

Application of firefly algorithm of solving equation group

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Abstract: An improved Firefly Algorithm is proposed for solving equation group. In the proposed algorithm, a heuristic exchange rule is introduced. Finally, numerical testing results are provided, and the comparisons demonstrate the effectiveness of the proposed algorithm for solving the equation groups.

1. Introduction

The equations is one of the basic problems in numerical linear algebra, many engineering application and scientific computing problems require the solution of equations finally, therefore, studying the equations has important significance^[1]. Linear equation group can be divided into the ill-conditioned linear equations and well-conditioned linear equations. The traditional method mainly has the Newton method^[2], iterative method, gradient method, evolutionary algorithms such as PSO algorithm^[3]. The traditional method has high accuracy, but the equations with high performance requirements, such as the requirements of continuous and differentiable equations. Evolutionary algorithm can improve the population size and the number of iterations to improve accuracy, the solving time is proportional to iterative times, time efficiency decreased obviously.

2. The basic principle of the firefly algorithm

2.1 The bionics principle of algorithm

The firefly algorithm is the development of simulating biological characteristics of the adults firefly, but the algorithm abandons some biological significance of luminous firefly, the firefly searches partners according to the search area only using the emission properties, and the firefly moves to position better firefly in the neighborhood structure, so as to realize the position evolution. In this algorithm, the firefly attracts one another reason depends on two factors, namely, its brightness and attraction. Among them, the firefly fluorescence brightness depends on the location of the target value, the brightness higher the location of the target value the better. The more light of fireflies has more attractive, it can attract brightness than its weak firefly in sight range to the direction of movement. If the fireflies have the same brightness, then they randomly move. The brightness and attraction are inversely proportional to the distance of the fireflies, they have increased as the distance decreases, which is equivalent to the simulation of the fluorescence characteristics of propagation in the space by the media absorption and fading. The firefly algorithm is adopted to simulate the firefly group behavior to construct a class of stochastic optimization algorithm. The bionic principle is: The points in the search space simulate the nature of firefly individuals, the search and optimization process simulation to attract and movement of the firefly individuals, the objective function of solving the problem is measured into the pros and cons of the location of the individual, the fittest of individual process for search and optimization process of a good analogy of feasible solutions to replace less feasible solutions.

2.2 Mathematical description and analysis of the algorithm

As mentioned above, the firefly algorithm includes two factors: the brightness and the degree of attraction. The brightness of the firefly reflects the pros and cons of location and determines its direction of movement, attraction degree determines the distance of the firefly mobile, constantly update the brightness and the attraction degree, so as to realize the goal of optimization. From a

mathematical perspective describes the Firefly algorithm optimization mechanism, the description as follows^[4,5].

(1) The relative fluorescence intensity of fireflies as:

$$h = \frac{h_0}{(1 + zr_{ij}^2)} \quad (1)$$

Among them: h_0 for the maximum fluorescence intensity of fireflies, that is itself ($r=0$) fluorescence brightness, associated with the value of the objective function, the objective function value is better, its brightness is higher; z as the light absorption coefficient, because the fluorescence decreases with the increasing of distance and media absorption^[6], so set the light absorption coefficient to reflect this characteristic, it can set to constant; r_{ij} is the space distance of the firefly i and the firefly j .

(2) The firefly attraction degree is:

$$\rho = \rho_0 * e^{-zr_{ij}^2} \quad (2)$$

Among them: ρ_0 as the biggest attraction, namely the light source ($r=0$) attraction degree; z , r_{ij} meaning as above.

(3) The firefly i is attracted to move to the firefly j , the location update by formula (3) decision.

$$x_i' = x_i + \rho * (x_j - x_i) + \alpha * (rand - 1/2) \quad (3)$$

Among them: x_i' is the individual i towards a brighter individual j update location, x_i, x_j are the firefly i and the firefly j located before the entire population renewal; α is the step factor, it is constant between 0 and 1, $rand$ is a random factor between 0 and 1, it obeys uniform distribution.

The process of optimization algorithm is: Firstly, the firefly populations randomly scattered in the solution space, each firefly has different fluorescence brightness at different positions, by comparing Eq.1, high brightness fireflies can attract low brightness fireflies to move, moving distance depending on the size of the attraction (according to Eq.2). In order to increase the search area, avoid getting into local optimization, and updated measures, according to the formula 3 to calculate the updated position. So by repeatedly movement, all individuals will be gathered in the highest brightness firefly position, so as to realize the optimization.

At present, it has been found that many insects existing Levy flight^[7], and Levy flight has been used in the field of optimization, and achieve the expected effect. In order to enhance the algorithm global search performance, avoid the population into a local optimum in the search process, in the firefly algorithm, if the individual is no better than their individual, choose to Levy flight instead of random flight in the original algorithm. In addition, the non - optimal those individuals in a population, the flight formula was improved: when they find more bright than their individual, first generates a random number q by the system, if q is less than 0.5, the formula (4) is updated; otherwise, still use the formula (3) to update the individual position.

$$x_i'' = x_i + \rho * (x_j - x_i) + \alpha * (rand - 1/2) \quad (4)$$

Among them,: x_j still expresses renewal position of individual j before the entire population, x_i' expresses the individual i toward the front of $j-1$ individuals than their bright individual after the update new position, x_i'' expresses x_i' toward than their bright individual j after update location, ρ expresses that individual j appeal to the individual i . As can be seen, formula (4) is updated in real time, formula (3) depends only on the entire population before moving. This flight update can increase the randomness of flight, It is helpful to keep the diversity of population, increasing the population search space.

In order to accelerate the convergence of the population, the paper proposes a method of α updating, which α gradually decreases with increasing number of iterations. Update formula is as follows:

$$\alpha = \alpha_0 - e^{-0.001*t} \quad (5)$$

In the formula (5): α_0 is 0.9, t is number of iterations.

The concrete steps of the algorithm are as follows:

1) Initialization the basic parameter of algorithm. Set the number of fireflies is s , the biggest attraction is ρ_0 , light absorption coefficient is z , the random parameters is α_0 , the maximum number of iterations is $t_{\max}=1000$, the evolving algebra $t=0$.

2) Randomly initialization firefly position, calculate the objective function value of firefly as the respective maximum fluorescence intensity of h_0 .

3) By formula 1 and formula 2 to calculate the relative brightness of h and attraction ρ , according to relative brightness of h to decide the movement direction of the firefly.

4) The formula 5 is used to update α , the individual, if there is more lighter than its individual, in accordance with the update is improved, by formula (4) to update the individual position; otherwise, the Levy flight is used to update the individual location.

5) According to the updated firefly position, to recalculate brightness of the firefly.

6) The system generates a random number, if the random number is less than the local search probability p , then local search of individuals of the population, and regenerate population.

7) Determine whether meet the conditions of termination of the algorithm. Such as the maximum number of iterations t_{\max} is 1000 or best solution stagnation does not change, turn to step 7, or $t = t + 1$, turn to step 3.

8) Output global extreme value point and optimal individual value.

3. The analysis of simulation

In order to verify the performance of the novel firefly algorithm of this paper, this paper compares the performance of the novel firefly algorithm of this paper with the basic firefly algorithm, and selects the following 5 typical benchmark functions to verify the optimization problem:

The first test function is Schwefel function:

$$f_1(x) = \sum_{i=1}^d |x_i| + \prod_{i=1}^d |x_i|, \quad -10 \leq x_i \leq 10, d = 50 . \text{The function obtains the global}$$

minimum value at the point $x_i=0$.

The second test function is Rastrigrin function:

$$f_2(x) = \sum_{i=1}^d [x_i^2 - 10 \cos(2\pi x_i) + 10], \quad -5.12 \leq x_i \leq 5.12, d = 50 . \text{The function}$$

obtains the global minimum value at the point $x_i=0$.

The third test function is Griewank function:

$$f_3(x) = \frac{1}{4000} \sum_{i=1}^d x_i^2 - \prod_{i=1}^d \cos \frac{x_i}{\sqrt{i}} + 1, \quad -600 \leq x_i \leq 600, d = 50 . \text{The function}$$

obtains the global minimum value at the point $x_i=0$.

The fourth test function is Rosenbrock function:

$$f_4(x) = \sum_{i=1}^{d-1} \left(100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \right), \quad -30 \leq x_i \leq 30, d = 50 . \text{The function}$$

obtains the global minimum value at the point $x_i=0$.

The fifth test function is Ackley function:

$$f_5(x) = 20 + e - 20 \exp\left(-0.2\sqrt{\frac{1}{d} \sum_{i=1}^d x_i^2}\right) - \exp\left(\frac{1}{d} \sum_{i=1}^d \cos(2\pi x_i)\right), \quad -30 \leq x_i \leq 30, d = 50$$

The function obtains the global minimum value at the point $x_i=0$.

Each test function is run 50 times, respectively, the optimal value, the difference, the average, the standard deviation and the average time consuming 5 parameters are compared. This experiment uses the computer for AMD Athlon (tm) IIX2B24, 2GB memory PC, programming software for the Matlab2010a, the results of the experiment are as shown in Table 1.

Tab 1 Compare of experimental result

Function	Algorithm	The optimal value	The worst value	Average value	Standard deviation	Average consuming time
f_1	Firefly algorithm	2.0017042e+002	3.7324562e+018	1.1987665e+017	5.3876567e+017	0.2098078
	Novel firefly algorithm	9.3876892e-005	19.1546398	1.7456403	4.1434543	0.7567564
f_2	Firefly algorithm	5.1674367e+002	6.6453769e+002	5.8645673e+002	28.3454310	0.1900877
	Novel firefly algorithm	1.0143653e-008	1.9789567e+002	39.7896543	17.1454309	0.7354680
f_3	Firefly algorithm	7.4532564e+002	1.2435432e+003	1.0897787e+003	92.0100987	0.2455432
	Novel firefly algorithm	0.4213431	1.5343214e+002	16.1565480	27.5430980	0.7987980
f_4	Firefly algorithm	3.0765743e+010	5.5987899e+010	4.6778950e+010	5.0709004e+009	0.1765890
	Novel firefly algorithm	0.0234213	6.5345645e+008	6.8906789e+007	1.3905644e+008	0.7165720
f_5	Firefly algorithm	0.0009785	19.5896754	14.7908978	5.2987898	0.2254365
	Novel firefly algorithm	1.6574326e-004	15.0567843	7.0005643	4.4656755	0.8453459

4. Conclusions

This paper proposes an improve firefly algorithm, and it is applied to equation groups. Due to firefly algorithm has the robustness and high computational efficiency, it is suitable for parallel computing, it has great practical value for solving large-scale. Experiments show that: It is feasible and effective to solve equation group.

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