Study on the Daily Exchange Rate Movement Based on the Model of Brownian Motion

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Abstract. Theory of Brownian motion is often used to describe the random phenomena of chaos and disorder. This paper tries to make an empirical study on the price of foreign exchange by using theory of Brownian motion. First of all, we use the theory to establish a model to describe the behavior of foreign currency prices.Then the data of the Euro against the US dollar, the Australian dollar against the US dollar and Gold against the US dollar closing price per hour is introduced to simulate the exchange rate by using the model. Finally, we will use the simulated price and the actual price to do a comparative analysis, including the error range, motion direction and inflection point. It was found that the simulation and the actual exchange rate very similar, the results show that this model can be used to forecast.

Introduction

With the rapid development of the foreign exchange market, more and more investors enter into the spot market for speculation. Facing the numerous and complicated movements in the exchange rate, investors only by personal experience is difficult to accurately determine the exchange rate movements to make a correct decision buy or sale. In this paper the exchange rate was simulated by Brown motion model, in order to provide a more scientific basis for decision making for speculators.

Foreign Studies on the Brownian motion is earlier, and widely used in the study of stocks, futures, gold and foreign exchange.In recent years, most people choose to use fractal Brownian theory. K. Ivanova , M. Ausloos[1] used the data for six years to make a empirical analysis of exchange rate movements, it is proved that the exchange rate is more consistent with the multi fractal Brownian motion. S. V. Muniandy, S. C. Lim, R. Murugan[2] used multi fractal Brownian motion model to do a empirical research on the influence of government's intervention policies on the exchange rate in the foreign exchange market in Malaysia. Most of the research on foreign exchange in China is about the structure of foreign exchange investment, investment objects and investment strategies. As for the analysis of the exchange rate trend, Tan Weiping[3] forecasted the trend through technology analysis. Zhang Yu[4] studied four major exchange rates of China foreign exchange market, found that the exchange rate time series is nonlinear. In the nonlinear system, Liu Xing Quan, Wang Keyu[5] use the fractional Brownian motion model to make empirical study about the US dollar against the yen in New York exchange market, found that estimates of H index values are in good agreement with the actual exchange rate fluctuations and trends.

Previous studies show that exchange rate is in line with the Brownian motion, the time span of data is always in a few years and most choices is the fractal Brownian motion model. As a global speculative market the foreign exchange market is affected by many factors, investors use day trading to prevent the loss caused by the incident of second day. Therefore, in order to avoid the shortcomings of previous studies in this field we select 24 hours of exchange rate which is more in line with the characteristics of speculative foreign exchange market and more instructive to speculators. Fractal Brownian motion model denies the efficient market hypothesis and developed on the basis of geometric Brownian motion.The research scope of this paper is the closing price per hour of a single day which will rapid and sensitive response to information on the exchange market on the day.It is in line with the efficient market hypothesis that the price reflects all the information

available. Therefore, this paper selects the geometric Brown motion model which can make this study more practical significance and application value.

The Model of Brownian Motion

The main factors of foreign exchange fluctuation are upward or downward trend and average volatility, The former depends on the length of time, the latter only depends on the random fluctuations caused by the movement of Brownian. We introduce a model describing the behavior of exchange rate:

$$ds_t=\mu s_t dt+\sigma s_t dw_t \quad (1)$$

s_t is the foreign exchange price for t, ds_t is the changes in exchange rate, It shall be composed of certain items (expectation) and volatility (random), μ is the average exchange rate(expected drift rate), σ is standard deviation (volatility), w_t is Standard Brownian motion.

At time intervals Δt, $\Delta w_t \sim N(0,\Delta t)$, $dw_t=\varepsilon\sqrt{dt}$,$\varepsilon \sim N(0,1)$.

According to Ito theorem:

$$d\ln s_t=(\mu-\frac{\sigma^2}{2})dt+\sigma dw_t \quad (2)$$

That is $d\ln s_t \sim N[(\mu-\frac{\sigma^2}{2})dt,\sigma^2 dt]$.

The discrete form of the stochastic differential equation can be expressed as:

$$\ln s_{t+\Delta t}-\ln s_t=(\mu-\frac{\sigma^2}{2})\Delta t+\sigma\varepsilon\sqrt{\Delta t} \quad (3)$$

By (2) and (3) can be calculated:

$$\mu=\frac{E(\ln s_{t+\Delta t}-\ln s_t)}{\Delta t}+\frac{\sigma^2}{2} \quad (4)$$

$$\sigma^2=\frac{Var(\ln s_{t+\Delta t}-\ln s_t)}{\Delta t} \quad (5)$$

Empirical Analysis

According to the different properties of foreign currencies, we select three currencies that are relatively stable and have different characteristics, including the safe haven currency and commodity currency. It can make data more representative.We chose closing prices per hour of the Euro against the US dollar, the Australian dollar against the U.S. dollar and Gold against the U.S. dollar of 24 hours in August 5, 2015. We take Δt is 1. According to historical data as well as formula (4), (5) calculate μ , σ, as table 1.

Table 1 Estimating Means and Standard Deviations of Three Currencies

	EUR / USD	AUD / USD	Gold / USD
μ	0.00016725985	-0.00010693263	-0.000035378392
σ	0.001057341	0.001116575	0.001212112

We randomly sampled$\varepsilon \sim N(0,1)$ through Excel, select 24 samples. According to the estimated mean and standard deviation of the various currencies, use the formula (3) to simulate the results, make a comparative analysis between the simulation and the real exchange rate about the maximum value, minimum value, mean, median, kurtosis, skewness and correlation. As table 2,3.

Table 2 Simulated Prices and Real Prices of Three Currency Pairs

Simulated Price1	Simulated Price2	Simulated Price3	Real Price1	Real Price2	Real Price3
1.085999998	0.735900000	1085.76000	1.0860	0.7359	1085.76
1.085029122	0.734996799	1084.40091	1.0859	0.7361	1086.66
1.084041940	0.734082424	1083.02385	1.0862	0.7356	1085.71
1.083665911	0.733605446	1082.34724	1.0867	0.7353	1086.46
1.084505319	0.733997354	1083.06229	1.0864	0.7354	1087.46
1.084208937	0.733577454	1082.47700	1.0872	0.7347	1087.61
1.085655894	0.734403047	1083.88697	1.0864	0.7362	1085.46
1.084426165	0.733316613	1082.23372	1.0855	0.7366	1085.50
1.085576931	0.733930238	1083.30420	1.0856	0.7359	1086.06
1.086333343	0.734262023	1083.92325	1.0869	0.7362	1087.31
1.087338451	0.734771052	1084.82649	1.0884	0.7370	1088.00
1.086381554	0.733880062	1083.48592	1.0897	0.7372	1089.36
1.084940632	0.732644381	1081.59286	1.0876	0.7347	1084.76
1.086040492	0.733220760	1082.60391	1.0867	0.7344	1084.01
1.087418831	0.733995298	1083.93284	1.0866	0.7342	1084.21
1.088423803	0.734503330	1084.83478	1.0872	0.7344	1085.83
1.091542525	0.736517121	1088.15170	1.0903	0.7353	1085.86

Simulated Price1	Simulated Price2	Simulated Price3	Real Price1	Real Price2	Real Price3
1.089245989	0.734672440	1085.28097	1.0898	0.7346	1084.58
1.088455730	0.733901416	1084.13202	1.0903	0.7354	1084.70
1.087711427	0.733163504	1083.03609	1.0903	0.7354	1084.49
1.088632758	0.733611245	1083.84152	1.0898	0.7345	1084.65
1.088599715	0.733379715	1083.55757	1.0900	0.7352	1085.16
1.087233629	0.732200192	1081.75310	1.0901	0.7348	1084.90
1.088299997	0.732750756	1082.72345	1.0911	0.7351	1084.10

Note：1 is EUR/USD, 2 is AUD/USD, 3 is Gold/USD.

Table 3 Results Statistical Table

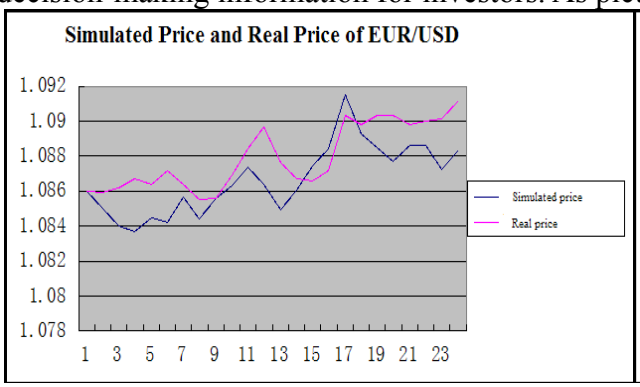
	Real Price1	Simulated Price1	Real Price2	Simulated Price2	Real Price3	Simulated Price3
Maximum Value	1.0911	1.0915	0.7372	0.7365	1089.36	1088.15
Minimum Value	1.0855	1.0837	0.7342	0.7322	1084.01	1081.59
Mean	1.0879	1.0867	0.7354	0.7340	1085.78	1083.67
Median	1.0872	1.0864	0.7354	0.7339	1085.60	1083.52
Kurtosis	-1.6102	-0.1056	-0.4299	1.2340	0.41161	2.91313
Skewness	0.3132	0.4599	0.5253	0.7541	0.90160	1.29517
Relevance	_	0.7524	_	0.2728	_	0.10128

Note： 1 is EUR/USD, 2 is AUD/USD, 3 is Gold/USD.

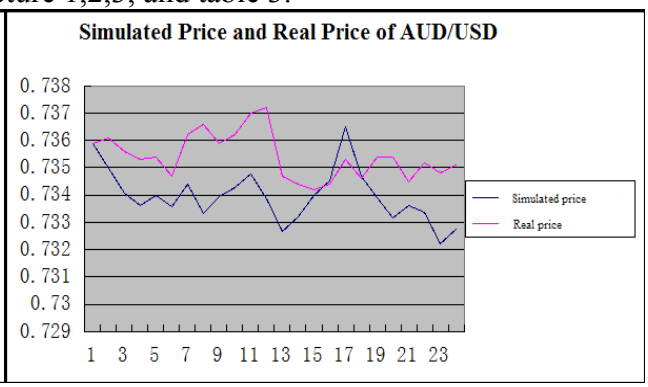
According to the comparison between simulated price and actual price about the maximum, minimum, mean and median, the simulated price of EUR/USD is very close to the actual price, even only 4/10000 of the gap and the gap about AUD/USD is only within 2/1000. As for Gold/USD because its marked prices are bigger than others, the estimated values show a large gap. The kurtosis of the simulated results of EUR/USD are relatively steady than real prices, the simulated results of AUD/USD and Gold/USD steeper than real prices. The real prices of three currency pairs skew to the right compare with normal distribution and the simulated results skew to the right compare with the real prices.The correlation of EUR/USD between simulation results and real prices is as high as 75%, the correlation of AUD/USD and Gold/USD between simulation results and real prices is relatively low, only 27% and 10%.

On the whole, simulation results based on Brownian motion are close to the actual prices of closing price in a single day. However, there is still a certain gap, reasons can be attributed to the following aspects: because the analysis basis on the data of the one-day exchange rate, a relatively small number may lead to the simulation results of some inaccuracy; the currency has its own particularity, a country's economic situation and policies have big impact on its currency, some complex factors can not be fully reflected in our model, in particular, both the gold and the U.S. dollar are the safe haven currency.

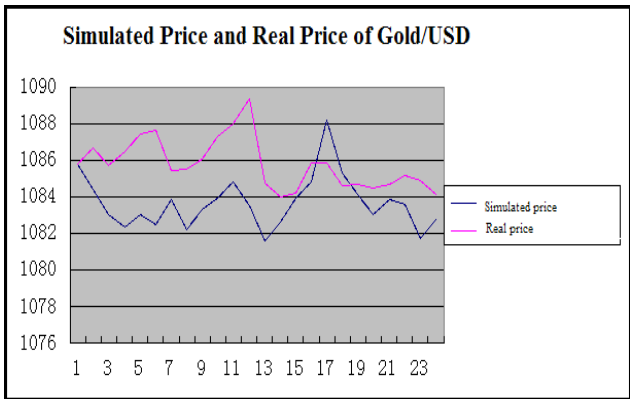
At the same time, the simulated curve based on Brownian motion can provide more decision-making information for investors. As picture 1,2,3, and table 3.



Picture 1 comparison of EUR/USD



Picture 2 comparison of AUD/USD



Picture 3 comparison of Gold/USD

Table 3 Peak and Valley of Simulated Price and Real Price

	Advance		Simultaneous		Lag	
	Valley	Peak	Valley	Peak	Valley	Peak
EUR/USD	8、13		22	1、18、21	4、6、20	5、7
AUD/USD	8、13	7、11、21、22	4、6、20、23	5、18		
Gold/USD	13、20	11		18	4	7

In the table, the advance or simultaneous peak and valley of EUR/USD are more than 60%, as for AUD/USD it reach 100% and Gold/USD also reach nearly 70%. It is obvious that the most of the peak and valley points are simultaneous even advance, there is a small part of the lag and the points of lag basically appear in the early stages of the simulation.

Therefore, we can combine the simulated prices with the peaks and valleys of simulation which is based on the Brown motion. First we should observe if there are advance or simultaneous points on the simulated curve, it is maybe a clear trading signal, while make sure the simulated price of the exchange rate to achieve which price, then according to the actual price estimate the risk to profit ratio and make a reasonable decision.

Conclusion

We simulated the exchange rates of three currency pairs of 24 hours according to the model of Brownian motion.Though a comparative analysis between the real price and the simulated price ,we find the two price very close.The method of the combination of peak or valley points with the prices has great guiding significance for speculators' trading.Comparing with the previous studies, we take into account the characteristics of the speculative foreign exchange market, select the closing price per hour of 24 hours as the sample and applied to the model of Brownian motion.It provide a objective analytical tool for the speculators to avoid that they can not get sufficient information.

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