# Effect of Wind Speed on Reaction Coefficient of Different Building Height 

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#### Abstract

The air-conditioning load will increase in the rooms with the same envelope size and same thermal performance, but in different height. Thermal reaction coefficient method is widely used in calculation of air-conditioning load. Change rule of thermal reaction coefficient can be confirmed by a calculation case in this paper. With the increase of the height of building, the reaction coefficient will increase.


## Introduction

Heat transfer on the building surface is greatly affected by corresponding wind speed. Especially in the high-rise building, building surface wind speed will increase with height, so that the surface heat transfer enhancement. Meanwhile additional heat loss is accompanied by increased air-conditioning load.

With the change rule of reaction coefficient, the air-conditioning load and corresponding capacity of air-conditioning system can be calculated accurately. The rule also can be used to analyze internal force and deformation in concrete structure building to reduce the risk of surface cracking produced by solar radiation and convection heat transfer.

## Wind Speed Distribution Law

At low altitude, the wind speed near to the ground surface will increase with height increasing due to surface friction. Below 100 m , the average wind speed profile is fitted by the logarithmic equation. In $100 \mathrm{~m}-300 \mathrm{~m}$ height, the average wind speed profile is fitted by exponential law. And the height above ground $300 \mathrm{~m}-500 \mathrm{~m}$, wind speed achieves maximum, because air flowing almost not be affected by ground surface friction but action of air pressure. Conclusion above also can be revealed in equation (1), (2). Wind speed variation presets logarithmic law in $10 \mathrm{~m}-100 \mathrm{~m}$ height, and presets index law in $100 \mathrm{~m}-300 \mathrm{~m}$ height.

Wind speed distribution in logarithmic law ( $10 \mathrm{~m}-100 \mathrm{~m}$ ) is shown in equation (1):

$$
\begin{equation*}
\frac{\overline{\mathrm{u}}}{\overline{\mathrm{u}}_{1}}=\frac{\ln \mathrm{z}-\ln \mathrm{z}_{0}}{\ln \mathrm{z}_{1}-\mathrm{z}_{0}} \quad \text { [1] } \tag{1}
\end{equation*}
$$

Wind speed distribution in index law $(100 \mathrm{~m}-300 \mathrm{~m})$ is shown in equation2:

$$
\begin{equation*}
\overline{\mathrm{u}}=\overline{\mathrm{u}}_{1}\left(\frac{\mathrm{Z}}{\mathrm{Z}_{1}}\right)^{\alpha} \tag{2}
\end{equation*}
$$

Where $Z_{0}$ is roughness height(In the neutral layer, roughness height is the height which vertical wind speed is approaching to zero, roughness height is 1.1 m in Beijing), $\mathrm{Z}_{1}$ is standard
height(Standard height is 10 m in most nations), $\overline{\mathrm{u}}$ is average speed in Z height, ${ }^{\overline{\mathrm{u}}}{ }_{1}$ is average speed in 10 m height, $\alpha$ is wind speed profile index.

## Convective Heat Transfer Coefficient in Different Height and Correspondent Wind Speed

Typically, convective heat transfer coefficient can be calculated and tested in natural convection state. But in fact, the wind speed on high-rise building surface increases with height. In the case of considering wind direction parallel to wall surface, the equation (3) is used to calculate convection heat transfer coefficient.

$$
\begin{equation*}
\alpha=5.7+3.8 \mathrm{v}[2] \tag{3}
\end{equation*}
$$

Where $\alpha$ is the heat transfer coefficient, $\mathrm{v}[\mathrm{m} / \mathrm{s}]$ is the velocity on wall surface.
According to equation (3), convective heat transfer coefficient in different height and correspondent wind speed in $10 \mathrm{~m}, 100 \mathrm{~m}, 200 \mathrm{~m}$ and 300 m height can be calculated in Table1.

Table1. convective heat transfer coefficient and correspondent wind speed in different height

| $\mathrm{Z}[\mathrm{m}]$ | 10 | 100 | 200 | 300 |
| :---: | :---: | :---: | :---: | :---: |
| Wind Speed $[\mathrm{m} / \mathrm{s}]$ | 4.5 | 9.19 | 12.1 | 13.9 |
| $\alpha\left[\mathrm{~W} / \mathrm{m}^{2} \cdot{ }^{\circ} \mathrm{C}\right]$ | 22.8 | 40.622 | 51.68 | 58.52 |

## Reaction Coefficient Method

The reaction coefficient method proposed by D. G. Stephenson and G. P. Mitalas in Canada in the late 60 's. In essence, reaction coefficient method integrates modern control theory into the Laplace transform and the invert transform. In the premise of considering the wall as a thermal system, the heat flux variation caused by the disturbance of triangle temperature wave cell is obtained by decomposing the boundary condition which varies continuously into a series of triangular pulses by multiplying the pulse height with the triangle pulse cell. In the view that each pulse is linearly independent and thermal system is linear system, the whole heat process can be obtained by application of superposition principle and convolution principle.

## The Transfer Matrix[3]

From the theory of heat transfer, the thermal field and heat flux variation trend can be obtained by solving the following partial differential equations which is shown in equation (4):

$$
\begin{cases}\frac{\partial(\mathrm{x}, \tau)}{\partial \tau}=\mathrm{a} \frac{\partial \mathrm{t}^{2}(\mathrm{x}, \tau)}{\partial \mathrm{x}^{2}} & (0<\mathrm{x}<1, \tau>0) \\ \mathrm{q}(\mathrm{x}, \tau)=-\lambda \frac{\partial \mathrm{t}(\mathrm{x}, \tau)}{\partial \mathrm{x}} & (0<\mathrm{x}<1, \tau>0) \\ \mathrm{t}(\mathrm{x}, 0)=0 & \end{cases}
$$

(4)

The equation (4) for Laplace transform is shown in equation (5):

$$
\left\{\begin{array}{l}
\mathrm{T}(\mathrm{x}, \mathrm{~s})=\operatorname{ch}\left(\sqrt{\frac{\mathrm{s}}{\mathrm{a}} \mathrm{x}}\right) \mathrm{T}(0, \mathrm{~s})-\operatorname{sh}\left(\sqrt{\frac{\mathrm{s}}{\mathrm{a}} \mathrm{x}}\right) \mathrm{Q}(0, \mathrm{~s}) /\left|\lambda \sqrt{\frac{\mathrm{s}}{\mathrm{a}}}\right|  \tag{5}\\
\mathrm{Q}(\mathrm{x}, \mathrm{~s})=-\lambda \sqrt{\frac{\mathrm{s}}{\mathrm{a}}} \operatorname{sh}\left(\sqrt{\frac{\mathrm{~s}}{\mathrm{a}} \mathrm{x}}\right) \mathrm{T}(0, \mathrm{~s})+\mathrm{ch}\left(\sqrt{\frac{\mathrm{~s}}{\mathrm{a}} \mathrm{x}}\right) \mathrm{Q}(0, \mathrm{~s})
\end{array}\right.
$$

Where $\mathrm{T}(\mathrm{x}, \mathrm{s})$ is the Laplace transform from temperature to time at the position $\mathrm{x}, \mathrm{Q}(\mathrm{x}, \mathrm{s})$ is the Laplace transform from heat flux to time at the x section, s is the Laplace transform of $\tau(\tau$ representing time), a is thermal diffusivity, $\mathrm{m}^{2} / \mathrm{s}, \lambda$ is thermal conductivity, $\mathrm{W} /(\mathrm{m} \cdot \mathrm{K}), \mathrm{x}[\mathrm{m}]$ is coordinates on the calculated surface, $1[\mathrm{~m}]$ is the thickness of wall.

From equation (4), (5), the temperature responds and heat flux responds in any position can be obtained by Laplace transform method once boundary condition $\mathrm{t}(0, \tau)$ and $\mathrm{q}(0, \tau)$ are known. And equation (5) can be solved by Laplace transform of $\mathrm{T}(\mathrm{x}, \mathrm{s})$ and $\mathrm{Q}(\mathrm{x}, \mathrm{s})$ which is shown in equation (6).

When $\mathrm{x}=1$, then:

$$
\left.\left|\begin{array}{c}
\mathrm{T}(1, \mathrm{~s})  \tag{6}\\
\mathrm{Q}(1, \mathrm{~s})
\end{array}\right|=\left|\begin{array}{cc}
\mathrm{A}(\mathrm{~s}) & -\mathrm{B}(\mathrm{~s}) \\
-\mathrm{C}(\mathrm{~s}) & \mathrm{D}(\mathrm{~s})
\end{array}\right| \begin{gathered}
\mathrm{T}(0, \mathrm{~s}) \\
\mathrm{Q}(0, \mathrm{~s})
\end{gathered} \right\rvert\,
$$

Where:

$$
\begin{aligned}
& \mathrm{A}(\mathrm{~s})=\operatorname{ch}(\sqrt{\mathrm{s} / \mathrm{a} \cdot 1}) ; \quad \mathrm{C}(\mathrm{~s})=\lambda \sqrt{\mathrm{s} / \mathrm{a}} \cdot \operatorname{sh}(\sqrt{\mathrm{~s} / \mathrm{a} \cdot 1}) \\
& \mathrm{B}(\mathrm{~s})=\operatorname{sh}(\sqrt{\mathrm{s} / \mathrm{a}} \cdot 1) /(\lambda \sqrt{\mathrm{s} / \mathrm{a}}) ; \quad \mathrm{D}(\mathrm{~s})=\operatorname{ch}(\sqrt{\mathrm{s} / \mathrm{a}} \cdot 1)
\end{aligned}
$$

When $[\mathrm{G}]=\left|\begin{array}{cc}\mathrm{A}(\mathrm{s}) & -\mathrm{B}(\mathrm{s}) \\ -\mathrm{C}(\mathrm{s}) & \mathrm{D}(\mathrm{s})\end{array}\right|,[\mathrm{G}]$ is transfer matrix of wall system representing thermal characteristics of wall system, and is not related to temperature and heat flux on both side of the wall.

## Heat Transfer Coefficient

In solving the heat transfer coefficient, $\mathrm{B}(\mathrm{s})=0$ in the transfer matrix, and $\alpha_{\mathrm{i}} \geq 100_{\text {in transcendental }}$ equation. Then heat transfer equation is shown in equation (7):

$$
\begin{array}{ll}
\mathrm{y}_{0}=\mathrm{K}+\sum_{\mathrm{i}=1}^{\infty} \frac{\mathrm{B}_{\mathrm{i}}}{\Delta \tau}\left(1-\mathrm{e}^{-\alpha_{\mathrm{i}} \Delta \tau}\right) & \mathrm{n}=0 \\
\mathrm{y}_{\mathrm{n}}=-\sum_{\mathrm{i}=1}^{\infty} \frac{\mathrm{B}_{\mathrm{i}}}{\Delta \tau}\left(1-\mathrm{e}^{-\alpha_{\mathrm{i}} \Delta \tau}\right)^{2} \mathrm{e}^{-(\mathrm{n}-1) \alpha_{\mathrm{i}} \Delta \tau} & \mathrm{n} \geq 1 \tag{7}
\end{array}
$$

Where $\Delta \tau$ is the discrete time interval, $\Delta \tau=1 \mathrm{~h}, \mathrm{Bi}$ is the coefficient, be equal to $-1 /\left[\alpha_{\mathrm{i}}{ }^{2} \mathrm{~B}^{\prime}\left(-\alpha_{\mathrm{i}}\right)\right]$, $\alpha_{\mathrm{i}}$ is the I-th root, n is the number of reaction coefficient, $\mathrm{n}=40, \mathrm{~K}$ is the heat transfer coefficient of wall, $\mathrm{W} /(\mathrm{m} 2 \cdot \mathrm{~K}), \Delta \tau$ is the discrete time interval, $\Delta \tau=1 \mathrm{~h}, \mathrm{Bi}$ is the coefficient, be equal to $-1 /\left[\alpha_{\mathrm{i}}^{2} \mathrm{~B}^{\prime}\left(-\alpha_{\mathrm{i}}\right)\right], \alpha_{\mathrm{i}}$ is the I-th root, n is the number of reaction coefficient, $\mathrm{n}=40, \mathrm{~K}$ is the heat transfer coefficient of wall, $\mathrm{W} /(\mathrm{m} 2 \cdot \mathrm{~K})$.

## Case Research

For one type of wall, the wall materials are shown in Table2:
Table 2. Wall materals

| Wall type | Thermal Diffusivity <br> $\left[\mathrm{W} / \mathrm{m}^{2} \cdot{ }^{\circ} \mathrm{C}\right]$ | Heat transfer <br> coefficient $\left[\mathrm{kg} / \mathrm{m}^{3}\right]$ |
| :---: | :---: | :---: |
| 1. brick setting $(240 \mathrm{~mm})$ | 0.00184 | 0.81 |
| 2. Lime plaster $(20 \mathrm{~mm})$ | 0.00179 | 0.7 |

## Root of Equation B(s)

Different height root distribution figures are shown in Fig.1-Fig.4, in which the horizontal coordinates are the roots of $\mathrm{B}(\mathrm{s})$, and the longitudinal coordinates are the values of $\mathrm{B}(\mathrm{s})$.


Fig. 1 Roots of $B(s)$ in 10 m height


Fig. 2 Roots of $B(s)$ in 100m height


Fig. 3 Roots of $B(s)$ in 200m height


Fig. 4 Roots of $B(s)$ in 300 m height

Represent the above figures by data in Table3:
Table3. The roots data of $B(s)$

| Height <br> 10 m | Roots | Height <br> 100 m | Roots | Height <br> 200 m | Roots | Height <br> 300 m | Roots |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -0.129 | 1 | -0.141 | 1 | -0.136 | 1 | -0.137 |
| 2 | -0.592 | 2 | -0.646 | 2 | -0.599 | 2 | -0.598 |
| 3 | -1.482 | 3 | -1.58 | 3 | -1.515 | 3 | -1.513 |
| 4 | -2.892 | 4 | -3.021 | 4 | -2.993 | 4 | -2.992 |
| 5 | -4.941 | 5 | -5.041 | 5 | -5.086 | 5 | -5.084 |
| 6 | -7.641 | 6 | -7.717 | 6 | -7.827 | 6 | -7.824 |
| 7 | -11.03 | 7 | -11.06 | 7 | -11.23 | 7 | -11.23 |
| 8 | -15.1 | 8 | -15.1 | 8 | -15.29 | 8 | -15.3 |
| 9 | -19.85 | 9 | -19.85 | 9 | -20.02 | 9 | -20.02 |
| 10 | -25.27 | 10 | -25.27 | 10 | -25.4 | 10 | -25.4 |
| 11 | -31.34 | 11 | -31.34 | 11 | -31.42 | 11 | -31.42 |
| 12 | -38.04 | 12 | -38.04 | 12 | -38.8 | 12 | -38.8 |
| 13 | -45.36 | 13 | -45.36 | 13 | -45.36 | 13 | -45.36 |
| 14 | -53.3 | 14 | -53.3 | 14 | -53.27 | 14 | -53.27 |
| 15 | -61.84 | 15 | -61.84 | 15 | -61.8 | 15 | -61.8 |
| 16 | -71 | 16 | -71 | 16 | -70.95 | 16 | -70.95 |
| 17 | -80.78 | 17 | -80.78 | 17 | -80.74 | 17 | -80.74 |
| 18 | -91.18 | 18 | -91.18 | 18 | -91.14 | 18 | -91.14 |

## Response Coefficient

According to equation (7), the reaction coefficient is calculated as Table4:
Table4. Reaction coefficient values in different heights

| Height 10m |  | Height 100 m |  | Height 200m |  | Height 300m |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| j | Y | j | Y | j | Y | j | Y |
| 1 | 0.0017 | 1 | 0.0067 | 1 | 0.0066 | 1 | 0.0083 |
| 2 | 0.0425 | 2 | 0.0556 | 2 | 0.0521 | 2 | 0.0536 |
| 3 | 0.1048 | 3 | 0.129 | 3 | 0.1208 | 3 | 0.1224 |
| 4 | 0.1481 | 4 | 0.1754 | 4 | 0.1662 | 4 | 0.1679 |
| 5 | 0.1661 | 5 | 0.1905 | 5 | 0.1834 | 5 | 0.185 |
| 6 | 0.167 | 6 | 0.1866 | 6 | 0.1822 | 6 | 0.1836 |
| 7 | 0.1587 | 7 | 0.1733 | 7 | 0.1715 | 7 | 0.1727 |
| 8 | 0.1462 | 8 | 0.1565 | 8 | 0.1565 | 8 | 0.1575 |
| 9 | 0.132 | 9 | 0.139 | 9 | 0.1404 | 9 | 0.1412 |
| 10 | 0.1182 | 10 | 0.1224 | 10 | 0.1247 | 10 | 0.1252 |
| 11 | 0.105 | 11 | 0.1072 | 11 | 0.1099 | 11 | 0.1103 |
| 12 | 0.093 | 12 | 0.0935 | 12 | 0.0966 | 12 | 0.0969 |
| 13 | 0.0821 | 13 | 0.0815 | 13 | 0.0847 | 13 | 0.0848 |
| 14 | 0.0723 | 14 | 0.0709 | 14 | 0.0741 | 14 | 0.0741 |

## Conclusions

Reaction coefficient appears to increasing trend which is accord to the actual situation in above analysis. It can be deduced that heating load and cooling load will increase with the increase of the building height. In the calculations of cooling load and heating load, the reaction coefficient should be adjusted with the building height regardless of simulation method or tabular computation. The adjusted route will help to improve the accuracy of design, especially be beneficial to optimize consultation.

## References

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