Integral Sliding Mode Control of Airship Pitch Channel simplified model

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Abstract. A kind of integral type sliding mode control law was designed for a kind of simplified pitch channel model of hypersonic aircraft which is widely discussed recently. The advantage of sliding mode control is that it has strong robustness. And the integral sliding mode control method can reduce steady state error. And with the help of constructing two Lyapunov functions, all signals of close system are proved to be stable. At last, detailed simulation was done to testify the rightness of the proposed method.

Introduction

The control problem of hypersonic vehicle has attracted the interest of many researchers in the world. It is mainly due to the strong nonlinearity[1-4], fast time-varying characteristics and the three channel coupling problems brings great challenge to the control theory. At present, the traditional methods such as traditional PID, state feedback and state space method[5-8] have been adopted by scholars at home and abroad, also variable structure, adaptive control and other modern control methods are tried in hypersonic vehicle control, even neural network, genetic algorithm, fuzzy control and other intelligent method are studied and applied. At present, the complexity of the control of hypersonic vehicle leads to the robustness of the design of the general control method can not be guaranteed, which means that the stability margin is not enough. In this paper, an integral sliding mode control method is proposed for a class of hypersonic vehicle pitching channel model, which can not only ensure the robustness, but also reduce the steady state error due to the introduction of integral.

Model Description

A kind of hypersonic vehicle can be simplified as below differential equations to described its pitch channel movement:

$$\dot{V} = \frac{T\cos\alpha - D}{m} - g\sin\gamma \tag{1}$$

$$\dot{\gamma} = \frac{L + T\sin\alpha}{mV} - \frac{g\cos\gamma}{V} \tag{2}$$

$$\dot{\alpha} = q - \dot{\gamma} \qquad (3)$$

$$\dot{q} = \frac{M}{I} \qquad (4)$$

$$\dot{h} = V \sin \gamma \qquad (5)$$

$$\ddot{\eta} = -2\varsigma \omega_n \dot{\eta} - \omega_n^2 \eta + \omega_n^2 \eta_c, \omega_n = 5, \varsigma = 0.7 \qquad (6)$$

Where aero coefficient is identified as

$$L = \bar{q} S C_L, C_L = C_L^\alpha \alpha + C_L^\delta \delta + C_L^0, \quad T = \bar{q} s C_T$$

$$D = \bar{q} S C_D, C_D = C_d^{\alpha^2} \alpha^2 + C_D^\alpha \alpha + C_D^{\delta^2} \delta^2 + C_D^\delta \delta + C_D^0$$

$$M = \bar{q} S \bar{c} [C_{M\alpha} + C_{M\delta} + C_{Mq}], C_{M\alpha} = C_{M\alpha}^{\alpha^2} \alpha^2 + C_{M\alpha}^\alpha \alpha + C_{M\alpha}^0,$$

$$C_{M\delta} = c_e (\delta_e - \alpha),$$

$$C_{M\alpha} = 10^{-4}(0.06 - e^{-M_a/3})(-2\alpha^2 + 120\alpha - 1)$$

$$C_{Mq} = \frac{\bar{c} q}{2V}(-0.025 M_a + 1.37)(-0.0021\alpha^2 + 0.0053\alpha - 0.23)$$

$$C_{M\delta} = 0.0292(\delta - \alpha)$$

$$C_L = \alpha(0.493 + 1.91 / M_a)$$

$$C_D = 0.0082(171\alpha^2 + 1.15\alpha + 2)(0.0012 M_a^2 - 0.054 M_a + 1)$$

$$C_T = \left\{ \begin{array}{l} 38[1 - 164(\alpha - \alpha_0)^2](1 + 17/M_a)(1 + 0.15)\eta, \eta < 1 \\ 38[1 - 164(\alpha - \alpha_0)^2](1 + 17/M_a)(1 + 0.15\eta), \eta < 1 \end{array} \right\}$$

And V is speed, γ is the speed angle, α is attack angle, Q is the attitude angle speed, h is the height. ϕ is the oil supplying factor, δ_c is the duck wing and δ_e is the lift rudder.

Sliding mode controller design

Define error variable as $e = \alpha - \alpha^d$, and choose a common integral sliding mode variable as

$$s = e + c_1 \int e dt \qquad (7)$$

Solve the derivative of sliding mode as

$$\dot{s} = \dot{e} + c_1 e \qquad (8)$$

Then it can be transformed as

$$\dot{s} = \dot{\alpha} + c_1 e = q - \dot{\gamma} + c_1 e \qquad (9)$$

By using the information of above model, it can be written as

$$\dot{s} = q - \frac{L + T \sin \alpha}{mV} + \frac{g \cos \gamma}{V} + c_1 e \qquad (10)$$

And it can be rewritten as

$$\dot{s} = q - \frac{T \sin \alpha}{mV} + \frac{g \cos \gamma}{V} + c_1 e - \frac{\bar{q} S C_L}{mV} \qquad (11)$$

Finally, we get

$$\dot{s} = q - \frac{T \sin \alpha}{mV} + \frac{g \cos \gamma}{V} + c_1 e - \frac{\bar{q} S (C_L^\alpha \alpha + C_L^\delta \delta + C_L^0)}{mV} \qquad (12)$$

And define new variables as follows

$$l_2(\alpha,q,e) = q - \frac{T\sin\alpha}{mV} + \frac{g\cos\gamma}{V} + c_1 e - \frac{\bar{q}S(C_L^\alpha\alpha + C_L^0)}{mV} \quad (13)$$

$$l_1 = -\frac{\bar{q}SC_L^\delta}{mV} \quad (14)$$

Then the derivative of sliding mode can be written as

$$\dot{s} = l_1\delta + l_2(\alpha,q,e) \quad (15)$$

Then the sliding mode control law can be designed as

$$\delta = -sign(l_1)[k_1 s + k_2\frac{s}{|s|+k_3} + k_4 s^{1/3} + k_5\int sdt] \quad (16)$$

Where $k_i > 0$, and assume there exist a big enough positive constant such k_6 and k_7 that

$$|l_2(\alpha,q,e)| < k_6|s| + k_7 \quad (17)$$

Choose a Lyapunov function as

$$V = \frac{1}{2}s^2 + \frac{k_5|l_1|}{2}\left(\int sdt\right)^2 \quad (18)$$

Then solve its derivative, it satisfies

$$\dot{V} = s\dot{s} + k_5|l_1|\left(s\int sdt\right)$$
$$= l_1\delta s + sl_2(\alpha,q,e) + k_5|l_1|\left(s\int sdt\right) \quad (19)$$

And it can be written as

$$\dot{V} = -|l_1|[k_1 s^2 + k_2\frac{s^2}{|s|+k_3} + k_4 s^{4/3} + k_5 s\int sdt] + sl_2(\alpha,q,e) + k_5|l_1|\left(s\int sdt\right) \quad (20)$$

With the help of inequality method, it can be rewritten as

$$\dot{V} \le -|l_1|[k_1 s^2 + k_2\frac{s^2}{|s|+k_3} + k_4 s^{4/3} + k_5 s\int sdt] + k_6 s^2 + k_7|s| + k_5|l_1|\left(s\int sdt\right)$$
$$= -|l_1|[k_1 s^2 + k_2\frac{s^2}{|s|+k_3} + k_4 s^{4/3}] + k_6 s^2 + k_7|s| \quad (21)$$

So it is easy to choose big enough parameters k_1, k_2 and k_3 such that

$$\dot{V} \le 0 \quad (22)$$

Then according to Lyapunov theory, we can prove $s \to 0$.

Since $c_1 > 0$, then it has

$$e = -c_1\int edt \quad (23)$$

Choose another Lyapunov function as

$$V_a = \left(\int edt\right)^2/2 \quad (24)$$

Then it holds

$$\dot{V}_a \le 0 \quad (25)$$

Then according to Lyapunov theory, we get $\int edt \to 0$, so we have $e \to 0$, then finally we can

prove that the system is stable.

Numerical Simulation and result analysis

Set model parameters as

$$I_{xx} = -7.1*10^{-5}m^2 + 19.1m - 59430$$

$$I_{yy} = -8.03*10^{-4}m^2 + 219.74m - 1690000$$

$$I_{zz} = -8.03*10^{-4}m^2 + 219.74m - 1690000$$

$$I_{yy0} = 1.23*10^7 \quad , \quad v_s = 3.017*10^2, \rho_a = 1.84*10^2,$$

$$g_a = 9.7147, \quad h = 30000$$

$$V = 4525, \eta_c = 0.15662$$

With above model and air coefficients and set initial condition as above paragraph and write a program with m language in Matlab software, then the simulation can be done with above control law where desired value of attack angle is set as 4 degree, and simulation results can be shown as following figures.
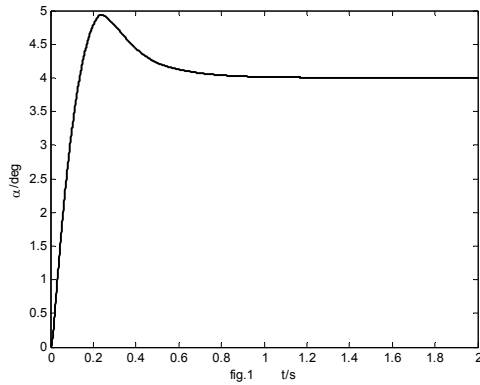

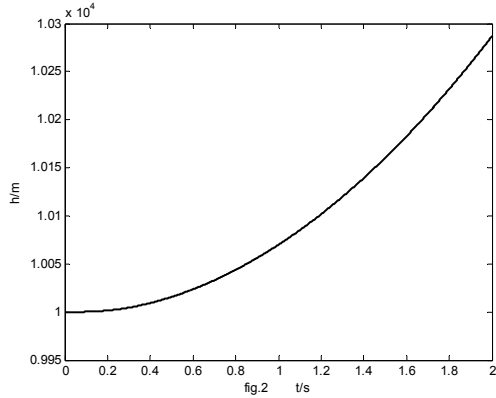Fig 1 The curve of attack angle

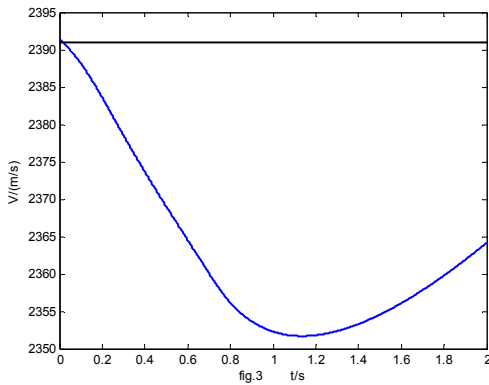
Fig 2 The curve of the height
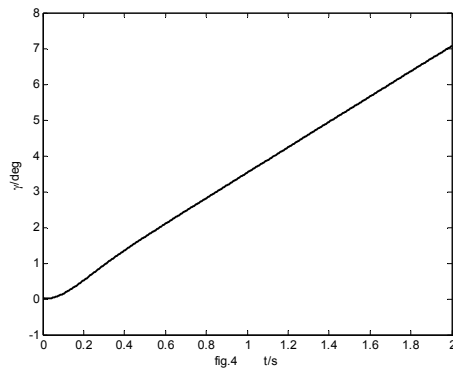

Fig 3 The curve of speed

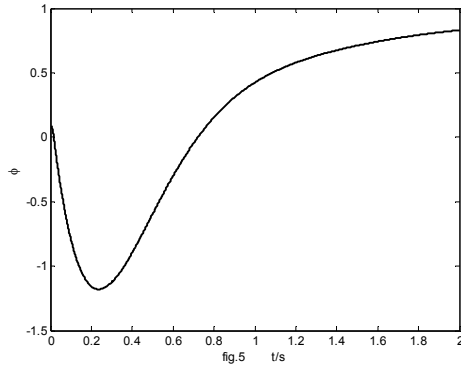

Fig 4 The curve of speed angle
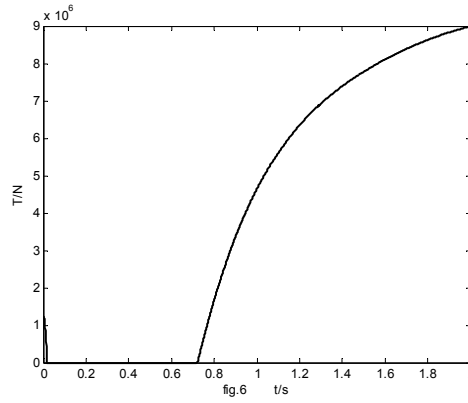
Fig 5 The curve of oil supplying factor   Fig 6 The curve of thrust

We can make a conclusion that the proposed method has good swiftness according to above simulation result, and the response time of attack angle is about 0.3s. And the height and speed of aircraft are in a normal range. So above simulation result shows the proposed sliding mode control method is effective for this kind of aircraft.

Conclusion

A kind of integral sliding mode control method is proposed to solve the attack angle tracking problem of a kind of simplified model of pitch channel of hypersonic aircraft systems. All signal of close system are guaranteed to be bounded and stable by constructing Lyapunov function method. At last, detailed numerical simulation result shows that the proposed method is right and effective, which also has good swiftness.

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