolley moves along the jib, the consultant force of the inertial force and the air resistance (the air resistant coefficient is *f*) is F_ρ, which is respectively decomposed as $F_{\theta\rho}$ and $F_{\phi\rho}$ along e_θ direction and e_φ direction.

$$F_{\theta\rho} = -(m\ddot{\rho} + f\dot{\rho})\cos\theta\cos\phi . \tag{3}$$

$$F_{\phi\rho} = (m\ddot{\rho} + f\dot{\rho})\sin\phi . \tag{4}$$

The convected inertial force, centrifugal force, and the air resistance caused by jib rotaion can be decomposed as $F_{\theta\psi}$ and $F_{\phi\psi}$ respectively along the e_θ and e_φ direction. Due to the amplitude of cargo sway with respect to the gyration radius of the cargo suspension point is much smaller during calculating the inertia force, centrifugal force and air resistance caused by the rotary motion, the gyration radius in e_θ direction can be neglected.

$$F_{\theta\psi} = -(m\rho\ddot{\psi} + f\rho\dot{\psi})\cos\theta\sin\phi + m\rho\dot{\psi}^2\cos\theta\cos\phi \tag{5}$$

$$F_{\phi\psi} = -(m\rho\ddot{\psi} + f\rho\dot{\psi})\cos\phi - m\rho\dot{\psi}^2\sin\phi \tag{6}$$

Analysis of Coriolis Forces. Transition and rotary motion produce Coriolis force that is decomposed as $F_{\theta\rho\psi}$ and $F_{\phi\rho\psi}$ respectively along e_θ direction and e_φ direction. They are

$$F_{\theta\rho\psi} = 2m\dot{\rho}\dot{\psi}\cos\theta\sin\phi . \tag{7}$$

$$F_{\phi\rho\psi} = 2m\dot{\rho}\dot{\psi}\cos\phi . \tag{8}$$

Coriolis force $F_{l\theta} = 2m\dot{l}\dot{\theta}$, along e_θ direction, results from the hoisting motion and the sway of the cargo in e_θ direction. The Coriolis force $F_{l\phi}$ induced from the hoisting movement and the oscillation in e_φ direction is $F_{l\phi} = 2m\dot{l}\dot{\phi}\sin\theta$. The oscillations in e_θ direction and e_φ direction produce Coriolis force $F_{\theta\phi}$ in e_φ direction, $F_{\theta\phi} = 2ml\dot{\theta}\dot{\phi}\cos\theta$.

Effect of Wind Loads. Given the wind speed in the o*x* direction and o*y* direction are respectively u_x and u_y, the wind resistance coefficient is *f*, the wind loads are f_{ux} and f_{uy}. The two loads are decomposed as $F_{\theta u}$ and $F_{\phi u}$ along the e_θ direction and e_φ direction.

$$F_{\theta u} = [fu_x\cos(\phi+\psi) + fu_y\sin(\phi+\psi)]\cos\theta . \tag{9}$$

$$F_{\phi u} = -fu_x\sin(\phi+\psi) + fu_y\cos(\phi+\psi) . \tag{10}$$

The circular motion of the cargo in e_φ direction produces centrifugal force F_ϕ whose component force is $F_\phi = ml\dot{\phi}^2\sin\theta\cos\theta$ along the e_θ direction. The sway of Cargo in e_θ direction causes the centrifugal force along the direction of hoisting cable.

The air resistance of the cargo movement in e_θ direction, $F_{f\theta} = -fl\dot{\theta}$, the air resistance of the cargo movement in e_φ direction $F_{f\phi} = -fl\sin\theta\dot{\phi}$. The inertial force from hoisting motion along the hoisting cable direction does not have an impact on the movements in e_θ direction and e_φ direction. According to the relationship of the oscillating torque balance in e_θ direction and in e_φ direction in the non-inertia reference system, the dynamic equations of cargo sway are modeled as following.

$$\begin{aligned} ml^2\ddot{\theta} = \sum M = &-mgl\sin\theta + ml^2\dot{\phi}^2\sin\theta\cos\theta - (ml\ddot{\rho} + fl\dot{\rho})\cos\theta\cos\phi \\ &-(m\rho l\ddot{\psi} + f\rho l\dot{\psi})\cos\theta\sin\phi + m\rho l\dot{\psi}^2\cos\theta\cos\phi - 2ml\dot{l}\dot{\theta} - 2ml\dot{\rho}\dot{\psi}\cos\theta\sin\phi . \\ &-fl^2\dot{\theta} + [fu_x\cos(\phi+\psi) + fu_y\sin(\phi+\psi)]l\cos\theta \end{aligned} \tag{11}$$

$$
\begin{aligned}
ml^2\sin^2\theta\ddot{\phi} = \sum M &= -2ml^2\dot{\theta}\dot{\phi}\sin\theta\cos\theta + (ml\ddot{\rho} + fl\dot{\rho})\sin\theta\sin\phi \\
&-(m\rho l\ddot{\psi} + f\rho l\dot{\psi})\sin\theta\cos\phi - m\rho l\dot{\psi}^2\sin\theta\sin\phi - 2ml\dot{l}\dot{\phi}\sin^2\theta - 2ml\dot{\rho}\dot{\psi}\sin\theta\cos\phi \\
&- fl^2\dot{\phi}\sin^2\theta + [-fu_x\sin(\phi+\psi) + fu_y\cos(\phi+\psi)]l\sin\theta
\end{aligned} \tag{12}
$$

The Eq. (1) and Eq. (2) are the models of cargo sway under the condition of the tower crane carrying out the transition, rotation, and hoisting motion at the same time.

Linear Models of Cargo Sway

The equilibrium states of the tower crane are achieved by ordering $\dot{\theta}(t)$=0, $\ddot{\theta}(t)$=0, $\dot{\phi}(t)$=0, $\ddot{\phi}(t)$=0. The linear models are obtained by inputting a small disturb in the vicinity of the system equilibrium states, ignoring the high order terms, and keeping the first order term.

$$\phi_e = \tan^{-1}\left(-\frac{\rho\ddot{\psi} + \dot{\rho}\dot{\psi}}{\rho\dot{\psi}^2 - \ddot{\rho}}\right). \tag{13}$$

$$\theta_e = \tan^{-1}\frac{a}{g}. \tag{14}$$

Where, $\sin\phi_e = -\dfrac{\rho\ddot{\psi} + \dot{\rho}\dot{\psi}}{a}$, $\cos\phi_e = \dfrac{\rho\dot{\psi}^2 - \ddot{\rho}}{a}$, $a = \sqrt{(\rho\ddot{\psi} + \dot{\rho}\dot{\psi})^2 + (\rho\dot{\psi}^2 - \ddot{\rho})^2}$.

The equilibrium state θ_e expresses the angle between the hoisting cable and the plumb line through the hanging point of the cargo and φ_e expresses the angle between the hoisting cable and the plane where the plumb line exists. θ_e is as a result of the interaction of inertia force, centrifugal force and Coriolis force. The magnitude of φ_e is the ratio of the consultant force of rotary inertia force and Coriolis force over the consultant force of rotating centrifugal force and inertial force. Due to the joint action of the inertia force, the centrifugal force and the Coriolis force, so that oscillating center line of the cargo deflects, just as the cargo does space pendulum in inclined gravity field as shown as in Fig. 2.

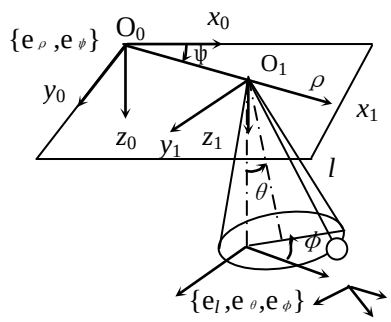


Fig. 2 Cargo sway when oscillation center line inclines

Linear models in the vicinity of the system equilibrium states are following.

$$l\delta\ddot{\theta} + 2\dot{l}\delta\dot{\theta} + \sqrt{g^2 + a^2}\,\delta\theta = -\cos\theta_e(\cos\phi_e\delta\ddot{\rho} + \sin\phi_e\delta\ddot{\psi}). \tag{15}$$

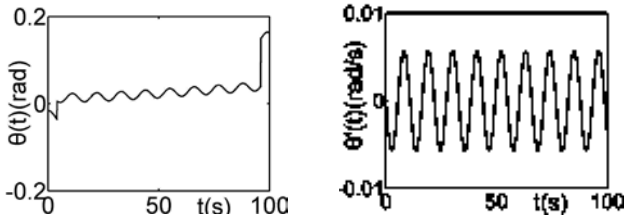
$$l\delta\ddot{\phi} + 2\dot{l}\delta\dot{\phi} + \sqrt{g^2 + a^2}\,\delta\phi = (\sin\phi_e\delta\ddot{\rho} - \rho\cos\phi_e\delta\ddot{\psi})/\sin\theta_e. \tag{16}$$

Eq. (15) and Eq. (16) are second-order differential equation about the pendulum angle θ and φ. The frequency of the sway is decided by the ratio of the synthetic acceleration over the hoisting cable length. And the sway amplitude is relative with transition inertial force and rotary inertia force. So oscillating frequency and amplitude can be controlled by the motion acceleration of the mechanisms.

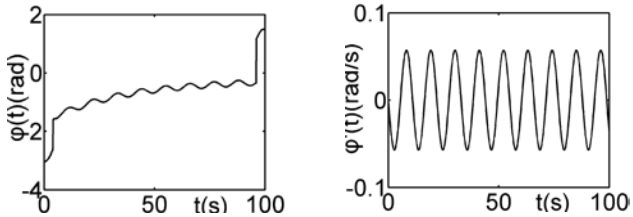
System Simulation

The cargo sway characteristics are carried out by the simulation test for the QTZ135tower crane controlled by frequency converter. Given the following conditions, the cargo mass m=2000kg, the rated motion speed of the trolley v=1m/s, the rated rotary speed of the rotary mechanism ω=0.0268rad/s, the rated hoisting speed of hoisting mechanism is 1m/s. Speed up for 4s based on the trapezoidal speed curve before reaching the rated speed, run for 92 s at a constant speed, decelerate

for 4s before stopping. The system initial states are 0. Fig. 3 shows the cargo sway angular displacement and velocity response curve when the crane transports and rotates at the same time and its oscillation amplitude and period is constant. Along with the transportation carrying on, the centrifugal force increases gradually, $\theta(t)$ deviates from the equilibrium position, the oscillating center line inclines and is no longer along the vertical direction.



(a) Cargo sway angle $\theta(t)$ (b) Cargo sway angular velocity $\dot{\theta}(t)$



(c) Cargo sway angle $\phi(t)$ (d) Cargo sway angular velocity $\dot{\phi}(t)$

Fig. 3 The cargo sway angle and angular velocity

Conclusion

In this paper, the nonlinear models of cargo sway are set up when the tower crane translates, rotates, and hoists and the models are linear near the equilibrium states to reveal that the characteristics of the cargo sway. Due to the inertia force or the centrifugal force, the center line of the cargo sway is tilted, just as the cargo does a space pendulum motion in the tilted gravity field whose sway model is the linear model and the deflection of the sway center line is the balance state values.

Acknowledgements

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