A delay-dependent approach to robust *H∞* control for stochastic system with interval time delay

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Abstract: This note deals with the problem of robust H∞ control for stochastic system with interval time delay. By using both, Itô's differential formula and Lyapunov function, a new design method is proposed. Meanwhile, design a feedback controller, such that solve the problem of robust stochastic stabilization, the expressions of feedback controlle ris given. Finally, an acdemic example is include to illustrate the effectiveness of result.

Introduction

During the past years, people's attention have been devoted to the study of time delay due to the fact that time delay is often the main cause for instability and poor performance of a control system[1, 2]. For example, delay-dependent stability criterion for systems with uncertain time-invariant delays was discussed in [3]. The H∞control problem for stochastic systems was studied, and a stochastic bounded real lemma was derived in [4] by using the Moons inequalities approach, [5] investigated the problem of delay dependent robust stochastic stabilization and H∞ control for stochastic systems with norm-bounded uncertainties and state delay.

For the case of uncertain linear systems, i.e. for proving the robust stability, the problem has been partially solved, either by using Lyapunov functionals[6], or robustness tools(quadratic separation [7]). All resulting stability conditions are based on Linear Matrix Inequality (LMI) and allow to conclude on stability region with respect to the delay. However, when revisiting this problem, we find that the lower bound of delay range for the most present research is limited to zero, the selected Lyapunov function mostly does not contain the information of the lower bound of delay.

Although much attention has been focused on the robustH∞ control for stochastic systems, the method therein still leaves much room for improvement.

This paper aims at design H∞ controller for stochastic system with interval time delay, which use proper Lyapunov function, Itô's formula and introducing free weighting matrix. Finally, an example illustrating the proposed approach is provided.

Problem Formulation

Consider the following stochastic system with state delay and input delay:

$dx(t)=[Ax(t)+A_\tau x(t-\tau(t))+Bu(t)+B_\tau u(t-\tau(t))+B_V v(t)]dt+[Ex(t)+E_\tau x(t-\tau(t))+E_V v(t)]d\omega(t)$(1)

$z(t) = Cx(t) + C_\tau x(t-\tau(t)) + Du(t) + D_\tau u(t-\tau(t))$(2)

$x(t) = \varphi(t), u(t) = \varphi(t), \forall t \in [-h_M, 0]$(3)

where $x(t) \in R^n$ is the state; $u(t) \in R^m$ is the control input; $v(t) \in R^p$ denotes for the disturbance which belongs to $L_2[0,\infty)$; $z(t) \in R^q$ stands for the controlled output; v(t) is a one-dimensional Brownian motion satisfying $\varepsilon\{d\omega(t)\} = 0$ and $\varepsilon\{d\omega(t)^2\} = dt$,A, A_τ ,B, B_τ,B_v,E,E_τ,E_v,C,C_τ,D,$D_\tau$are known real constant matrices matrices, and $\tau(t)$is the time-varying delay satisfying

$$h_m < \tau(t) < h_M, \dot{\tau} \le u \quad (4)$$

where h_m and$h_M$are real constant scalars.

Lemma 1. For any constant matrix $M > 0$, any scalars a and b with $a < b$, and a vector function

$x(t): [a,b] \rightarrow R^n$ such that the integrals concerned are well defined the following holds:

$$\left[\int_a^b x(s)ds\right]^T M \left[\int_a^b x(s)ds\right] \le (b-a)\int_a^b x^T(s)Mx(s)ds \qquad (5)$$

H∞ Performance Analysis

We design a feedback gain matrix $u(t) = Kx(t)$, the closed-loop stochastic system consisting of (1) and (2) can be rewritten in the following form:

$$x(t) = \left[\bar{A}_K x(t) + \bar{A}_{\tau_K} x(t-\tau(t)) + B_v v(t)\right]dt + \left[Ex(t) + E_\tau x\big(t-\tau(t)\big) + E_v v(t)\right]d\omega(t) \qquad (6)$$

$$z(t) = C_K x(t) + D_{\tau K} u(t - \tau(t)) \qquad (7)$$

where

$$\bar{A}_K = A + BK \qquad \bar{A}_{\tau K} = A_\tau + B_\tau K$$

$$E = E \qquad E_\tau = E_\tau$$

$$C_K = C + DK \qquad D_{\tau K} = C_\tau + D_\tau K$$

In this section, a delay-dependent approach is proposed to solve the problem of robust stochastic stabilization with disturbance attenuation level. We first assume that the feedback gain matrix K is known. For robust $H\infty$ performance analysis of the system (6-7), we have the following result.

Theorem 1. Consider the closed-loop system (6)-(7). For given scalars $\gamma > 0$, $h_m > 0, h_M > 0$, μ > 0, and feedback gain K, the stochastic system (6) and (7) is robustly stochastically stable with disturbance attenuations γ for any $\tau(t)$ satisfying $h_m < \tau(t) < h_M, \dot{\tau}(t) \le u$, if there exist matrices $P > 0, Q_q > 0, Z_1 > 0, Z_2 > 0, N_p, G_p, M_q, S_q, (p\ 1,2, q = 1,2,3)$, satisfying the following LMI

$$\begin{bmatrix}
\varphi_{11} & \varphi_{12} & H_1 & -G_1 & \varphi_{15} & \varphi_{16} & -N_1 & -G_1 & -H_1 & \varphi_{1,10} & \varphi_{1,11} \\
* & \varphi_{22} & H_2 & -G_2 & \varphi_{25} & \varphi_{26} & -N_2 & -G_2 & -H_2 & \varphi_{2,10} & \varphi_{2,11} \\
* & * & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \varphi_{55} & 0 & 0 & 0 & 0 & \varphi_{5,10} & 0 \\
* & * & * & * & * & \varphi_{66} & 0 & 0 & 0 & \varphi_{6,10} & 0 \\
* & * & * & * & * & * & -\frac{Z_1}{h_M} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\frac{Z_1+Z_2}{h_M-h_m} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\frac{Z_2}{h_M-h_m} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\gamma^2 I & 0 \\
* & * & * & * & * & * & * & * & * & * & -I
\end{bmatrix}$$

$\varphi_{11} = Q_1 + Q_2 + Q_3 + N_1 + N_1^T + M_1(A+BK) + (A+BK)^T M_1^T + S_1 E + E^T S_1^T$

$\varphi_{12} = -N_1 + N_2^T + G_1 - H_1 + M_1(A_\tau + B_\tau K) + (A+BK)^T M_2^T + S_1 E_\tau + E_\tau^T S_1^T$

$\varphi_{22} = -(1-u)Q_3 - N_2 - N_2^T + G_2 - H_2 - H_2^T + M_2(A_\tau + B_\tau K) + (A_\tau + B_\tau K)^T$
$M_2^T + S_2 E_\tau + E_\tau^T S_2^T$

$\varphi_{15} = P - M_1 + (A+BK)^T M_3^T$

$\varphi_{16} = -S_1 + E^T S_3^T$

$\varphi_{1,10} = M_1 B_V + S_1 E_V$

$\varphi_{1,11} = C^T + K^T D^T$

$\varphi_{25} = -M_2 + (A_\tau + B_\tau K)^T M_3^T$

$\varphi_{26} = -S_2 + E_\tau^T S_3^T$

$\varphi_{2,10} = M_2 B_v + S_2 E_v$

$\varphi_{2,11} = C_\tau^T + K^T D_\tau^T$

$\varphi_{55} = h_M Z_1 + (h_M - h_m)Z_2 - M_3 - M_3^T$

$\varphi_{66} = P - S_3 - S_3^T$

$\varphi_{5,10} = M_3 B_v$
$\varphi_{6,10} = S_3 E_v$

Proof. First, we define two new state variables

$$y(t) = \bar{A}_k x(t) + \bar{A}_{\tau K} x\big(t - \tau(t)\big) + B_v v(t) \tag{9}$$

$$g(t) = E x(t) + E_\tau x\big(t - \tau(t)\big) + E_v v(t) \tag{10}$$

Then, the closed-loop system (6) can be represented as

$dx(t) = y(t)d(t) + g(t)d\omega(t)$(11)

Now, choose a Lyapunov-Krasovskii functional as

$$\begin{aligned} V(x_t, t) &= x^T(t)Px(t) \\ &+ \int_{t-h_M}^{t} x^T(s)Q_1 x(s)ds + \int_{t-h_m}^{t} x^T(s)Q_2 x(s)ds + \int_{t-\tau(t)}^{t} x^T(s)Q_3 x(s)ds \\ &+ \int_{-h_M}^{0}\int_{t+\theta}^{t} y^T(s)Z_1 y(s)dsd\theta + \int_{-h_M}^{-h_m}\int_{t+\theta}^{t} y^T(s)Z_2 y(s)dsd\theta \end{aligned} \tag{12}$$

Then, by Itô's formula we can obtain the stochastic differential as

$$dV(x_t, t) = \mathfrak{I}V(x_t, t)dt + 2x^T(t)Pg(t)d\omega(t)$$

where

$$\begin{aligned} LV(x_t, t) = {} & 2x^T(t)Py(t) + g^T(t)Pg(t) + x^T(t)\left(\sum_{i=1}^{3} Q_i\right)x(t) - x^T(t - h_M)Q_1 x(t - h_M) \\ & - x^T(t - h_m)Q_2 x(t - h_m) - (1 - \dot{\tau}(t))x^T\big(t - \tau(t)\big)Q_3 x\big(t - \tau(t)\big) \\ & + y^T(t)[h_M Z_1 + (h_M - h_m)Z_2]y(t) - \int_{t-\tau(t)}^{t} y^T(s)Z_1 y(s)ds \\ & - \int_{t-h_M}^{t-\tau(t)} y^T(s)(Z_1 + Z_2)y(s)ds - \int_{t-\tau(t)}^{t-h_m} y^T(s)Z_2 y(s)ds \\ & \leq 2x^T(t)Py(t) + g^T(t)Pg(t) + x^T(t)\left(\sum_{i=1}^{3} Q_i\right)x(t) - x^T(t - h_M)Q_1 x(t - h_M) \\ & - x^T(t - h_m)Q_2 x(t - h_m) - (1 - u)x^T(t - \tau(t))Q_3 x\big(t - \tau(t)\big) \\ & + y^T(t)[h_M Z_1 + (h_M - h_m)Z_2]y(t) - \int_{t-\tau(t)}^{t} y^T(s)Z_1 y(s)ds \\ & - \int_{t-h_M}^{t-\tau(t)} y^T(s)(Z_1 + Z_2)y(s)ds - \int_{t-\tau(t)}^{t-h_m} y^T(s)Z_2 y(s)ds \end{aligned} \tag{13}$$

Then, it follows from Lemma 1, that

$$\begin{aligned} & -\int_{t-\tau(t)}^{t} y^T(s)Z_1 y(s)ds \\ & \leq -\frac{1}{h_M}\left[\int_{t-\tau(t)}^{t} y(s)ds\right]^T Z_1 \left[\int_{t-\tau(t)}^{t} y(s)ds\right] - \int_{t-h_M}^{t-\tau(t)} y^T(s)(Z_1 + Z_2)y(s)ds \\ & \leq -\frac{1}{h_M - h_m}\left[\int_{t-h_M}^{t-\tau(t)} y(s)ds\right]^T (Z_1 + Z_2) \times \left[\int_{t-h_M}^{t-\tau(t)} y(s)ds\right] \\ & - \int_{t-\tau(t)}^{t-h_m} y^T(s)Z_2 y(s)ds \leq -\frac{1}{h_M - h_m}\left[\int_{t-\tau(t)}^{t-h_m} y(s)ds\right]^T Z_2 \left[\int_{t-\tau(t)}^{t-h_m} y(s)ds\right] \end{aligned} \tag{16}$$

Now, define the new vector $e_1(t)$ as

$$e_1^T(t) = [x^T(t)x^T(t - \tau(t))]$$

For free-weighting matrices N,G and H, the following equalities hold:

$\eta_1(t) = 2e_1^T(t)N\left[x(t) - x\big(t-\tau(t)\big) - \int_{t-\tau(t)}^{t} y(s)ds - \int_{t-\tau(t)}^{t} g(s)d\omega(s)\right] = 0$ (17)

$\eta_2(t) = 2e_1^T(t)G\left[x\big(t-\tau(t)\big) - x(t-h_M) - \int_{t-h_M}^{t-\tau(t)} y(s)ds - \int_{t-h_M}^{t-\tau(t)} g(s)d\omega(s)\right] = 0$ (18)

$\eta_3(t) = 2e_1^T(t)H\left[x(t-h_m) - x\big(t-\tau(t)\big) - \int_{t-\tau(t)}^{t-h_m} y(s)ds - \int_{t-\tau(t)}^{t-h_m} g(s)dw(s)\right] = 0$ (19)

Where

$N = [N_1^T, N_2^T]^T, G = [G_1^T, G_2^T]^T, H = [H_1^T, H_2^T]^T$

Similarly, for matrix $M = [M_1^T, M_2^T, M_3^T]^T, S = [S_1^T, S_2^T, S_3^T]^T$ with compatible dimensions, the following equalities hold:

$\eta_4(t) = 2e_2^T(t)M\left[\bar{A}_K x(t) + \bar{A}_{\tau K} x\big(t-\tau(t)\big) + B_v v(t) - y(t)\right] = 0$ (20)

$\eta_5(t) = 2e_3^T(t)S\left[Ex(t) + E_\tau x\big(t-\tau(t)\big) + E_v v(t) - g(t)\right] = 0$(21)

Where

$e_2^T(t) = [x^T(t)x^T(t-\tau(t))y^T(t)]$ (22)

$e_3^T(t) = [x^T(t)x^T(t-\tau(t))g^T(t)]$ (23)

Let$F\big(d\omega(t)\big) =$

$-2e_1^T(t)N\int_{t-\tau(t)}^{t} g(s)d\omega(s) - 2e_3^T(t)G\int_{t-h_M}^{t-\tau(t)} g(s)dw(s) - 2e_3^T(t)H\int_{t-\tau(t)}^{t-h_m} g(s)d\omega(s)$.Note that the mathematical expectation of $F(d\omega(t))$ equals$\varepsilon F(d\omega(t)) = 0$ .Then, combining (13)-(21) gives

$$\varepsilon LV(x_t, t) \le \xi^T(t)\Lambda_1\xi(t)$$

where

$$\Lambda_1 = \begin{bmatrix} \tilde{\varphi}_{11} & \tilde{\varphi}_{12} & H_1 & -G_1 & \varphi_{15} & \varphi_{16} & -N_1 & -G_1 & -H_1 & \varphi_{1,10} \\ * & \tilde{\varphi}_{22} & H_2 & -G_2 & \varphi_{25} & \varphi_{26} & -N_2 & -G_2 & -H_2 & \varphi_{2,10} \\ * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \varphi_{55} & 0 & 0 & 0 & 0 & \varphi_{5,10} \\ * & * & * & * & * & \varphi_{66} & 0 & 0 & 0 & \varphi_{6,10} \\ * & * & * & * & * & * & -\frac{Z_1}{h_M} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\frac{Z_1+Z_2}{h_M-h_m} & 0 & 0 \\ * & * & * & * & * & * & * & * & -\frac{Z_2}{h_M-h_m} & 0 \\ * & * & * & * & * & * & * & * & * & 0 \end{bmatrix}$$

$$\xi^T(t) = \Big[x^T(t)x^T(t-\tau(t))x^T(t-h_m)x^T(t - h_M)y^T(t)g^T(t)\int_{t-\tau(t)}^{t} y^T(s)ds\int_{t-h_M}^{t-\tau(t)} y^T(s)ds\int_{t-\tau(t)}^{t-h_m} y^T(s)ds\, v^T(t)\Big]$$

Where

$\tilde{\varphi}_{11} = Q_1 + Q_2 + Q_3 + N_1 + N_1^T + M_1(A+BK) + (A+BK)^T M_1^T + S_1E + E^T S_1^T$

$\tilde{\varphi}_{12} = -N_1 + N_2^T + G_1 - H_1 + M_1(A_\tau + B_\tau K) + (A+BK)^T M_2^T + S_1E_\tau + E^T S_2^T$

$\varphi_{22} = -(1-u)Q_3 - N_2 - N_2^T + G_2 + G_2^T - H_2 - H_2^T + M_2(A_\tau + B_\tau K) + (A_\tau + B_\tau K)^T M_2^T + S_2E_\tau + E_\tau^T S_2^T$

Now, we set

$$J(t) = \varepsilon\left\{\int_0^t [z^T(s)z(s) - \gamma^2 v^T(s)v(s)]ds\right\}$$

where $t > 0$. Because $V(\varphi(t), 0) = 0$ under zero initial conditions, that is, $\varphi(t) = 0$ for $t \in [-h_M, 0]$, then, by Itô's formula, we derive

$$J(t)=\varepsilon\left\{\int_0^t [z^T(s)z(s)-\gamma^2 v^T(s)v(s)]ds\right\}$$
$$=\varepsilon\left\{\int_0^t [z^T(s)z(s)-\gamma^2 v^T(s)v(s)+\Im V(x_s,s)]ds\right\}-\varepsilon V(X_t,t)$$
$$\leq \varepsilon\left\{\int_0^t [z^T(s)z(s)-\gamma^2 v^T(s)v(s)+\Im V(x_s,s)]\,ds\right\}\leq \varepsilon\left\{\int_0^t [\xi^T(s)\Xi\xi(s)]ds\right\}$$

Where

$$\Xi=\begin{bmatrix}
\tilde{\varphi}_{11} & \tilde{\varphi}_{12} & H_1 & -G_1 & \varphi_{15} & \varphi_{16} & -N_1 & -G_1 & -H_1 & \varphi_{1,10}\\
* & \tilde{\varphi}_{22} & H_2 & -G_2 & \varphi_{25} & \varphi_{26} & -N_2 & -G_2 & -H_2 & \varphi_{2,10}\\
* & * & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & \varphi_{55} & 0 & 0 & 0 & 0 & \varphi_{5,10}\\
* & * & * & * & * & \varphi_{66} & 0 & 0 & 0 & \varphi_{6,10}\\
* & * & * & * & * & * & -\frac{Z_1}{h_M} & 0 & 0 & 0\\
* & * & * & * & * & * & * & -\frac{Z_1+Z_2}{h_M-h_m} & 0 & 0\\
* & * & * & * & * & * & * & * & -\frac{Z_2}{h_M-h_m} & 0\\
* & * & * & * & * & * & * & * & * & -\gamma^2 I
\end{bmatrix}$$

$\varphi_{11}=\varphi_{11}+(C+DK)^T(C+DK)$
$\varphi_{12}=\varphi_{12}+(C+DK)^T(C_\tau+D_\tau K)$
$\varphi_{22}=\varphi_{22}+(C_\tau+D_\tau K)^T(C_\tau+D_\tau K)$

Applying Schurs complement to (8), we know that $\Xi<0$, Moreover, $J(t)<0$, for all $t>0$. Consequently, $\| z(t)\|_{E_2}<\gamma\| v(t)\|_2$, holds for any nonzero $v(t)\in L_2[0,\infty)$. Furthermore, when $v(t)=0$, we can conclude that the trivial solution of the resulting closed-loop system (6) is robustly stochastically stable.The proof is completed.

Robust H∞ Control

This section is devoted to the design of the feedback gain matrix K such that the resulting closed-loop system (6) and (7) is robustly stochastically stable with disturbance attenuation γ. For this problem, under the assumption of zero initial conditions,we have the following result.

Theorem 2. Consider the closed-loop system (6) and (7).For given scalars $\gamma>0, h_m, h_M, \mu, a_q, b_q, (q=1,2,3)$,the stochastic system (6) and (7) is robustly stochastically stabilizable with disturbance attenuation γ for any $\tau(t)$ satisfying $h_m<\tau(t)<h_M, \dot{\tau}(t)\leq u$, if there exist matrices $X, L, \bar{P}>0, \bar{Q}_q>0, \bar{Z}_1>0, \bar{Z}_2>0, \bar{G}_q, \bar{H}_P, \bar{N}_P (p=1,2, q=1,2,3)$, satisfying the following LMI:

$$\begin{bmatrix} \varphi_{11} & \varphi_{12} & \bar{H}_1 & -\bar{G}_1 & \varphi_{15} & \varphi_{16} & -\bar{N}_1 & -\bar{G}_1 & -\bar{H}_1 & \varphi_{1,10} & \varphi_{1,11} \\ * & \varphi_{22} & \bar{H}_2 & -\bar{G}_2 & \varphi_{25} & \varphi_{26} & -\bar{N}_2 & -\bar{G}_2 & -\bar{H}_2 & \varphi_{2,10} & \varphi_{2,11} \\ * & * & -\bar{Q}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{Q}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \varphi_{55} & 0 & 0 & 0 & 0 & \varphi_{5,10} & 0 \\ * & * & * & * & * & \varphi_{66} & 0 & 0 & 0 & \varphi_{6,10} & 0 \\ * & * & * & * & * & * & -\frac{\bar{Z}_1}{h_M} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \frac{-(\bar{Z}_1+\bar{Z}_2)}{h_M-h_m} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\frac{\bar{Z}_2}{h_M-h_m} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \qquad (24)$$

$\phi_{11} = \bar{Q}_1 + \bar{Q}_2 + \bar{Q}_3 + \bar{N}_1 + \bar{N}_1^T + a_1 A X^T + a_1 X A^T + a_1 B L + a_1 L^T B^T + \mathrm{b}_1 E X^T + \mathrm{b}_1 X E^T$

$\phi_{12} = -\bar{N}_1 + \bar{N}_2^T + \bar{G}_1 - \bar{H}_1 + a_1 A_\tau X^T + a_2 X A^T + a_1 B_\tau L + a_2 L^T B^T + \mathrm{b}_1 E_\tau X^T + \mathrm{b}_2 X E^T$

$\phi_{22} = -(1-u)\bar{Q}_3 - \bar{N}_2 - \bar{N}_2^T + \bar{G}_2 + \bar{G}_2^T - \bar{H}_2 - \bar{H}_2^T + a_2 A_\tau X^T + a_2 X A_\tau^T + a_2 B_\tau L + a_2 L^T B_\tau^T + \mathrm{b}_2 E_\tau X^T + \mathrm{b}_2 X E_\tau^T$

$\phi_{15} = \bar{P} - a_1 X^T + a_3 X A^T + a_3 L^T B^T$

$\phi_{16} = -b_1 X^T + b_3 X E^T$

$\phi_{25} = -a_2 X^T + a_3 X A_\tau^T + a_3 L^T B_\tau^T$

$\phi_{26} = -b_2 X^T + b_3 X E_\tau^T$

$\phi_{1,10} = a_1 B_V + b_1 E_v \phi_{1,11} = X C^T + L^T D^T \phi_{2,10} = a_2 B_V + b_2 E_v \phi_{2,11} = X C_\tau^T + L^T D_\tau^T$

$\phi_{55} = h_m \bar{Z}_1 + (h_M - h_m)\bar{Z}_2 - a_3 X^T - X a_3$

$\phi_{5,10} = a_3 B_V \phi_{6,10} = b_3 E_v$

$\phi_{66} = \bar{P} - b_3 X - b_3 X^T$

Moreover,the feedback gain matrix K is given by

$K = L X^{-T}$

proof. According to Theorem 1, if we pre-and postmultiply (8) by

$$\Omega = diag(X \;\; X \;\; X \;\; X \;\; X \;\; X \;\; X \;\; X \;\; X \;\; I \;\; I)$$

and its transpose, we have that $\Omega(8)\Omega^T < 0$. Now, wedefine new variables in $\Omega(8)\Omega^T < 0$ as follows:

$$M_q = a_q X^{-1} S_q = b_q X^{-1} \bar{P} = X P X^T \bar{Q}_q = X Q_q X^T H_p = X H X^T$$

$$\bar{N}_p = X N_P X^T \bar{G}_p = X \bar{G} X^T \qquad L = K X^T \qquad p = 1,2 \qquad q = 1,2,3$$

with X being an invertible matrix.Then (28) follows immediately by applying Schurs complement to$\Omega(8)\Omega^T < 0$.The proof is completed.In this section,the following example is used to demonstrate the effectiveness of the proposed method.

Numerical Examples

Consider the uncertain stochastic system (1),

$$\mathrm{A} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} A_\tau = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix} B_v = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathrm{C} = [0 \quad 1] E_v = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix} \mathrm{H} = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}$$

$$\mathrm{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \mathrm{D} = 0.1 B_\tau = 0 C_\tau = 0 D_\tau = 0$$

$$\mathrm{E} = \begin{bmatrix} 0.1 & 0.2 \\ 0 & -0.1 \end{bmatrix} \mathrm{E}_\tau = \begin{bmatrix} 0.1 & 0.2 \\ 0 & -0.1 \end{bmatrix}$$

Then, for $\gamma = 1$, by using Theorem 2 with $a_1 = a_2 = 0.04, a_3 = 0.08, b_1 = b_2 = 0.04, b_3 = 0.3$ *and* $u = 0$. it has been found that the maximal interval delay is$h_m = 0.01$,$h_M = 0.56$. and the feedback gain matrix is computed as

$$K = [-0.0232 \quad -7.9424]$$

Conclusion

In this paper, the problem of a delay-dependent approach to robust H∞ control has been addressed. The freeweighting matrix technique has been used to develop the delay-dependent stability conditions, and a robust H∞ controller has been constructed. The numerical example has been given to illustrate the effectiveness of our results, and the simulations show that our results are less conservative than the existing ones.

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