Attitude Control of Rigid-Flexible Coupling Solar Sail

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Abstract. The main characteristics of solar sails are their large size and the extreme flexibility which have a prominent effect on the attitude control of the solar sail. Most existing researches on dynamic analysis and the control law design of solar sails are only based on the rigid dynamic models. In this paper, a rigid-flexible coupling model is proposed for describing dynamics of solar sails in consideration of the flexible characteristics of solar sails. Then an active robust technique is developed for attitude control of the new model. Simulation results demonstrate the stability and robustness of the solar sail overall control system. The final results of the developed dynamic model and the attitude control system of the solar sail spacecraft could be significant for a sail spaceflight experiment in future.

Introduction

The solar sail spacecraft is a novel conceptual space propellant system The outstanding advantages of solar sails such as incessant acceleration utilizing the sun photons and consumption of little fuel are of interest on the future applications of interstellar probe in solar system and beyond. Those applications include plenty of technical challenges, some of which are significantly associated with accurate dynamic modeling and optimal attitude maneuver control .

Wie[1] has given the dynamic models of solar sail configurations above respectively. Kolk and Flandro [2] have established the attitude dynamic model of solar sails by using the four elements of the quaternion formulation. The common point of these dynamic models is that they are approximately assumed to be the rigid body.

However, the main characteristics of solar sails are their large size and the extreme flexibility. For example, the data of ST7 solar sail show that the sail area is 1400 square meters, the areal density is 0.111 kilogram per square meter, and the sail film thickness is only 7.5 microns, where the extreme flexible characteristic can be found distinctly. During sailing in the space, the shape of the large sail film of solar sail will be changed as billow, then the vibrations of flexible modes are likely to be excited. Therefore the extreme flexibility of solar sail has a prominent effect on the attitude control of the solar sail in the space.

In this paper, we considered the solar sail as a rigid body coupled with flexible modes which is a more practical model to be developed. Accordingly, the robust $H\infty$ technique was employed to design the control law for the rigid- flexible coupling model of the solar sail.

A Rigid-Flexible Coupling Model For Attitude Control of the Solar Sail

We consider the practical dynamic model of the solar sail as a model of a rigid body coupled with the flexible modes. Then the hybrid coordinates dynamic equations[5] are employed to formulate the dynamic model, which combining Euler parameter coordinates with flexible mode coordinates. Thus we obtain the pitch axis equations of motion based on the rigid-flexible coupling model:

$$\left[J_s + \frac{m_s m_p b(b+l)}{m}\right]\ddot{\alpha} + \frac{m_s m_p bl}{m}\ddot{\delta} + \mathbf{p}^{\mathbf{T}}\ddot{\mathbf{q}} = -\frac{m_p b}{m}F_t - T_g \tag{1}$$

$$\left[J_p + \frac{m_s m_p l(b+l)}{m}\right]\ddot{\alpha} + \left[J_p + \frac{m_s m_p l^2}{m}\right]\ddot{\delta} + \mathbf{p}^{\mathbf{T}}\ddot{\mathbf{q}} = -\frac{m_p l}{m}F_t - \frac{m_p l}{m}F_n\delta + T_g \tag{2}$$

$$\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} + \mathbf{p}\ddot{\alpha} = 0 \tag{3}$$

where $\mathbf{p} = [p_1, p_2, \cdots, p_n]^T$ is the coefficient vector of rigid-flexible coupling, $\mathbf{q} = [q_1, q_2, \cdots, q_n]^T$ is the n-order flexible mode coordinates vector, $\mathbf{C} = 2 \cdot diag[\zeta_1 \cdot \omega_1, \zeta_2 \cdot \omega_2, \cdots, \zeta_n \cdot \omega_n]$ and $\mathbf{K} = diag[\omega_1^2, \omega_2^2, \cdots, \omega_n^2]$ are diagonal matrix with constant coefficients, $\mathbf{\Psi} = diag[\zeta_1, \zeta_2, \cdots, \zeta_n]$ is the damping coefficient diagonal matrix, $\mathbf{\Omega} = diag[\omega_1, \omega_2, \cdots, \omega_n]$ is the n-order flexible mode frequency matrix. (F_n, F_t) are the solar radiation pressure (SRP) force components.

Defining state variables $\mathbf{x} = [\dot{\alpha} \quad \alpha \quad \dot{\delta} \quad \delta \quad \dot{\mathbf{q}} \quad \mathbf{q}]^T$, then a linear state space equation for control design can be obtained as:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}T_g \tag{4}$$

Attitude Control Strategy

The standard description of a closed-loop system using the $H\infty$ control technique is illustrated as follows: G represents the augmented plant that to be controlled, and K represents the controller designed to stabilize the plant G. ω represents the input signals outside the system, such as the reference signal, the disturbance signal and sensor noises. z represents the controlled output signals, such as the tracking error signal, the regulating error signal and the actuating signal. u represents the control input signal comes from the controller K.

Then G can be divided and on the assumption that $(I - G_{22}K)$ is an invertible real rational matrix, then the transfer function matrix $T_{z\omega}$ from ω to z can be expressed as:

$$T_{z\omega} = F_l(G, K) = G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21} \tag{5}$$

Thus, the $H\infty$ control problem can be described as a mathematical issue, and the key point of solving the $H\infty$ control problem is equivalent to minimize the infinite norm of the transfer function from the disturbance input signal ω to the error output signal z, namely to minimize $\|T_{z\omega}\|_\infty$ by designing a real rational stabilizing controller K.

Based on the robust $H\infty$ control technique, we obtain the overall closed loop control system, in which W_u is the weighting function selected to penalize the controller output signal u in order to avoid the output saturation of the controller, and $W_{e_\theta}(s)$ is the weighting function selected to penalize the error of the attitude angle θ. $W_{e_i}(s) \quad (i = 1, 2, \ldots, n)$ are the weighting functions selected to penalize the i-order flexible mode output e_i respectively.

As we consider both the outputs of the attitude angle and the n-order flexible modes as the inputs of the robust $H\infty$ controller, the controller is a system with multiple inputs and single output. Therefore, the designed control system is used to perform the optimal control objective – tracking and achieving the optimal attitude angle asymptotically and suppressing the flexible vibrations robustly.

Then we can also obtain the optimal control objective as follows:

$$\min J = \min_{C_\theta(s), C_1(s), \cdots, C_n(s)} \sup \left\{ \left\| W_{e_\theta} (r - \theta) \right\|_2^2 + \|W_u u\|_2^2 + \left\| W_{e_1} e_1 \right\|_2^2 + \cdots + \left\| W_{e_n} e_n \right\|_2^2 \right\} \tag{6}$$

Simulation Results

We employed the parameters according as the solar sail configuration for Space Technology 7 proposed by Jet Propulsion Laboratory [1]. The first and second order flexible modes of the solar sail are considered for simplification [6] with the finite element analysis method. We selected the sun angle of 35 degree which can provide the effective thrust control of solar sails in sun-centered orbits [3]. The following figures are given to show the simulation results of the overall control system, where the weighting functions W_u =0.01, $W_{e\theta} = 12 \times (s+5)/(s+0.05)$, W_{e_1} =10, W_{e_2} =20 are selected to penalize the control signal and the error signals respectively.

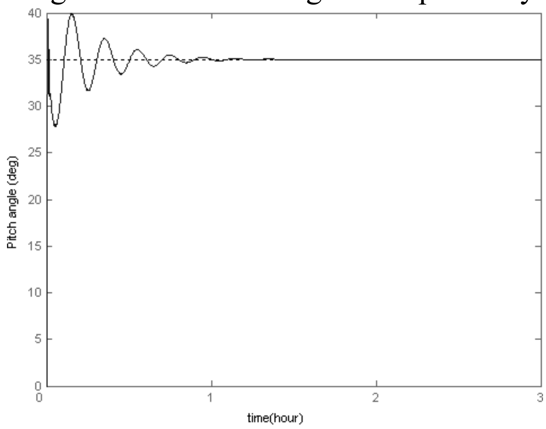

Fig. 1 The response of the pitch angle tracking to the optimal sun angle

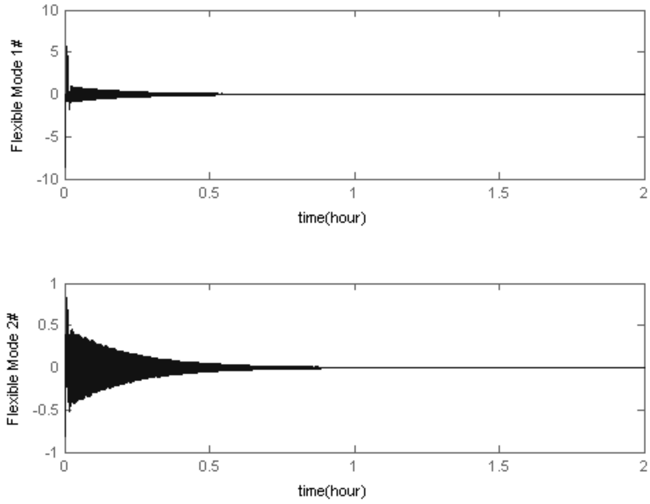

Fig. 2 The 1st and 2nd flexible mode responses with the robust $H\infty$ controller

As the simulation results illustrated here, we can distinctly find that the pitch angle controlled by the robust $H\infty$ control technique can approach desired value asymptotically and reach the stable state robustly. Therefore, the robust control law we designed is well demonstrated.

Fig.2 illustrates the suppressive effects on vibrations of the first and second flexible modes by the robust $H\infty$ control technique in the overall control system. From the figures we can easily find that the magnitudes of vibrations excited by the two flexible modes are both reduced gradually, and responses of the two flexible modes are stable ultimately with their magnitudes approach zero.

Conclusions

As a solar sail is virtually a spacecraft with a large flexible structure, the impact of vibrations excited by those flexible modes on solar sail can be tremendous. Therefore we considered the solar sail as a rigid body coupled with flexible modes. In order to suppress the vibrations, we developed a robust control strategy to guarantee the stability of the solar sail system.

The simulation results we presented in the paper demonstrated the overall effectiveness and practicality of a robust control system based on a rigid body coupled with the flexible modes. The final results of the developed dynamic model and the attitude control system of the solar sail spacecraft could be significant for a sail spaceflight experiment in future.

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