Study on Image Thinking in HigherEducation

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Abstract—This paper has discussed what is Image Thinking in High education especially in Advanced Mathematics. At the same time, its four characteristics、 the relation between Image Thinking and Logical Thinking are given by some typical examples and intuitive graphics. Research shows that Cross Nature and Rich Culture of Image Thinking in Advanced Mathematics among them are the bright points of this paper. Therefore, Image Thinking should be strengthened for college students to cultivate Innovative Spirit in order to improve undergraduate teaching quality.

Keywords-Image Thinking; Logical Thinking; High Education; Advanced Mathematics; Intuition

I. INTRODUCTION

When college students study university courses especially Advanced Mathematics made of Calculus、 Linear Algebra and Statistics, they feel them difficult because these courses are very abstract. So, Professor Uri Leron,a famous Israeli Mathematics, asked a question: "In cases where natural and mathematical thinking clash, what can we as math educators do to help students create peaceful coexistence between the two?[1] " This paper can give some examples to college students during the university teaching process in order to study and use Image Thkinging in High Education. Thus, it will be beneficial for college students to cultivate Innovative Spirit and for universities improve undergraduate teaching quality.

II. IMAGE THINKING IN HIGH EDUCATION

We will talk about the mathematics image because Image Thkinging is often used in Advanced Mathematics during High Education.Firstly, Image Thinking is that an object reflects the image in the human brain, and this kind of image can be reproduced out with the image of the materialization and perceived. As everyone knows, there are three kinds of images: felt image、 practice image and idea image, so its characteristic is the image、 integrity and jumping[2]. Secondly, Image Thinking in Advanced Mathematics means explaining the meaning of the mathematics image. This is a image summarized by the abstract mathematics concept, which is one kind of idea images. This kind of more general images such as geometric figure、 intuitive graph、 mathematics language、 mathematics symbol、 mathematics model and so on are all idea image[3].

Thus, the above indicated Image Thinking in Advanced Mathematics is the process of applying the mathematics image thinking to Advanced Mathematics courses such as Calculus、 Linear Algebra and Statistics. As we all know, when the German mathematician Gauss expressed his Algebra results, he abandoned the before expression of mathematics image thinking and led to that future generations couldn't figure out Gauss how to obtain these results. This counter-example illustrated the importance of Image Thinking in Advanced Mathematics. Based on detailed investigation of Dean's Office and teaching practice of Advanced Mathematics for many years, the university mathematics teaching team of Huazhong Agricultural University of the people's Republic of China found: duo to poor abstract thinking ability, many college students during the learning process were only formally accepted, imitated and done, while they did not seize the algebraic nature of the problem, and slowly lost learning interest.Consequently, how to make mathematics teaching intuitive and visual is a topic of great practical significance to enhance undergraduate teaching quality.

III. CHARACTERISTICS AND COUNTERMEASURES

Image Thinking in Algebra is different from a kind of thinking in image of literature、 art and other thinking in image, which has own notable characteristics. By way of examples and discussions, we will summarize some of its characteristics and give the corresponding solution method.

A. High Abstraction

In Advanced Mathematics, although some knowledge points such as maximum image being "mountain peak" and minimum image being "mountain valley" is intuitive, but most content is highly abstract. For instance, most students don't understand what is the linear correlation and linear independence even if they know how to judge the two by their definition. In fact, they feel Advanced Mathematics very abstract and can't take good understanding of the Vector Theory. If we give the example of the cube, college students will know that the intersect of two or three lines （vectors） is the linear independent while the form of parallel lines （vectors） means the linear correlation（Figure 1）. So，examples of the relationships of intersect and parallel lines in the cube will facilitate the college students a better study and comprehension of the vector content. As Immanuel Kant said: all human knowledge starts from the intuition into the concept and to the end of the concept. Thus, almost all of the great mathematicians have emphasized the importance of intuitive mathematics[4].
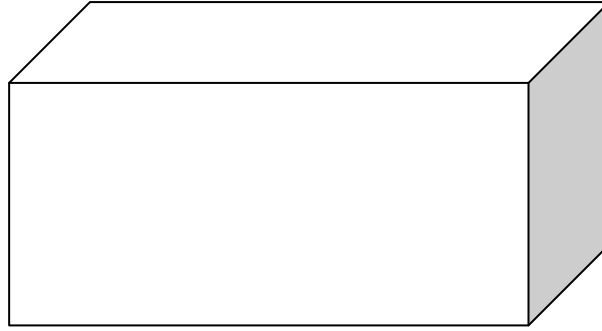
Figure 1. The relationships of intersect and parallel lines in the cube

B. Great Generality

Advanced Mathematics has generality. Taking Linear Algebra as an example, we must catch the matrix firmly which is the thread during the process of Image Thinking. We can emphasize the importance of matrix representation and matix（linear）transformation when the college students study Linear Algebra.But they often confuse what is matrix representation or transformation on earth, we may tell them rotation transformation in the plane is a good example. Then, we will simplify the function $x_1 x_2 = 1$ to the standard form $y_1^2 - y_2^2 = 1/2$ with their curve graph （Figure 2）. Therefore, Image Thinking in Advanced Mathematics can help college students understand matrix applications.
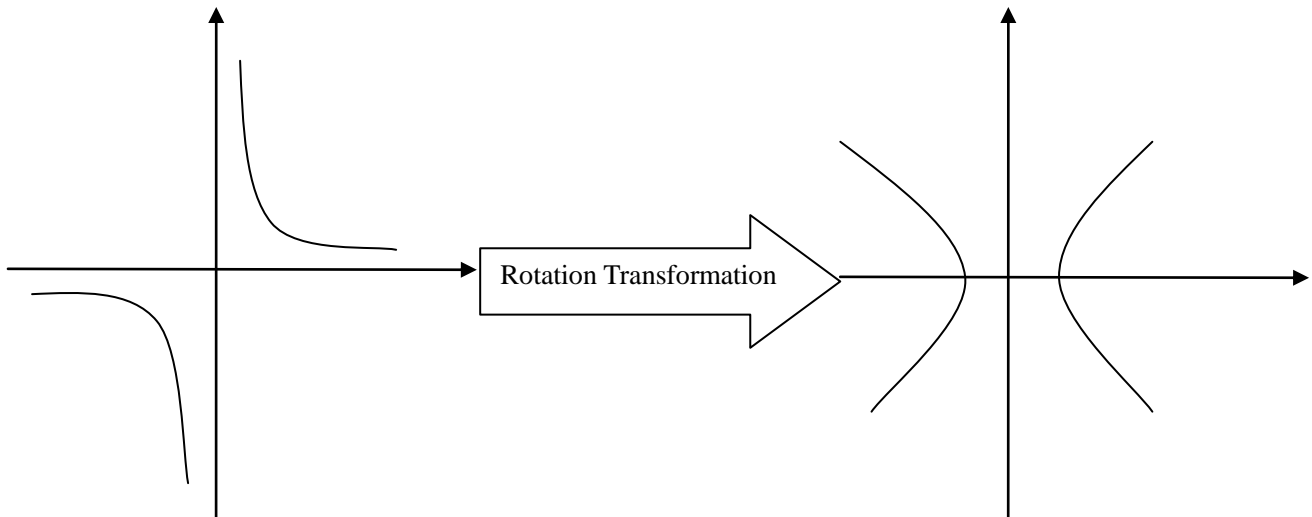

Rotation Transformation

Figure 2.   The standardization of the function $x_1 x_2 = 1$

C. Cross Nature

Image Thinking in Advanced Mathematics differring from the general logical thinking is a multi-loop motion, and we can use the theory of different mathematics branches and the knowledge of different disciplines to solve some problems about Calculus、Linear Algebra and Statistics. After we gain the characteristic polynomial whose times is greater than 3, we will find that it is very difficult to solve the eigenvalue if you take use of the secondary school mathematics method.But the derivation in Calculus or "testing root method" of Advanced Algebra is helpful for the solution of the eigenvalue in a certain extent. These are good illustrations of exercising your own problem-solving skills advocated by Professor Zhang Pingtong who is one of the Four American 2010 Distinguished Professors selected by U.S. Carnegie Foundation for the Advancement of Teaching[5]. Actually, this process reflects the cross nature of different mathematics branches under the guidance of Image Thinking in Advanced Mathematics. In addition, the use of Matlab、Lingo and R Software dealing with Linear Programming problems by Mathematical Modeling is also a useful attempt in different subjects.

D. Rich Culture

The thinking of any subject has a rich cultural heritage and the development of Advanced Mathematics courses is a cultural phenomenon. Mathematical culture expresses mathematical ideas、spirits and methods, naturally Image Thinking in Advanced Mathematics has the rich culture.For instance, 24-point game of 2010 Math Cultural Festival Chaired by Professor Zou Tingrong in Huazhong Agricultural University presents a "2 × 2 matrix" operation intuitively and enhances our students'

sensitivity of the digital[6] .Not only that, it is helpful for UK Education Sector to carry out the national numeracy campaign to raise the level of mathematics education and revitalize the country's economy. Another example, the great mathematician Euler skillfully transformed famous Seven Bridges Problem into "a stroke problem" and his application of Image Thinking created a precedent for the study of Graph Theory in Algebra（Figure 3）[7, 8].
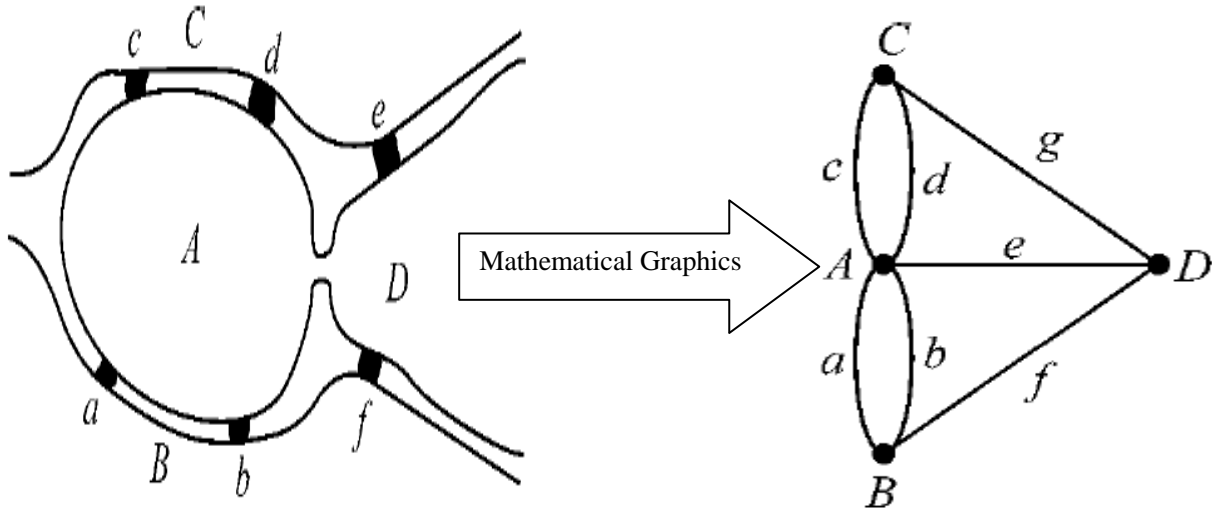

Figure 3.　Seven Bridges Problem and it's Graphics

IV.　RELATIONSHIPS BETWEEN IMAGE THINKING AND LOGICAL THINKING IN HIGHER EDUCATION

On one hand, in Advanced Mathematics courses include Calculus、Linear Algebra and Statistics, Logical thinking is an important mode of thinking in the primary importance while Image Thinking is a kind of auxiliary mode of thinking, but they complement and promote each other. So, it is not enough to only grasp Logical Thinking, and Image Thinking should also be master. On the other hand, Image Thinking and Logical Thinking communicate and relate with each other closely. Drawing the support from Image Thinking, logical reasoning can obtain the best result and effect.

For instance, when college students study Matrix Operation including Addition、Multiplication、Inversion and so on, they often feel fear for abstract mathematical symbols and complex operational rules. Therefore, after giving them the definition of Vector Inner Product, we introduce Matrix Multiplication and it is greatly convenient for our students to understand the operational rule of Multiplication[9]. Apparently, it is not difficult to find that it is a very good method for them to derive Matrix Multiplication from Logical thinking after we explain the operational rule of Vector Inner Product intuitively from Image Thinking（Figure 4）.
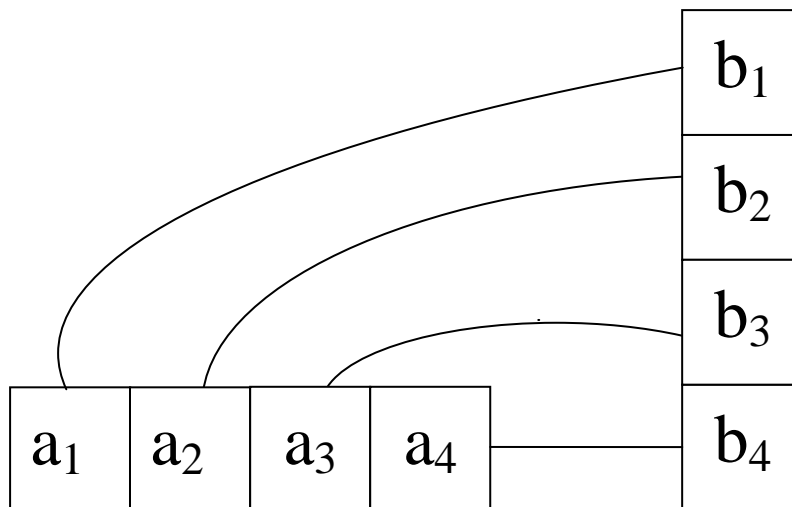

Figure 4. Operational Rule of Vector Inner Product

Furthermore, Logical Thinking and Image Thinking in Advanced Mathematics often interweave and take shape together for the creative thinking of mathematics, because Mathematical Image Thinking itself is the rational result and not the things of sense organ nature. Therefore, we can affirm that it is not possible for our mathematicians to create mathematical thoughts、rigorous methods and rich culture without Image Thinking. As everyone knows, in the 19th century, the great Mathematician and Physicist Hamilton thought at first that the image of the new number was 3 dimension and named for $\alpha + \beta\, i + \gamma\, j$. Through reasoning demonstration of 15 yews and a lot of failure, the collision of a spark produced a new mathematics number "a+ bi+ cj+ dk" which was called Quaternions. It is worth noting that Quaternions Theory is widely used in Spacecraft Flight、Big Data Processing and other fields since Ken Shoemake introduced Quaternions into computer graphics in1985[10].

V. CONCLUSIONS

From the above, Image Thinking in Advanced Mathematics is help for as a result of its four characteristics, especially Cross Nature and Rich Culture. In that way, we should strengthen Image Thinking training for college students to cultivate Innovative Spirit in order to improve undergraduate teaching quality.

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REFERENCES

[1] Uri Leron, "The Power of Natural Thinking: Applications of Cognitive Psychology to Mathematics Education," Plenary Lecture, Proceedings of PME-NA, Columbus, Ohio, 2010.

[2] Chen Zhiquan, "Talk about the culture of mathematical thinking in images," JIAO YU JIAO XUE LUN TAN, vol. 3, Jan. 2010, pp. 59–59,doi: 10.3969/j.issn.1674-9324.2010.03.043.

[3] Zheng Longxin, "The image, inspiration, aesthetic and mathematics creation," Wuhan: Hubei Education Press, 1990.

[4] Shi Ningzhong, "Talking about the basic idea of mathematics," CHINA UNIVERSITY TEACHING, vol. 7, Jul. 2011, pp. 9–11, doi: 10.3969/j.issn.1005-0450.2011.07.004.

[5] Guo Yingjian, "New model of the 2010 Distinguished Professor of Undergraduate Education", Science Times, vol. 11, Nov. 2010.

[6] Zou Tingrong, Xia Jingbo and Li Nana, "Curricular and Extend of Mathematical Culture of Huazhong Agricultural University," Journal of Mathematics Education, vol. 21, Feb. 2012, pp. 4–4.

[7] Zou Tingrong, Xia Jingbo, Shen Jingfang and Hu Donggang, "Mathematics Cultural Appreciation," Wuhan: Wuhan University Press, 2013.

[8] Sun Pei, Liu Kai, Zeng Junjie and Yang Benchao, "Teaching reform of mathematical modeling thought in the course of graph theory," University Education, vol.8, Aug. 2015, pp. 118–119.

[9] Shao Maomin, "Discussion on the image thinking in Linear Algebra," EDUCATION SCIENCE & CULTURE MAGAZINE, vol. 1, Jan. 2009, pp. 131–132, doi: 10.3969/j.issn. 1672-7894.2009. 01.096.

[10] Xia Xiwang, Du Han and Liu Hanbing, "Discussion on the Transformation Between Attitude Quaternion and Euler Angles in Large-angle Maneuver Scenario," MISSILES AND SPACE VEHICLES, vol. 5, May. 2012, pp. 47–53.