Discussion on the choosing of Lyapunov function for adaptive parameter identification problem

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Keywords: adaptive control, parameter identification, one order system, final value, stability

Abstract: The selection of Lyapunov function is depth studied with adaptive parameter identification, where a kind of simple first order system with two unknown parameters is taken as an example. The study shows that the selection of Lyapunov function is completely free, and the proportion of the parameter identification and the steady-state error part can be matched arbitrarily. But the above two coefficients should be set to be positive, and the different proportion coefficient setting method will lead to different result of steady state error and parameter identification.

1. Introduction

Parameter identification[1-4] provides a kind of solution for the system parameters calculation. In 1962, Zadeh first time proposed the word system identification. According to the definition of Zadeh: "the system identification is based on the input and output data, from a set of given model classes, to determine a system equivalent to the test model.

According to the identification theory, the identification method can be divided into two categories[5-8], the classical identification method and the modern identification method. Adaptive parameter identification[9-12] is a small branch of modern identification method.

In this paper, we discuss the free degree problem of Lyapunov function selection with the adaptive parameter identification method, and the coefficient of the parameter identification and error convergence of the Lyapunov function can be arbitrarily selected. The final study shows that, as long as both parameters are positive, the Lyapunov function is completely free. But with different proportion, the parameter identification and error convergence effect are different.

2. Problem description

An one order system can be written as：

$$\dot{x} = a_1 x + a_2 \sin x + u \qquad \text{（Eq.1）}$$

where a_1 and a_2 are unknown constant parameter, the goal is designing a controller $u = h(x, \hat{a}_1, \hat{a}_2)$ such that

the system state x can trace the expected value x^d and $\hat{a}_i$ can converge to a_i.

3. Design of adaptive identification controller

An ordinary adaptive control method is used as follows, define a error variable as $z_1 = x_1 - x_1^d$, then

$$\dot{z}_1 = \dot{x}_1 - \dot{x}_1^d = a_1 x + a_2 \sin x + u \qquad \text{（Eq.2）}$$

Design state feedback control law as:

$$u = -\hat{a}_1 x - \hat{a}_2 \sin x - \sum_{i=1}^{n} k_i f_i(z_1) \qquad \text{（Eq.3）}$$

Choose $n = 5$, $k_i > 0$

$$f_1(z_1) = z_1, \quad f_2(z_1) = z_1^3, \quad f_3(z_1) = z_1^{1/3} \qquad \text{（Eq.4）}$$

$$f_4(z_1) = \frac{z_1}{|z_1| + \varepsilon}, \quad \varepsilon = 0.2, \qquad \text{（Eq.5）}$$

$$f_5(z_1) = \frac{1 - e^{-\tau z_1}}{1 + e^{-\tau z_1}}, \quad \tau = 0.5 \qquad \text{（Eq.6）}$$

where $f_3(z_1)$ is Terminal attractor, and $f_5(z_1)$ is Sigmoid function, $f_4(z_1)$ and $f_5(z_1)$ are both bounded, Obviously, $f_i(z_1)$ satisfies $z_1 f_i(z_1) \geq 0$, then

$$\dot{z}_1 = \tilde{a}_1 x + \tilde{a}_2 \sin x - \sum_{i=1}^{n} k_i f_i(z_1) \qquad \text{（Eq.7）}$$

where the error variable $\tilde{a}_i$ can be defined as:

$$\tilde{a}_1 = a_1 - \hat{a}_1, \quad \tilde{a}_2 = a_2 - \hat{a}_2 \qquad \text{（Eq.8）}$$

Design regulating law as:

$$\dot{\hat{a}}_1 = \Gamma_1 z_1 x, \quad \dot{\hat{a}}_2 = \Gamma_2 z_1 \sin x \qquad \text{（Eq.9）}$$

where $\hat{a}_i$ is unknown estimated parameter value, choose initial value $\hat{a}_i(0) = 0$, then

$$\dot{\tilde{a}}_1 = -\dot{\hat{a}}_1, \quad \dot{\tilde{a}}_2 = -\dot{\hat{a}}_2 \qquad \text{（Eq.10）}$$

Choose a Lyapunov function as:

$$V = \frac{1}{2} z_1^2 + \frac{1}{2\Gamma_1} \tilde{a}_1^{\,2} + \frac{1}{2\Gamma_2} \tilde{a}_2^{\,2} \qquad \text{（Eq.11）}$$

Then

$$\dot{V} = z_1\dot{z}_1 + \frac{1}{\Gamma_1}\tilde{a}_1\dot{\tilde{a}}_1 + \frac{1}{\Gamma_2}\tilde{a}_2\dot{\tilde{a}}_2 \qquad \text{（Eq.12）}$$

Then:

$$\begin{aligned}\dot{V} &= z_1\tilde{a}_1 x + z_1\tilde{a}_2 \sin x - \sum_{i=1}^{n} k_i z_1 f_i(z_1) - \frac{1}{\Gamma_1}\tilde{a}_1\Gamma_1 z_1 x - \frac{1}{\Gamma_2}\tilde{a}_2\Gamma_2 z_1 \sin x \\ &= -\sum_{i=1}^{n} k_i z_1 f_i(z_1) \le 0\end{aligned} \qquad \text{（Eq.12）}$$

So according to Lyapunov theory we get $z_1 \to 0$.

4. Parameter identification result analysis

When $z_1 \to 0$, where $u = -\hat{a}x$, then

$$\dot{z}_1 = a_1 x - \hat{a}_1 x + a_2 \sin x - \hat{a}_2 \sin x = \tilde{a}_1 x + \tilde{a}_2 \sin x \qquad \text{（Eq.13）}$$

When $z_1 \to 0$, there is $\dot{z}_1 \to 0$, then there is $\dot{z}_1 = \tilde{a}_1 x + \tilde{a}_2 \sin x = 0$, so the parameter can not be identified.

5. Numerical simulation

According to above model, choose unknown parameters as $a_1 = 5$, $a_2 = -3$,set expected value as $x_1^d = 2$, and write m language program as follows:

```
clc;clear; k11=-10;k12=-5;k13=-5;k14=-5;k15=-5;k16=-5;esten1=0.2;taox=3;
```

And use Simulink to construct a program structure as following figures:

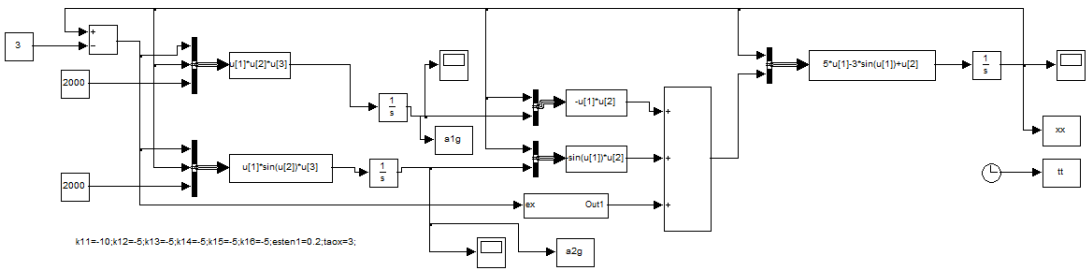

Fig.1 Figure of Simulink Program

Write program as follows to plot figure:

```
figure(1);plot(tt,xx,'k');xlabel('t/s');ylabel('state xx');
figure(2);plot(tt,a1g,'k');xlabel('t/s');ylabel('state a1g');
figure(3);plot(tt,a2g,'k');xlabel('t/s');ylabel('state a2g');
```

Choose the speed of unknown parameter as $\Gamma_1 = -50$, $\Gamma_2 = 50$,then the simulation result is as following figures:

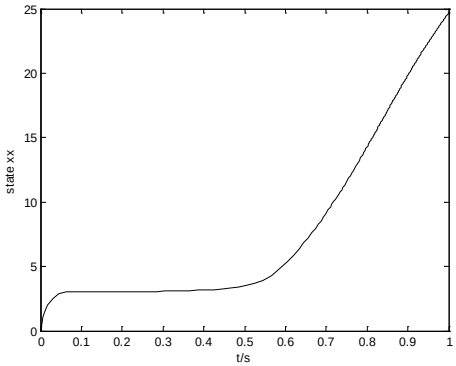

Fig.2 state x1

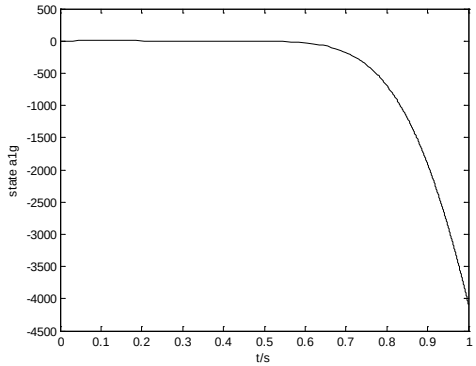

Fig.3 state a1g

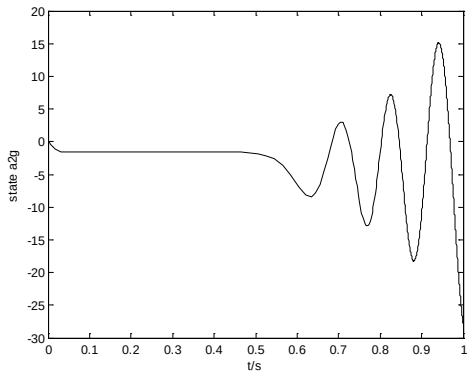

Fig.4 state a2g

The simulation results show that the identification speed of improved integral control is lower than no-integral control.

6. Conclusion

According to above discussion, a conclusion can be made as follows: the choosing of Lyapunov function is totally free. And in other word, the speed of adaptive law is free but it is need to be positive. But if the speed of adaptive law is too big, then the system response will has oscillations. And if the speed is too slow, then the steady state error of the system will converged to zero to quick such that the parameters are not identified yet. In all, a slow speed is good for the identification of unknown parameters and the performance of control is not depended on the adaptive parameter identification very much.

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