

Research on the Sun Shadow Positioning Technology

Bin Yang

School of Electric Power Engineering, North China Electric Power University, Baoding 071000, China.

1422714487@qq.com

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Abstract. In this paper, we will establish practical and reliable mathematical model to determine the video filming locations and dates by analyzing the shadow variations of the object in the video. At first, we convert the video into image sequences which can be read by MATLAB at regular intervals. So we can analyze the image sequences to obtain coordinates of the end of the shadow in the video; in the second place, we adopt the method of data transformation based on three-dimensional reconstruction to obtain the actual coordinates of the end of the shadow; next, taking advantage of the parametric equations of the shadow and the actual coordinates of the end of the shadow, we establish goal programming model to make the sum of the error of the shadow length and deviation angle variance of the coordinate system minimum. And we can obtain solar hour angle, declination and angle latitude; Finally, we analyze the conversion relation of the solar hour angle with longitude and declination with dates to determine the video filming locations and dates.

1. Introduction

Determining the video filming locations and dates according to the video is an important aspect of analyzing video statistic, and has important significance in investigation on the surface and space exploration. Sun shadow location technology is a method of determining the video filming location and dates by analyzing the shadow variations of the object in the video, so researching sun shadow location technology has important practical significance.

2. Assumptions

As to simplify the problem, we make the following assumptions in the process of building model in this paper:

- We make sunlight shine on the Earth in parallel rays, under the situation that we assume that the Earth is a sphere and we ignore the atmospheric refraction of sunlight;
- The ground is flat and the shadow will not deform;
- Ignoring revolution of the earth in one day.

3. Symbols, Terminology Definitions

3.1 Symbols and Definitions

Table 1 Parameters in the process of obtaining coordinates of the end of the shadow

Parameters	Meaning
$r_{1\sim 9}$, t_x , t_y , t_z	Undetermined parameters
(δ_x, δ_y)	Distortion of coordinates
X_d , Y_d	The image coordinates
f	Effective focal length

Table 2 Parameters in the process of determining the location of the sun

Parameters	Meaning
h	solar altitude
A	solar azimuth
δ	declination of the sun location
Ω	the solar hour angle of the sun location
H	the length of the object
L	the length of the shadow
α	the angle between the Coordinate system in video and the Coordinate system in reality

4. The establishment of the model

4.1 Obtaining the coordinates of the end of the shadow in the video

We convert the video into image sequences which can be read by MATLAB such as the time interval and obtain coordinates of the end of the shadow in the video. We use simple Cartesian coordinate system to establish the coordinate system in the video: The x axis in the horizontal direction, vertical direction for the y axis.

4.2 Obtaining the actual coordinates of the end of the shadow in reality

Adopting the method of data transformation based on three-dimensional reconstruction, we derive the conversion relation of the actual coordinates with the coordinates in the video. The specific formula is defined as follow:

$$\left\{ \begin{array}{l} \delta_x = X_d \cdot (k_1 \cdot \rho^2 + k_2 \cdot \rho^4) + k_3 \cdot (3X_d^2 + Y_d^2) + 2k_4 \cdot X_d \cdot Y_d \\ \delta_y = Y_d \cdot (k_1 \cdot \rho^2 + k_2 \cdot \rho^4) + 2k_3 \cdot X_d \cdot Y_d + k_4 \cdot (X_d^2 + 3Y_d^2) \\ X_d + \delta_x = f \frac{r_1 x_w + r_2 y_w + r_3 z_w + t_x}{r_7 x_w + r_8 y_w + r_9 z_w + t_z} \\ Y_d + \delta_y = f \frac{r_4 x_w + r_5 y_w + r_6 z_w + t_y}{r_7 x_w + r_8 y_w + r_9 z_w + t_z} \\ X_d = X_u (1 + k^2) \\ Y_d = Y_u (1 + k^2) \end{array} \right. \quad (1)$$

In this paper, without considering the distortion of the camera lens (namely, δ_x 、 δ_y 、 k is zero), we simplify the formula as follow:

$$\left\{ \begin{array}{l} X_d = X_u = \frac{R_1 x_w + R_2 y_w + R_3 z_w + T_x}{r_7 x_w + r_8 y_w + r_9 z_w + t_z} \\ Y_d = Y_u = \frac{R_4 x_w + R_5 y_w + R_6 z_w + T_y}{r_7 x_w + r_8 y_w + r_9 z_w + t_z} \end{array} \right. \quad (2)$$

Among them $R_{1 \sim 6} = f \cdot r_{1 \sim 6}$ 、 $T_x = f \cdot t_x$ 、 $T_y = f \cdot t_y$ 、 $T_z = f \cdot t_z$.

At this point, we obtain the actual coordinates in reality by adopting the method of data transformation based on three-dimensional reconstruction.

4.3 Determining the video filming location and dates

4.3.1 Analyzing the length and coordinates of the shadow

Based on the ‘‘Celestial theory’’^[1], we adopt the principle of relative motion to analyze the relative position of the earth and the sun: we assume that the earth is the center of the solar system and the sun revolve around the earth. We come to the conclusion that the sun relative to the location of the earth has great importance in the length of object and obtain some formula of parameters, so we obtain the formula about the length of the shadow as follow:

$$\left\{ \begin{array}{l} L = H \cdot \cot[\sin^{-1}(\sin \varphi \sin \delta + \cos \varphi \cos \delta \cos \Omega)] \\ x = (H \cdot \cot h) \times \cos A \\ y = (H \cdot \cot h) \times \sin A \end{array} \right. \quad (3)$$

Among them, we use simple Cartesian coordinate system to establish the actual coordinate system in reality: The x axis in the latitude direction, longitude direction for the y axis.

4.3.2 Building the goal programming goal

Due to the video's direction is not given, there is angle of deviation between the coordinate system in the video and the actual coordinate system. The concrete structure is shown in the figure below:

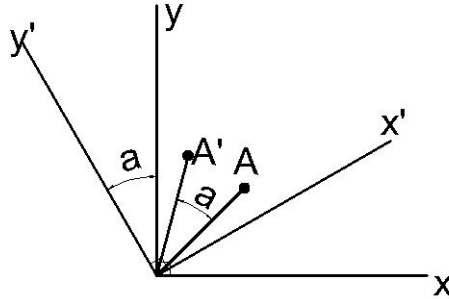


Figure 1 The angle between the Coordinate system in video and the Coordinate system in reality

Among them, the x-y coordinate system is defined in reality, the x'-y' coordinate system is defined in the video, α is defined as the angle of deviation.

Although there is angle of deviation between the two coordinate system, the angle of deviation is fixed which should be " α ".so we adopt the discrete degree of Angle to measure the error of the two coordinate system, and take the conclusion that the length of object is the same in different coordinate system into consideration to establish goal programming model. After solving the goal programming model, we determine the video filming locations and dates. The goal programming model is defined as follow:

$$\begin{aligned} \min \quad & z = \text{var}(a) + (L - L')^2 \\ \text{s.t} \quad & \begin{cases} a_i = \frac{L^2 + L'^2 - \sqrt{(x_i - x'_i)^2 + (y_i - y'_i)^2}}{2 \cdot L \cdot L'} \\ L' = \sqrt{x_i'^2 + y_i'^2} \\ L = \sqrt{x_i^2 + y_i^2} \\ x_a > 0 \\ -\frac{\pi}{2} \leq x_b \leq \frac{\pi}{2} \end{cases} \end{aligned} \quad (4)$$

4.4 Solving the model

In this paper, we use Particle Swarm optimization to solve the goal programming goal and obtain solar hour angle、declination and angle latitude. By analyzing the conversion relation of the solar hour angle with longitude and declination with dates we finally determine the video filming locations and dates. We take as an example of Beijing to make the image to describe the change of the sun's shadow in October 22, 2015 as follow:

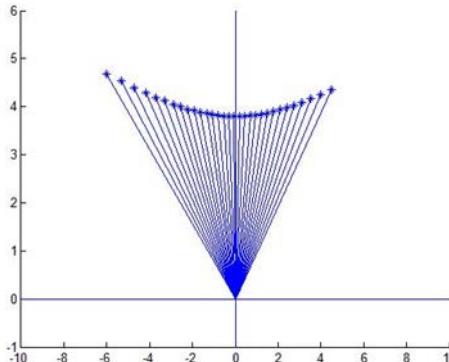


Figure 2 The change of the sun's shadow

Reference

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