

## Discussion on optimization of flight delays

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**Abstract.** Hong Kong South China Morning network according to FlightStats.com of statistics said: China of flights delays most serious, according to this problem, paper proposed has a optimization flights of scheduling optimization algorithm. Taking a certain period of time and a certain airport as the research object, according to reality, subjecting to some conditions, reasonable arrangements flights, making flights delays time reached minimum, last obtained for this airport of most reasonable of flights arrangements. In the process of solving the model, as in the target weight of another, greatly simplifies the process and reduces the amount of computation. lastly, we chose an airport for example, delay time reduction is obtained using the model

### 1. Introduction

With the improvement of people's living standard, more and more people who travel, sharp increases in demand for flights, huge pressure on the airlines, because People travel with seasonal characteristics, holiday flights of vacancies more serious. Becoming an increasingly serious problem of flight delays, flight delays recently has brought great inconvenience to travellers and huge property losses to Airline company, first of all, It is necessary for flight scheduling became a problem to solve.

### 2. scheduling optimization model

#### 2.1 Delay time model

Air corridor into Airways of departure and arrival in each airport air corridor, we can study the Dynamic process of departure and arrival, so as to minimize the delay time of the goal.

Set the time for  $T$  Will Divided into  $N$  time interval  $\Delta$ . That is  $T = N\Delta$ .

Set  $I = \{1, 2 \cdots N\}$  Sequence of time intervals,  $J = \{1, 2 \cdots n_a\}$  and  $K = \{1, 2 \cdots n_b\}$  is the departure air corridor sequence and arrival air corridor sequence, in which  $n_a$  And  $n_b$  Representing free number of departure air corridor and arrival air corridor in the vicinity of the airport. And that variable must satisfy following conditions:

(1) Air corridor traffic balancing

$$X_{i+1}^j = X_i^j + a_i^j - w_i^j, \quad Y_{i+1}^k = Y_i^k + b_i^k - z_i^k \quad (1)$$

Which  $X_{i+1}^j$  means  $i+1$  time interval and  $j$  Arrival corridor the number of delay ariport,  $a_i^j$  means  $i$  time interval  $j$  Arrival corridor the number of flight demand,  $w_i^j$  means  $i$  time period  $j$  Reach the Airways flights were to leave. These relationships can be represented by the following figure

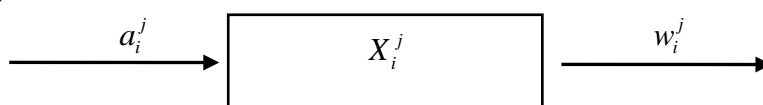


Figure 1. Subsection A time periodOne to the Airways flight flow diagram

(2) Flight delay queue the nonnegativity constraints.

$$X_{i+1}^j \geq 0, \quad Y_{i+1}^k \geq 0 \quad (2)$$

(3) Subsection  $i+1$  At the beginning of a time period, total arrival flight delay and total departure

delays were.

$$X_{i+1} = \sum_{j=1}^{n_a} X_{i+1}^j, \quad Y_{i+1} = \sum_{k=1}^{n_b} Y_{i+1}^k \quad (3)$$

(4) Meanwhile, total reaches capacity  $w_i$  Not from the airport to reach capacity  $u_i$  Total paper capacity  $z_i$  No more than run to start volume  $v_i$ .

$$w_i \leq u_i, \quad z_i \leq v_i \quad (4)$$

(5) based on the airport runway capacity and reach the capacity to meet the nonlinear relationship.

$$v_i = \phi_i(u_i) \quad (5)$$

Their relationship and airport-related, for each airport, the function is known. For example, the runway of the capital international airport in Beijing based on capacity and reach a capacity of relationships as shown in the following figure:

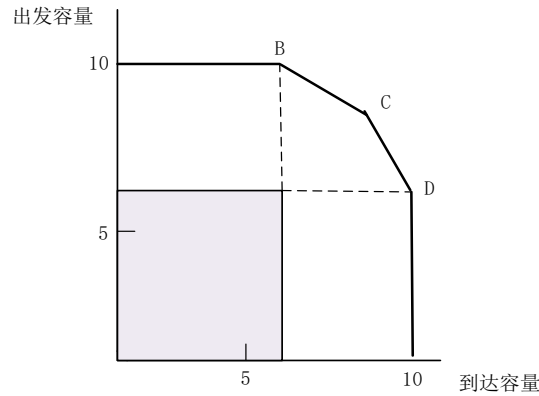


Figure 2. Airport capacity and reach capacity

(6) Airport runway to reach capacity  $u_i$  various Airways flight capacity  $w_i^j$  and  $z_i^k$  are non-negative.

$$u_i \geq 0, w_i^j \geq 0, z_i^k \geq 0 \quad (6)$$

(7) the objective function. We need to plan time T the total time of flight delays in at least, that is the objective function is

$$\min D_T = \left( \sum_{i=1}^N X_{i+1} + \sum_{i=1}^N Y_{i+1} \right) \cdot \Delta \quad (7)$$

Among them,  $D_T$  for the total delay time,  $\sum_{i=1}^N X_{i+1}$  And ,respectively, for the first  $i+1$  a period of time the arrival of the cumulative delay flight number and departure delays flight number.

Because we believe that in case of delay, delay time for  $\Delta$  integer times, the minimum delay time is equivalent to the smallest total flight queue, therefore, the objective function can be written as

$$\min_{u,w,z} D_T = \left( \sum_{i=1}^N X_{i+1} + \sum_{i=1}^N Y_{i+1} \right) \quad (8)$$

Among them, the  $u = \{u_i\}$ ,  $w = \{w_i^j\}$ ,  $z = \{z_i^k\}$  for decision variables set.

Conciusion: established for a particular airport flight flow dynamic programming model,the objective function for the type(8), constraint condition is as follows:

$$X_i^j + a_i^j - w_i^j \geq 0, \quad Y_i^k + b_i^k - z_i^k \geq 0, \quad w_i \leq u_i, z_i \leq \phi_i(u_i), u_i \geq 0, w_i^j \geq 0, z_i^k \geq 0 \quad (9)$$

## 2.2 Delay time model

### The first step, the expression Variant

Type (1) for deformation available:

$$X_{i+1}^j = X_1^j + \sum_{p=1}^i a_p^j - \sum_{p=1}^i w_p^j, \quad Y_{i+1}^k = Y_1^k + \sum_{p=1}^i b_p^k - \sum_{p=1}^i z_p^k \quad (10)$$

Therefore, (9) the first constraint can be rewritten as

$$X_1^j + \sum_{p=1}^i a_p^j \geq \sum_{p=1}^i w_p^j, \quad Y_1^k + \sum_{p=1}^i b_p^k \geq \sum_{p=1}^i z_p^k \quad (11)$$

Combined (7), the objective function (8) can be rewritten as

$$\max_{u, w, z} \sum_{i=1}^N \left[ \sum_{p=1}^i \left( \alpha \sum_{j=1}^{n_a} w_p^j + (1-\alpha) \sum_{k=1}^{n_b} z_p^k \right) \right] = \max_{u, w, z} \sum_{i=1}^N (N-i+1) \left( \alpha \sum_{j=1}^{n_a} w_i^j + (1-\alpha) \sum_{k=1}^{n_b} z_i^k \right) \quad (12)$$

So, delayed flights queue delay minimum total time of the problem becomes a flight maximum flow problem, and consider the weight of the flow arrival and departure flight flow.

### The second step, new optimization algorithm

In type (12), the flight will arrive per interval flow and flight departure flow weighted and

$\left( \alpha \sum_{j=1}^{n_a} w_i^j + (1-\alpha) \sum_{k=1}^{n_b} z_i^k \right)$  as an element, the  $(N-i+1)$  as the weight of the element. So, it is equivalent to a maximum of the weighted element, make each element of the largest. Method as follows:

(1) First, to optimize the first time interval, that initial delay is zero, When meet the objective function (12) results:  $u_1, w_1^j, z_1^k$ .

(2) 1-2 optimize the time interval, use the result in the previous step to get  $u_2, w_2^j, z_2^k$ .

(3) And so on, until the 1-N time to optimize the obtained set of decision variables  $u = \{u_i\}$ ,  $w = \{w_i^j\}$ ,  $z = \{z_i^k\}$ .

In this new optimization method can greatly reduce the amount of calculation and computation time required is significantly reduced, and improve work efficiency.

### 3. Delay time model examples

There are 7 corridor around the capital international airport, 2 corridor for military corridor, the remaining six for the civil aviation in the corridor, into the left already separate, one-way corridor. No. 1 and no. 3, and 5 to reach the air corridor, from no. 1 into the corridor of aircraft accounted for 80%, 3, 6, each accounted for 10%; No. 4, 6, 7 for starting air corridor, corridor of the plane from 4 out of the corridor about 40% 10%, 6, 7, 50%. Each corridor has a fixed route direction, for example, in taiyuan, capital of wei country, potow, tianjin and other directions planes, from no. 1 corridor into [9].

Beijing capital international airport air corridor of total capacity of 60. Assuming that the air corridor is directly proportional to the current capacity and demand, is the capacity of an air corridor are shown in table (1) as shown in the table below:

Table 1. The air corridor capacity

空中走廊序号	容量
1	24
到达空中走廊	3
5	3
4	3
出发空中走廊	12
7	15
军用走廊	未知

Take the capital international airport is an hour's flight as the object of study, the time period T is divided into 10 times, finishing by the airport and the surrounding area the demand of each period is as follows:

Table 2. The aviation needs of the airport and the surrounding area  
According to its requirements, within the guarantee period T the air corridor total flights to meet

时间段	到达空中走廊			出发空中走廊		
	1	3	5	4	6	7
1	10	2	2	2	10	14
2	26	4	4	3	10	12
3	30	4	4	2	15	17
4	16	7	8	6	10	19
5	30	2	1	2	14	12
6	10	1	1	6	13	15
7	14	0	2	1	10	11
8	16	1	2	2	14	12
9	26	4	2	2	6	14
10	20	1	1	2	10	13
总计	198	26	27	28	112	139

demand under the premise of the distribution of each flight departure and arrival time. When calculating result table shown in (3).

According to the arrangement of table 3, minimizing the total time delays the time period. Within the time period T, total time delays can reduce the time of 214 units, which is 1284 minutes, the average 2.42 minutes to reduce flight delays.

It shows that the model can obviously reduce the total time of flight delays.

Table 3. The Airways flights

时间段	到达空中走廊			出发空中走廊		
	1	3	5	4	6	7
1	10	2	2	2	10	14
2	24	3	3	3	10	12
3	24	3	3	2	12	15
4	24	3	3	3	12	15
5	24	3	3	3	12	15
6	16	3	3	3	12	15
7	14	3	3	3	12	14
8	16	1	3	3	12	12
9	24	3	3	3	10	14
10	22	2	2	2	10	13
总计	198	26	27	28	112	139

#### 4. Summary

This model combined with the flight information such as the type and capacity on one hand, through the reasonable arrangement of the plane on the runway debut order, make to minimize the economic loss of flight delays; On the other hand, combined with the air traffic demand and the relationship between the airport capacity, dynamic adjustment of flight capacity and airport capacity, for airlines flight arrangement and management provides a balance between efficiency and the accuracy of the optimization algorithm, and realize the goal of flight delay time shortest.

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