

Approach to Arbitrary Transportation of suspended particles Based on Ultrasonic composite Field

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Abstract: A particle suspension transport method was proposed based on ultrasonic composite field. Firstly, the array focusing model and accumulated force equation were established based on phased array ultrasonic cells delay, then integrated with the standing wave ultrasonic field radiation force calculation method, the particle motion control model and driving force formul under the combined ultrasound field were introduced. Through the MATLAB simulation of complex ultrasound field, the factors that affected the focusing performance and control performance were analysed. Finally, arbitrary control scheme of particle motion was put forward by compound ultrasonic driving field.

1. Introduction

The inner machining technology based on ultrasonic transport the machining energy or tool into the body, and directly to machine the internal structure. In our team, we used to put forward to machine the internal structure in the non-transparent material that was ultrasonic standing wave technology^[1,8], and here are several advantages comparing to laser technology, the non-transparent material can be machined^[2,9,10,11], furthermore, the several energy effects can be used also, but the problem was that the machining energy or the particle can move only between the standing wave node, cannot move arbitrarily in a sequential space, it is difficult to meet the control precision requirement. In this study, the array focusing model and accumulated force equation were established based on phased array ultrasonic cells delay, then integrated with the standing wave ultrasonic field radiation force calculation method, the particle motion control model and driving force formul under the combined ultrasound field were introduced.

2. the principle of phased array and the study of focused ultrasound field

Here we introduced ultrasonic array phased focus method^[3], derived the compound sound pressure formula.

2.1 the principle of ultrasonic phased array focus

The model of ultrasonic phased array focusing model is shown in Fig.1, if the number of array elements is n , and suppose the sound pressure of i -th element in point p is P_i , then we can derive the gross pressure as:

$$P = \sum_{i=1}^n P_i$$

Due to the distance between each array element to focus point is different, we can control the focal signal depending on the delay time, so all the signal can reach the same focus point simultaneously, similar to the principle of concave spherical transducer, as the study of the concave spherical transducer, the focus depth can be shown as:

$$F_e = 8.16\lambda \left(\frac{F}{D} \right)^2 = \frac{8.16\lambda F^2}{D^2}, 2F/D > 1 \quad (1)$$

Where λ is the wave length; F is focal; D is the width of the transducer. focus can be achieved if

the phase difference is zero, the time difference and phase difference can be expressed as:

$$\Delta\varphi = 2\pi f_0 \times \Delta t$$

2.2 the expression of delay time of phased array

Phased array focusing is achieved through the controlling of the delay time of phased array^[10,12], the relationship can be obtained from the geometric relationship, it can be shown in Fig.2

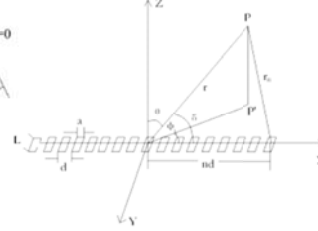
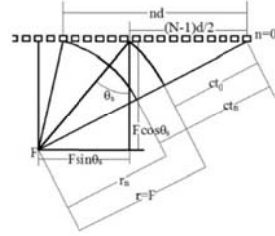
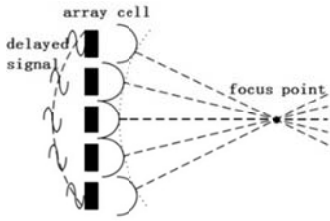


Fig.1 phased array focal model

Fig.2 phased array delay time geometry

Fig.3 schematic of sound pressure

$$\text{It derive that: } (F \cos \theta_s)^2 + \left[F \sin \theta_s - \left(nd - \frac{N-1}{2} d \right) \right]^2 = [F - (t_n - t_0) c]^2$$

here, N is amount of array element; $n = 0, 1, 2, 3, \dots, N-1$; d is the center space between the array cell; D is the center distance of adjective array element.

$$\text{then: } t_n = \frac{F}{c} \left\{ 1 - \left[1 + \left(\frac{d}{F} \left(n - \frac{N-1}{2} \right) \right)^2 - 2 \sin \theta_s \frac{d}{F} \left(n - \frac{N-1}{2} \right) \right]^{1/2} \right\} + t_0$$

the delay time between the n th and $(n-1)$ cell is: $\Delta\tau_n = t_n - t_{n-1}$

$$\text{It can be simplified to: } \Delta\tau_n = \Delta\tau_0 + \frac{c\Delta\tau_0^2}{2F \tan^2 \theta_s} (N - 2n)$$

$$\text{here: } \Delta\tau_0 = \frac{d \sin \theta_s}{c}$$

2.3 computing model of sound pressure of ultrasonic phased array focusing

The phased array sound pressure is built as Fig.3. The distribution of sound pressure of per array can be derived as below^[4]:

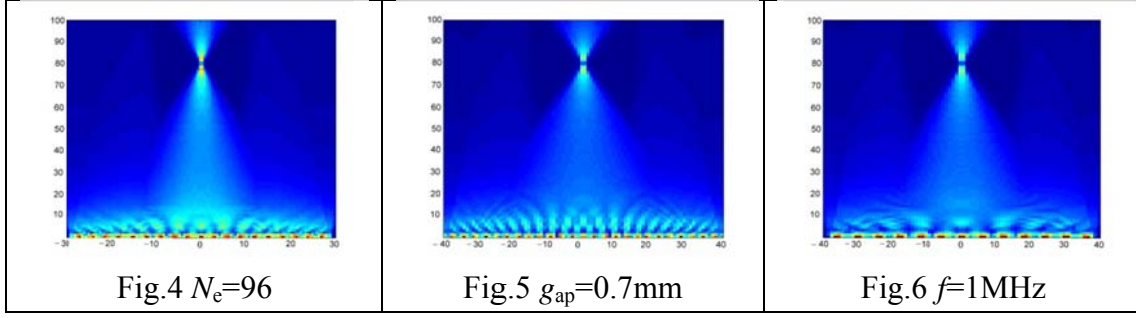
$$p = \frac{jkc u \rho_0}{2\pi r} aL \frac{\sin\left(\frac{kL \sin \theta \sin \phi}{2}\right)}{kL \sin \theta \sin \phi} \frac{\sin\left(\frac{ka \sin \theta \cos \phi}{2}\right)}{ka \sin \theta \cos \phi} e^{j(\omega t - kr)}$$

the superposition sum pressure of all the array cell is written as:

$$p_A = \sum_{n=0}^{n=N-1} p_n = \sum_{n=0}^{n=N-1} \frac{jkc u \rho_0}{2\pi r} aL \frac{\sin\left(\frac{kL \sin \theta \sin \phi}{2}\right)}{kL \sin \theta \sin \phi} \frac{\sin\left(\frac{ka \sin \theta \cos \phi}{2}\right)}{ka \sin \theta \cos \phi} e^{j(\omega t_n - kr_n)} \quad (2)$$

2.4 the simulation of focus performance of ultrasonic phased array

In order to find the influence of per parameter acting on the sound pressure of phased focal array, some parameter should be determined, such as: the original dimension of array element (3mm × 0.7mm), the quantity of array element ($N_e=64$), the spacing between array element ($g_{ap}=0.5$ mm), the initial emitted frequency ($f=0.8$ MHz), the focal depth was 80mm. the quantity of simulation array element was set to $N_e=32, 64$, and 96; the spacing was set to $g_{ap}=0.3$ mm, 0.5mm, and 0.7mm; the frequency was set to 0.6 MHz, 0.8 MHz and 1MHz, the sound field in different parameter is shown in Fig.4 ($N_e=96$), Fig.5 ($g_{ap}=0.7$ mm), Fig.6 ($f=1$ MHz).



the above simulation result can derive the following conclusion: Increasing the number of array elements, decreasing the array element spacing and the higher transmitter frequency can improve the performance of the phased focus.

3. driving method of ultrasonic composite field

3.1 ultrasonic focal radiated force

we can assume the single array element as a point sound source, the spherule suffered from radiation force in the sound field, the force direction went along radial-direction, it is shown in Fig. 7. According to the superposed theory^[4], the force acting on the spherule was the vector sum of each radial force^[5,6].

3.1.1 point-p on the axis-z

We constructed the force relationship as Fig. 8, we assume that the centre distance of array element was d , the number of array element was n , the spherule located at point-p, the radius of spherule was R_p , the plane coordinate centres o (the center of the phased array). we analysed and calculated the force acting on the point-p which is respectively on the axis-z and arbitrary position of plane-xoz.

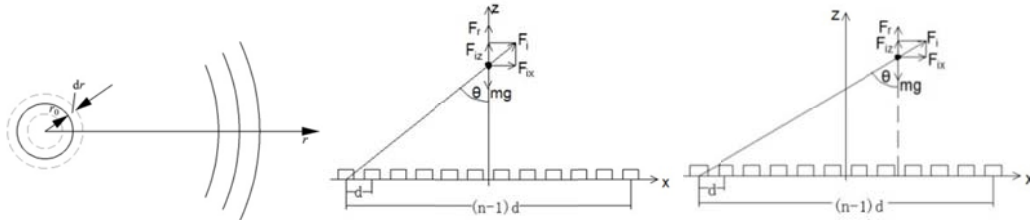


Fig. 7 point sound source Fig. 8 force on axis-z Fig. 9 force on non axis-z

According to the Fig. 9, the gravity and the levitation force generated by standing wave are balance in the vertical direction, the levitation force which is generated by array element, it can be divided into two sub-force (axis-z and axis-x force), the resultant goes along the vertical direction, and the vertical force can be written as:

$$F_{iz} = \frac{b}{\sqrt{L_i^2 + b^2}} F_i$$

Here: $F_i = 2\pi\rho_0 |A|^2 (kR_p)^6 \frac{9 + (1 - \lambda_p)}{9(2 + \lambda_p)^2}$; $L_i = \left| -\frac{(N-1)}{2}d + (i-1)d \right|$; $\lambda_p = \rho_0 / \rho_p$;

Where b is coordinate value on z ; A is acoustic wave amplitude; R_p is the radius of particle

the driving force derived by array can be written as:

$$F_{pz} = \sum_{i=1}^N F_{iz} = \sum_{i=1}^N \frac{b}{\sqrt{L_i^2 + b^2}} F_i = \sum_{i=1}^N \frac{b}{\sqrt{L_i^2 + b^2}} 2\pi\rho_0 |A|^2 (kR_p)^6 \frac{9 + (1 - \lambda_p)}{9(2 + \lambda_p)^2} \quad (3)$$

3.1.2 point-P on the z-axis

When the spherule is in the xoz plane but outside of the Z-axis, certainly, the force relationship has been analysed based on the force diagram in Fig. 6. the force acted on the spherule can be divided into axis-z and axis-x force, and the axis-z force supplied the driving force of right transportation, it can be written as:

$$F_{ix} = \frac{L_i}{\sqrt{L_i^2 + b^2}} F_i$$

here:

$$F_i = 2\pi\rho_0 |A|^2 (kR_p)^6 \frac{9 + (1 - \lambda_p)}{9(2 + \lambda_p)^2}, \quad L_i = a + \frac{(N-1)}{2}d - (i-1)d$$

The right driving force can be written similarly as:

$$F_{px} = \sum_{i=1}^M F_{ix} = \sum_{i=1}^M \frac{L_i}{\sqrt{L_i^2 + b^2}} F_i = \sum_{i=1}^M \frac{L_i}{\sqrt{L_i^2 + b^2}} 2\pi\rho_0 |A|^2 (kR_p)^6 \frac{9 + (1 - \lambda_p)}{9(2 + \lambda_p)^2} \quad (4)$$

where M is the quantity of array element; we defined that only the left array element work; we can derive that:

$$a + \frac{(N-1)}{2}d - (i-1)d \geq 0 \quad (5)$$

Then we can get: $i \leq \frac{a}{d} + \frac{N+1}{2}$, where $M = \lfloor \frac{a}{d} + \frac{N+1}{2} \rfloor$, that is the upper limit. where, $\lfloor \cdot \rfloor$ is shown the maximum integer which does not exceed the number.

Similarly, we can derive the axis-z driving force as below:

$$F_{pz} = \sum_{i=1}^M F_{iz} = \sum_{i=1}^M \frac{b}{\sqrt{L_i^2 + b^2}} F_i = \sum_{i=1}^M \frac{b}{\sqrt{L_i^2 + b^2}} 2\pi\rho_0 |A|^2 (kR_p)^6 \frac{9 + (1 - \lambda_p)}{9(2 + \lambda_p)^2}$$

If the spherule suffered the right side driving force that generated by right side array element, similarly, we can get the left side driving force.

3.2 the caculation of standing waveradiation force of particle

Combining the expression of the sound pressrue with the condition of boundary, we can obtain the expression of radiation force. KING^[6], Yosioka and Kawasima^[7] have demonstrated the theory of sound radiation force, and the expression can be written as:

$$F_r = - \left(\frac{\pi p_0^2 V_p \beta_m}{2 \lambda} \right) \phi(\beta, \rho) \sin(2kx); \quad (6)$$

$$\phi(\beta, \rho) = \frac{5\rho_p - 2\rho_m}{2\rho_p + \rho_m} - \frac{\beta_p}{\beta_m}$$

Where, V_p is the volume of spherule, ρ_p is density of spherule, ρ_m is the density of medium, β_p is the compressible coefficient of spherule, β_m is the compressible coefficient of medium, and the compressible coefficient is related with the speed of wave velocity of medium, it can be written as:

$$= \beta / \rho c^2$$

3.3 physical model of ultrasonic composite field

the array element of high-frequency ultrasonic standing wave was embedded on the end-surface of ultrasonic amplitude amplifier of low-frequency (in the Fig.10), this setup formed a ultrasonic composite field (UCF), and the radiation force on which acted on the spacial particle in the UCF is the vector sum of standing wave and phased array element, that is:

$$F_c = F_r + F_p \quad (7)$$

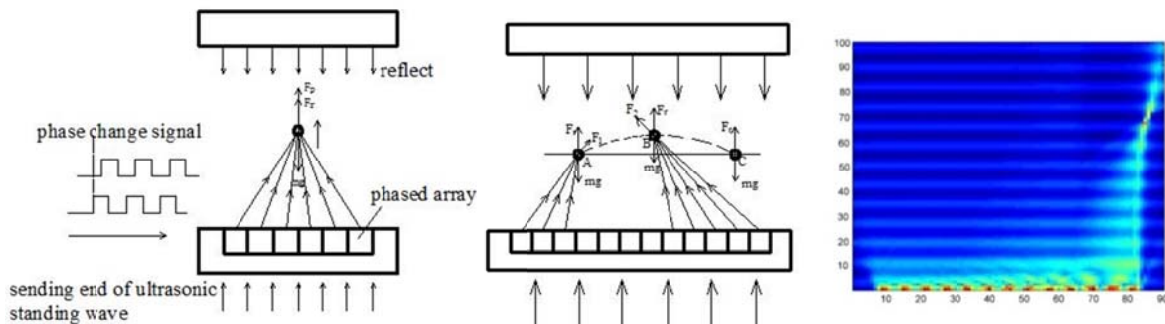


Fig.10 vertical motion Fig.11 horizontal motion Fig.12 horizontal direction driving

4. study of particle transportation in the UCF

4.1 particle transportation in vertical direction

We can adjust the phase and frequency of the standing wave to control the particle motion. we can adjust the parameter of the standing wave and the phased array as well.

4.2 particle transportation in horizontal direction

the array element located in the opposite direction of particle motion are triggered, according to the above study (in the Fig. 10) the particle suffered the force from the vertical and horizontal direction. the spherule will keep dynamic stable in the vertical direction due to the restoring force will balance the standing wave force. then the horizontal force is the main driving force, the superposed horizontal force generated by parts of the array element will drive the spherule motion, it is shown in the below Fig. 11. we can adjust the delay time to control the particle motion in the UCF, it is shown in the Fig. 12, the simulation demonstrates that the controllable sound pressure can be achieved near the focal point, and this can adjust dynamically the force situation acting on the spherule.

5. conclusion

This paper introduced the UCF theory, the simulation result proved that the implementation of a wide range of space suspension transportation is workable. the arbitrary focus characteristic of phased array will balance the restoring force, establish dynamic balance expression and achieve levitation driving of particle, the driving method can achieve arbitrary suspension transportation, and the shift of displacement was obtained by adjusting the delay time, the UCF technology will simplify the controlling algorithm, improve the controlling accuracy of transportation, provide greater flexibility and handling for the inner machining technology, the next work we will construct the experiment system and analyze the phased delay arithmetic^[13].

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