

The Sea Air Defense Combat Method Based on Graph Theory Design and Analysis

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Abstract—According to the characteristics of modern air defense combat at sea, on the analysis of the “cooperative engagement capability (CEC)”, on the basis of using the knowledge of graph theory, we design a maritime defense -- small world network (WS), the connection between the network nodes is realized through communication link, finally, the network structure performance is analyzed. Research results to achieve real sea air defense combat network to provide certain reference and reference.

Keywords—*Maritime Air Defense Combat; The Network Model; Graph Theory*

I. INTRODUCTION

Sea fleet mission by various platforms (planes, ships and submarines) launch anti-ship missile threats, including air missiles, anti-radiation missiles, cruise missiles and tactical ballistic missiles, etc., the air and the cruise missile is the biggest threat. Developed by the U.S. navy “cooperative engagement capability (CEC),” using the computer, communication and network technology, the sea fleet of the ship target detection system, command and control systems, weapon system and an organic network awacs systems, it within a very short time delay is allowed to share the platform all the data obtained by the detection system, thus the whole fleet got behind a whole, can greatly increase the efficiency of the fleet of air defense missile defense. “Cooperative engagement capability (CEC)” is the “network centric warfare (cnw-2 under)” typical application in the navy. The article is based on graph theory to design a maritime defense network.

II. A COMPLEX NETWORK OF RESEARCH WAS REVIEWED

Complex networks since 1960 by Erdos and Renyi (ER) as a random network topology structure research, has been attracting the attention of people. A complex network of related models are:

Rules of network and stochastic network; Small world network (WS); Scale-free networks and other networks model.

A. Statistical Feature of Complex Network of Network Parameters

(1) the average path length

Complex network research, for a composed of N nodes in the network, I generally define two nodes, the

distance between $l(i, j)$ for the connection both the node number of the shortest path; The diameter of the network is the maximum distance between any two points; The average path length \bar{l} of the network is the average of the distance between all nodes on, it describes the degree of separation between the nodes in the network.

$$\bar{l} = \frac{1}{N} \sum_{i=1}^N \left[\frac{1}{N} \sum_{j=1}^N l(i, j) \right] \quad (1)$$

(2) Gathered Coefficient

Aggregation coefficient C is used to describe the gathering for nodes in the network, the network has more closely. The calculation method for: I assume that node i by side connected to other nodes k_i , if the nodes k_i are interconnected, they should exist between $k_i \frac{(k_i - 1)}{2}$ side, while the actual number of edges between nodes k_i only have E_i , that $k_i \frac{(k_i - 1)}{2}$ is the aggregation coefficient of node i , namely:

$$C_i = \frac{2E_i}{k_i(k_i - 1)} \quad (2)$$

Network clustering coefficient are the average aggregation coefficient of all the nodes in the network. Obviously, only in the total network (each node is connected to the rest of the all nodes), aggregation coefficient is equal to 1, generally less than 1.

3) Degree Distribution

i nodes in graph theory for the node i connect the edge of the total number of all the nodes i referred to as a network of the average degrees k_i , defined as $\langle k \rangle$. Network node degree distribution with the distribution function $P(k)$, its meaning is an arbitrary choice of the probability of nodes are well k edge, is equal to the number of degrees of k nodes in the network of the ratio of the total number of network nodes.

4) Betweenness

Betweenness is divided into edge betweenness and node betweenness. Node betweenness for all the shortest path in the network after the node number in proportion; The edge betweenness has similar meanings. The

corresponding node or edge betweenness reflect in the role and influence in the whole network.

B. Assumptions

Maritime defense network the key to improve the air defense missile operational effectiveness is: improve the ability of real-time information sharing, improve the ability of multi-objective interception, implements the centralized type of ship to air missile command and over-the-horizon interception, then expand the ship to air missile combat area.

Maritime defense network design, the following assumptions:

- 1) In the network node number is limited, $n \leq M$. M as an integer, such as $M=10$;
- 2) In the network node implementations of relay communication ability is limited, can't unlimited relay communication services, the degree of a node is limited;

C. Network Design

Reference design idea of CEC system network structure, on maritime defense network design:

- 1) For the rapid information sharing ability (namely small world, the smaller the shortest path) and the perturbation resistance strong random attack (i.e. larger clusters, relatively large aggregation coefficient, small world network model is used to build (as shown in figure 1);

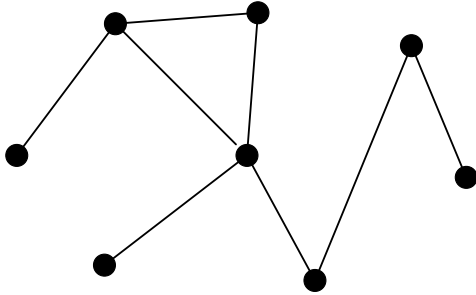


Figure 1. Sea Air Defense Combat Effectiveness Node

- 2) to improve the efficiency of the accused node command, accused of nodes in the network of 1 - the center of gravity;
- 3) to improve the survivability of the network, the current allegations center nodes were destroyed, a standby charges under the network automatically enabled center.
- 4) in order to ensure the communication between nodes in a network connectivity, accused of nodes using the algorithm of dynamic restructuring, to ensure that when a member node and charges cannot be established communication link, can with a backup charges node communication link is established.

III. NETWORK PERFORMANCE ANALYSIS

A. the Execution Time

Now suppose there is a composed of command and control node in the network is used to coordinate a key time. The nodes of each node to complete several

information processing tasks. If λ_i for the node i completed all of its mission, the average length of time of complete the task are assumed to be exponential distribution, in this way, $F_i(t)$ for accomplishing all tasks in a node i at time t .

$$F_i(t) = \begin{cases} 1 - e^{-t/\lambda_i} & t > 0 \\ 0 & t \leq 0 \end{cases} \quad (3)$$

Generally speaking, in the network to support the action will have several parallel and sequential node. A total of M has such a function set of nodes. In the simplest case, a critical path consists of N nodes, where N is a subset of M , as shown in figure 2.

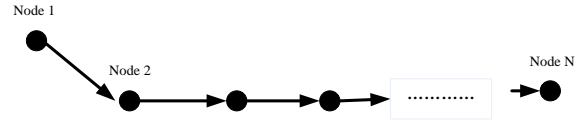


Figure 2. the Key Path

Define the critical path the total execution time for the path waiting time (time), the sum of all nodes plus terminal firing weapons and target in time. In the case of the sequential, the total time is the critical path is supposed to be performed on the expectation of node execution time plus the sum of time:

$$T = \sum_{i=1}^N \lambda_i + t_m \quad (4)$$

If there are both node serial and parallel on the critical path node, as shown in the following ways to illustrate the means of processing.

In figure 3 this example are as follows:

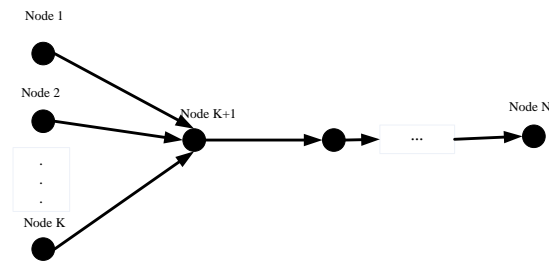


Figure 3. Parallel Node of the Critical Path

$$T = \max(\lambda_1, \lambda_2, \dots, \lambda_k) + \lambda_{k+1} + \dots + \lambda_N + t_m \quad 4-3$$

The article's centre of gravity is not refers to the geometric center of the network, but to achieve a certain performance or efficiency (e.g., shortest transmission delay) and determine a network node. In this paper, we will charge the node as the center of gravity of the network, in order to realize rapid information sharing ability and strong random disturbance resistance. The following discussion network the center of gravity of the problem.

Has undirected networks $N = (V, E, W, Q)$, including W for edge weights, distance or length, $W = \{\omega_{ij}\}$ ($i, j \in E$ commonly, Q for weights on the

vertices, $Q = \{q_i\}$ and the q_i corresponding vertex weights v_i . The network can be expressed as the center of gravity of the problem

$$\min_z M(z) = \sum_i q_i d(z, i) \quad (5)$$

$d(z, i)$ is the network from z to the vertices v_i of the short circuit. Z may be on the network vertices, may also be one side between the peaks, is determined by the shortest path; Like in the z is located in the edge (a, b) , and the proportion of center point a , is θ , that

$$d(z, i) = \min \{ \theta d_{ba} + d(a, i), (1 - \theta) d_{ab} + d(b, i) \} \quad (6)$$

If the network as directed, for the network (a, b) in an arc, then $d_{ba} = \infty$, type the first you can skip some consider at this time.

With N to the network, the edge of Z^* have a point (a, b) , V_A is the vertex v_i to Z^* the shortest path through the vertices of a collection of all the vertices v_i . Unless a vertex Z^* to the shortest path through a (that is, through b), then the vertices do not belong to V_A , and to belong to V_B , so:

$$V_A \cup V_B = V, \quad V_B = V \setminus V_A$$

$$\begin{aligned} \text{Thus there are} \\ M(z^*) &= \sum_{i \in V_A} q_i d(z^*, i) + \sum_{i \in V_B} q_i d(z^*, i) \\ &= \sum_{i \in V_A} q_i [d(a, i) + \theta d_{ba}] + \sum_{i \in V_B} q_i [d(b, i) + (1 - \theta) d_{ab}] \\ &= \theta \{ \sum_{i \in V_A} q_i [d(a, i) + d_{ab}] + \sum_{i \in V_B} q_i d(b, i) \} \\ &\quad + (1 - \theta) \{ \sum_{i \in V_A} q_i d(a, i) + \sum_{i \in V_B} q_i [d(b, i) + d_{ab}] \} \end{aligned} \quad (7)$$

By the relation

$$d(a, i) + d_{ab} \geq d(b, i),$$

$$d(b, i) + d_{ab} \geq d(a, i)$$

$$M(z^*) \geq \theta \sum_{i \in V} q_i d(b, i) + (1 - \theta) \sum_{i \in V} q_i d(a, i) \quad (8)$$

$$= \theta M(b) + (1 - \theta) M(a)$$

As a result, there will be $M(a) \leq M(z^*)$ or $M(b) \leq M(z^*)$, for any Z^* , there is always a vertex. i^* the center of gravity of the optimal solution which indicate that the network often one vertex in the network. Also suggests that each node in the maritime defense network exists in a satisfied optimal command and control tasks (rapid information sharing ability and strong random disturbance resistance) of the node, the node is the center of gravity of the network.

B. Ability to Survive

Discussed in this paper, the network survivability the maritime air defense network structure of connected

ability oneself, does not include the anti-damage ability of each node, the kill probability, etc.

By the knowledge of graph theory, the connectivity can be used to measure the strength of a connected graph, it shows how much a figure at least remove the edge or vertex cannot destroy the connectivity, or, can remove most figure of how many edges or vertex connectivity can remain. Figure two parameters are available "connectivity" and "connectivity" to reflect the size of the figure of connecting strength. Connectivity and edge connectivity, the greater the network more reliable. In this paper, through the discussion of "connectivity" to study the connecting ability of the network.

Equipped with air defense network diagram $G = (V, E)$, V for the node set, E for edge set, the following research, any two nodes in the network S network connectivity probability between T .

There are k path between node A and node S , and the path i connected probability is $A_i (i = 1, 2 \dots k)$, expressed in A node S , T connectivity.

Path between a node S , T , k , if and only if at least one path connecting the two nodes are interconnected, it is

$$A = A_1 \cup A_2 \cup \dots \cup A_k \quad (9)$$

Nodes are obtained by the independence of the events, S , T between the connected probability:

$$\begin{aligned} P(A) &= P(A_1 \cup A_2 \cup \dots \cup A_k) \\ &= \sum_{i=1}^k P(A_i) - \sum_{i < j < k} P(A_i A_j) + \dots + (-1)^{k-1} P(A_1 A_2 \dots A_k) \end{aligned} \quad (10)$$

$$P(A_1 A_2 \dots A_k) = P(A_1) P(A_2) \dots P(A_k) \quad (11)$$

Type: $A_i = L_{i1} L_{i2} \dots L_{ij} \in E$; L_{ij} is the first article I path j article links

$$P(A_i) = P(L_{i1} L_{i2} \dots L_{ij}) = P(L_{i1}) P(L_{i2}) \dots P(L_{ij}) \quad (12)$$

$P(L_{ij})$ Article j said in the first article i path link connectivity probability.

The reliability of the link between the available link connectivity probability reflects, link failure rate are assumed to be exponential distribution, are:

$$P(t) = \begin{cases} 1 - e^{-t/\theta_j} & t > 0 \\ 0 & t \leq 0 \end{cases} \quad (13)$$

Therefore, the reliability of the link

$$Q = 1 - P(t) \quad (14)$$

IV. CONCLUSION

Maritime defense network by the detector, charge network and weapon network, the network is an organic whole repeatedly through the communication system. Article on the basis of graph theory, we design a maritime defense network model -- small world network, the network connection is through the communication link between. Articles not discussed the performance of network node itself, and the main structural characteristics are studied for the networks themselves. For maritime air defense combat network, the network

structure is the precondition of network warfare. To establish a real sea air defense network, there are many technical problems need to study, such as space and time consistency technology, command and control function of dynamic reconfiguration technology, collaborative guidance technology, point trace synthesis and information fusion technology, etc.

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