# Influence of Rotation Angle Error on Motion Platform Poses Error for Simulator in 6-RSPS Platform without Z-axis Rotation Washout 

Yang Chunjie ${ }^{\text {a* }}$, Li Jianjun ${ }^{\text {b }}$, Yin Niandong ${ }^{\text {a }}$, Lin Shaojiang ${ }^{a}$<br>${ }^{\text {a }}$ School of Mechanical and Electrical Engineering , Hubei Polytechnic University, Huangshi, 435003, China<br>${ }^{\mathrm{b}}$ College of Mechanical and Electrical Engineering, China Jiliang University, Hangzhou, 310027, China<br>yachuje@163.com

Keywords: parallel structure; simulator platform; 6-RSPS; error


#### Abstract

This paper proposes a new 6-RSPS structure of vehicle simulation that can successive rotation, and studies the affect relationship and regularity for rotation angle error effect on motion platform pose error. We built an error model according to independent function principle of error and differential derivation of kinematics formulation. It results that the rotation angle error has greater impact on the position error of the platform, while it has less affected the orientation error of the platform. As such, the new structure increased the fidelity of simulated driving compared to the classical 6-SPS structure based vehicle simulator.


## Introduction

Simulator platform is the foundation platform of the driving simulator that has used extensively in the $\mathrm{R} \& \mathrm{D}$ of new vehicle and driving-training [1]. Classic 6-SPS structure is the basic of most simulator platform which can make the motion platform realize six free degrees motion in three coordinates system [2]. Classic Stewart platform is used for helicopter simulator early [3], and is used gradually for ship and vehicle simulator [4-5]. But the vehicle simulator today still can't reappear completely the movement of a vehicle because of the limitation of the length of member results in the smaller work space [6-7]. High-frequency and low frequency signals among the output signals from the reality dynamic model devices must be filtered out in a special algorithm after a movement is finished to realize the continuous sense of reality when it revert to the middle position slowly to make ensure the simulator platform has enough stroke to the next motion simulation [8-9]. The purpose of washout algorithm is to make the motion platform in a limit space reappear the specific force and angular velocity that driver can feel in the true environment.

However, studies show that washout algorithm just try to reduce the drivers unrealistic sense during the motion platform return to middle position other than make drivers have none awareness[10]. Motion platform usually is satisfactory of the need when the pitch and incline of motion platform less than 30 degrees. However, the simulator rotation angle is not satisfied actual demand when the vehicle makes a wide-angle accelerate continually turn result in decrease the sense of reality of turning of drivers in the simulation. Therefore, proposed the structure 6-RSPS which rotates around the Z axis infinitely as the fundamental of simulator platform. Z axis washout algorithm of the new structure is eliminated to reduce the requirements of washout algorithm and effectively improve fidelity of driving simulation. This paper focused on studying the effective factors and regulations of the error of rotation angular relative to the error of motion platform.

## Description of 6-RSPS Parallel Simulator Platform <br> Description of Model

6-RSPS platform is consisted of static platform and motion platform which are connected by six links in variable length. Two spherical hinges are fixed on the both ends of each link to connect the motion platform and link as well as the link and slider. The sliders are fitted on the circular guide rail which is mounted on the static platform. The servo motor is fixed on slider through connected with reducer and its output shaft drives the pinion fitted on the slider. Each pinion engages the circular gear of static platform. So, the under-hinges of the links rotate around Z axis on static platform. The under-hinges have relative displacement to each other through control independent motor for satisfactory of special requirements. The structural model is shown in Figure 1. and 3D simulation schematic drawing in Figure 2.


Figure 1. Theoretical model of 6-RSPS structure


Figure 2. 3D simulation schematic drawing
Firstly, a static coordinate system $\mathrm{B}-\mathrm{X}_{\mathrm{B}} \mathrm{Y}_{\mathrm{B}} \mathrm{Z}_{\mathrm{B}}$ lie at the center of static platform and the moving coordinate system $\mathrm{M}-\mathrm{X}_{\mathrm{A}} \mathrm{Y}_{\mathrm{A}} \mathrm{Z}_{\mathrm{A}}$ lie at the center of motion platform. Then build the coordinated system $\mathrm{O}_{\mathrm{I}}-\mathrm{X}_{\mathrm{I}} \mathrm{Y}_{\mathrm{I}} \mathrm{Z}_{\mathrm{I}}$ which lied at the center of six hinges in the original position of stationary platform. Build subsequently the coordinated system $\mathrm{O}_{\text {II }}-\mathrm{X}_{I I} \mathrm{Y}_{\|} \mathrm{Z}_{\| I}$ on the rotation position relative to the original position of stationary platform. Last, build coordinated system $\mathrm{O}_{\text {III }} \mathrm{X}_{\text {III }} \mathrm{Y}_{\text {III }} \mathrm{Z}_{\text {III }}$ which origin is lied in the center of the six hinges on the motion platform. Such as Fig.1, the theoretic center of six hinges before rotation in the static platform is $O_{i}\left(x_{i l}, y_{i}, z_{i l}\right)(i=1,2, \ldots, 6)$, the center of six hinges after rotation is $B_{i}\left(x_{\text {III }}, y_{\text {II }}, z_{i l}\right)(i=1,2, \ldots, 6)$.

## Position Analysis

The position and direction vector of the motion platform is $(x, y, z, \phi, \varphi)$. When $\varphi=0$, the theoretical original positions of the under-hinge on the six links are not rotating, but there are the different deviation angle ${ }^{\delta_{i}}$ relative to theoretical initial position of each under-hinge on the links in
the actual rotation, as shown in Figure 3.


Figure 3. Under-hinge without common rotation
When $\varphi \neq 0$, the theoretical initial positions are rotating in $\varphi$, and actual positions are rotating different angle $\delta_{i}$, as shown in Figure 4.


Figure 4. Under-hinge with common rotation
Where: $\alpha$ is the angle-bisect of the adjacent hinge on the moving platform; $\beta$ is the angle-bisect of the theoretical initial position of the adjacent hinge on the stationary platform; $\delta_{i}$ is the angle of deviation from the theoretical position of under-hinge on the i-th link.
If $\varphi, ~ \mathbf{B}_{i}(i=1,3,5)$ clockwise rotation is positive, and $\mathbf{B}_{i}(i=2,4,6)$ clockwise rotation is negative, then the coordinates of $O_{i}$ is:

$$
\left[\begin{array}{llll}
{ }^{B} X_{l i} & { }^{B} Y_{l i} & { }^{B} Z_{l i}
\end{array}\right]^{T}=r_{B}\left[\begin{array}{lll}
\cos \left(\theta_{0 i}\right) & \sin \left(\theta_{0 i}\right) & 0 \tag{1}
\end{array}\right]^{T}
$$

Where:
$\theta_{O i}=\left\{\begin{array}{cc}\frac{i \pi}{3}-(\varphi+\beta) & (i=1,3,5) \\ \frac{(i-1) \pi}{3}-\varphi+\beta & (i=2,4,6)\end{array}\right.$
$B_{i}$ coordinates in the static coordinate system is:
$\left[\begin{array}{lll}{ }^{B} X_{I I} & { }^{B} Y_{I I} & { }^{B} Z_{I I}\end{array}\right]^{T}=r_{B}\left[\begin{array}{lll}\cos \left(\theta_{B i}\right) & \sin \left(\theta_{B i}\right) & 0\end{array}\right]^{T}$

Where:
$\theta_{B i}=\left\{\begin{array}{cc}\frac{i \pi}{3}-\left(\beta+\varphi+\delta_{i}\right) & (i=1,3,5) \\ \frac{(i-1) \pi}{3}-\varphi+\left(\beta+\delta_{i}\right) & (i=2,4,6)\end{array}\right.$
Ai coordinates in the motion coordinate system is:
$\left[\begin{array}{lll}{ }^{A} X_{I I I i} & { }^{A} Y_{I I I i} & { }^{A} Z_{I I I I}\end{array}\right]^{T}=r_{A}\left[\sin \left(\theta_{A i}\right) \sin \left(\theta_{A i}\right) \quad 0\right]^{T}$
Where:
$\theta_{A i}=\left\{\begin{array}{c}\frac{(i-1) \pi}{2}+\alpha \quad(i=1,3,5) \\ \frac{i \pi}{2}-\alpha \quad(i=2,4,6)\end{array}\right.$
Position vector of link i-th can be expressed as formula (4) below according to Figure 5.

$$
\begin{equation*}
L_{i}=P+{ }^{B} r_{A i}-{ }^{B} r_{B i} \tag{4}
\end{equation*}
$$



Figure 5. Nominal vector chain for link i-th
Where: ${ }^{B} r_{A i}={ }^{A-B} q \cdot{ }^{A} r_{A i}$
Where: ${ }^{A-B} q=R_{z} \cdot R_{y} \cdot R_{x}=$
$\left[\begin{array}{ccc}\mathrm{c} \varphi & -\mathrm{s} \varphi & 0 \\ \mathrm{~s} \varphi & \mathrm{c} \varphi & 0 \\ 0 & 0 & 1\end{array}\right\rfloor\left[\begin{array}{ccc}\mathrm{c} \phi & 0 & \mathrm{~s} \phi \\ 0 & 1 & 0 \\ -\mathrm{s} \phi & 0 & \mathrm{c} \phi\end{array}\right\rfloor\left[\begin{array}{ccc}1 & 0 & 0 \\ 0 & \mathrm{c} \Psi & -\mathrm{s} \Psi \\ 0 & \mathrm{~s} \Psi & \mathrm{c} \Psi\end{array}\right\rfloor$
Where: $s$ represent sin; $c$ represent $\cos ;{ }^{A-B} q$ represent rotation matrix; $R_{z}, R_{y}, R_{x}$ represent $X, ~ Y, ~$ Z axis rotation coefficient matrix;
$\stackrel{\text { Lo }}{i}=P+{ }^{A-B}{ }_{q} \cdot{ }^{A} r_{A i}-{ }^{B} r_{B i}$

## The influence of rotation angle error on moving platform poses error

According to the formulation (4), get:

$$
\begin{equation*}
{ }^{A-B} q \cdot{ }^{A} r_{A i}-{ }^{B} r_{B i}=L_{i}-P \tag{5}
\end{equation*}
$$

$\left[\begin{array}{lll}q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33}\end{array}\right]\left[\begin{array}{l}{ }^{A} X_{I I I i} \\ { }^{A} Y_{I I I i} \\ { }^{A} Z_{I I I i}\end{array}\right]-\left[\begin{array}{c}{ }^{B} X_{I I i} \\ { }^{B} Y_{I I i} \\ { }^{B} Z_{I I i}\end{array}\right]=\left[\begin{array}{l}L_{i x} \\ L_{i y} \\ L_{i z}\end{array}\right]+\left[\begin{array}{c}X \\ Y \\ Z\end{array}\right]$
Expansion formula as below:

$$
\begin{aligned}
& \left(q_{11}{ }^{M} X_{I I I i}+q_{12}{ }^{M} Y_{I I I i}+q_{13}{ }^{M} Z_{I I I i}-{ }^{B} X_{I I i}\right)^{2}+\left(q_{21}{ }^{M} X_{I I I i}+q_{22}{ }^{M} Y_{I I I i}+q_{23}{ }^{M} Z_{I I I i}-{ }^{B} Y_{I I i}\right)^{2}+ \\
& \left(q_{31}{ }^{M} X_{I I I i}+q_{32}{ }^{M} Y_{I I I i}+q_{33}{ }^{M} Z_{I I I i}-{ }^{B} Z_{I I i}\right)^{2}=\left(L_{i x}+X\right)^{2}+\left(L_{i y}+Y\right)^{2}+\left(L_{i z}+Z\right)^{2}
\end{aligned}
$$

That is:
$\left(q_{11}{ }^{M} X_{I I I i}+q_{12}{ }^{M} Y_{I I I i}+q_{13}{ }^{M} Z_{I I I i}-r_{B} \cos \theta_{B i}\right)^{2}+$
$\left(q_{21}{ }^{M} X_{I I I i}+q_{22}{ }^{M} Y_{I I I I}+q_{23}{ }^{M} Z_{I I I I}-r_{B} \sin \theta_{B i}\right)^{2}+\left(q_{31}{ }^{M} X_{I I I i}+q_{32}{ }^{M} Y_{I I I i}+q_{33}{ }^{M} Z_{I I I i}\right)^{2}=$
$\left(L_{i x}+X\right)^{2}+\left(L_{i y}+Y\right)^{2}+\left(L_{i z}+Z\right)^{2}$
Because of the pose error of motion platform is completely caused by rotation angle error, that is, the error found in $\theta_{B i}$ results in producing the position error and pose error of motion platform, so, regard
${ }^{A} X_{I I I i}, ~{ }^{A} Y_{I I I I}, ~{ }^{A} Z_{I I I i}, ~ l_{i}$ as constants and $X, ~ Y, ~ Z, ~ \psi, ~ \phi, ~ \varphi, ~ \Delta \theta_{B i}$ as variable quantity, arrange the differential formula (8), then get to:
$F_{i 1} \Delta x+F_{i 2} \Delta y+F_{i 3} \Delta z+F_{i 4} \Delta \psi+F_{i 5} \Delta \phi+F_{i 6} \Delta \varphi=G_{i} \Delta \theta_{B i}$
Transform the pose error to matrix array such as below:
$\left[\begin{array}{llllll}F_{11} & F_{12} & F_{13} & F_{14} & F_{15} & F_{16} \\ F_{21} & F_{22} & F_{23} & F_{24} & F_{25} & F_{26} \\ F_{31} & F_{32} & F_{33} & F_{34} & F_{35} & F_{36} \\ F_{41} & F_{42} & F_{43} & F_{44} & F_{45} & F_{46} \\ F_{51} & F_{52} & F_{53} & F_{54} & F_{55} & F_{56} \\ F_{61} & F_{62} & F_{63} & F_{64} & F_{65} & F_{66}\end{array}\right]\left[\begin{array}{c}\Delta x \\ \Delta y \\ \Delta z \\ \Delta \psi \\ \Delta \phi \\ \Delta \varphi\end{array}\right]=$
$\left[\begin{array}{l}G_{1} \\ G_{2} \\ G_{3} \\ G_{4} \\ G_{5} \\ G_{6}\end{array}\right]\left[\begin{array}{llllll}\Delta \theta_{B 1} & \Delta \theta_{B 2} & \Delta \theta_{B 3} & \Delta \theta_{B 4} & \Delta \theta_{B 5} & \Delta \theta_{B 6}\end{array}\right]\left[\begin{array}{cccccc}1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1\end{array}\right]\left[\begin{array}{l}1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1\end{array}\right]$
Transform the pose error to matrix array such as below:
$\left[\begin{array}{llllll}F_{11} & F_{12} & F_{13} & F_{14} & F_{15} & F_{16} \\ F_{21} & F_{22} & F_{23} & F_{24} & F_{25} & F_{26} \\ F_{31} & F_{32} & F_{33} & F_{34} & F_{35} & F_{36} \\ F_{41} & F_{42} & F_{43} & F_{44} & F_{45} & F_{46} \\ F_{51} & F_{52} & F_{53} & F_{54} & F_{55} & F_{56} \\ F_{61} & F_{62} & F_{63} & F_{64} & F_{65} & F_{66}\end{array}\right]\left[\begin{array}{c}\Delta x \\ \Delta y \\ \Delta z \\ \Delta \psi \\ \Delta \phi \\ \Delta \varphi\end{array}\right]=$
$\left[\begin{array}{l}G_{1} \\ G_{2} \\ G_{3} \\ G_{4} \\ G_{5} \\ G_{6}\end{array}\right]\left[\begin{array}{llllll}\Delta \theta_{B 1} & \Delta \theta_{B 2} & \Delta \theta_{B 3} & \Delta \theta_{B 4} & \Delta \theta_{B 5} & \Delta \theta_{B 6}\end{array}\right]$
$\left[\begin{array}{llllll}1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1\end{array}\right]\left[\begin{array}{l}1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1\end{array}\right]$
make:
$F=\left[\begin{array}{llllll}F_{11} & F_{12} & F_{13} & F_{14} & F_{15} & F_{16} \\ F_{21} & F_{22} & F_{23} & F_{24} & F_{25} & F_{26} \\ F_{31} & F_{32} & F_{33} & F_{34} & F_{35} & F_{36} \\ F_{41} & F_{42} & F_{43} & F_{44} & F_{45} & F_{46} \\ F_{51} & F_{52} & F_{53} & F_{54} & F_{55} & F_{56} \\ F_{61} & F_{62} & F_{63} & F_{64} & F_{65} & F_{66}\end{array}\right]$
$G=\left[\begin{array}{llllll}G_{1} & G_{2} & G_{3} & G_{4} & G_{5} & G_{6}\end{array}\right]^{T}$
Get pose error of motion platform caused by rotation angle error as formula below:
$\Delta \Pi_{1}=\left[\begin{array}{l}\Delta x \\ \Delta y \\ \Delta z \\ \Delta \psi \\ \Delta \phi \\ \Delta \varphi\end{array}\right]=F^{-1} G \quad \quad \cdot\left[\begin{array}{llllll}\Delta \theta_{B 1} & \Delta \theta_{B 2} & \Delta \theta_{B 3} & \Delta \theta_{B 4} & \Delta \theta_{B 5} & \Delta \theta_{B 6}\end{array}\right] I^{6 \times 6}\left[\begin{array}{l}1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1\end{array}\right]$
According to the formula (11), we will find the pose error when the rotation angle error of the rotation link is given.

## Analyzing and Simulation

The error influence of motion platform cause by each segment is relative to the theoretical values as listed in the previous content. For easy to compare, we set initial theoretical pose, while the length of telescoping links and the rotation angle of rotation links are in a certain situation.

Define the length of each telescoping link and the rotation angle of each rotation link according to preset theoretical pose of the motion platform.

Setting the rotation angle error of rotation link 1fluctuates in sine and there are not rotation angle errors with others rotation links, result in the moving errors in the three direction of $\mathrm{X}, ~ \mathrm{Y}, ~ \mathrm{Z}$ of motion platform shown in Figure 6. It can be claimed by observing the figure 7, in general, the moving error in three directions almost fluctuates in sine, and appears obviously amplification effect. The maximum error which is 31 mm is the moving error of Y axis when the rotation angle error is $1^{\circ}$.

The influence of angle error of rotation link relative to the angle error in the direction $\mathrm{X}, ~ \mathrm{Y}, ~ \mathrm{Z}$ of motion platform as shown in Fig.6, in general, the angle error in three directions almost fluctuates in sine, and appears obviously reduce effect. The maximum error which is $0.782^{\circ}$ is the angle error of Z axis when the rotation angle error is $1^{\circ}$ while the minimum error which is $0.086^{\circ}$ is the angle error of Y axis.


Figure 6. Influence of rotation angle error to motion error in three directions


Figure 7. Influence of rotation angle error to angle error in three directions

## Conclusion

The effect factors in pose precision of 6-RSPS simulator platform include the length error of link, hinge error of static platform and hinge error of motion platform. This paper focused on researching the influence of rotation angle error relative to the precision of motion platform. Build error model according to independent function principle of error and differential derivation of kinematics formulation. According to the error model, the pose precision of the structure 6-RSPS should be evaluated in a more perfect way and proposed a new theoretical foundation for further improve the pose error.

The conclusion:
(1) Defined the error effect relation and regulation of rotation angle error relative to the pose error of platform according to the characters of 6-RSPS simulator platform.
(2) Found the rotation angle error of rotation link having a more effect on the position error of platform and having obviously amplification effect.
(3) Found the rotation angle error of rotation link having a less effect on the posture error of platform and having obviously reduction of effect.
(4) The news structure utilized six under-hinges which possess independent motion controlled by each motor result in increased the complexity of control motion but the relative independent motion between six under-hinges in a special range, so, it is easy to control and superior to the classic SPS structure that the under fulcrums is fixed on a same circle frame result in more uncontrolled huge moment of inertia.
(5) The new structure 6-RSPS proposed in this paper is proved that it is superior to the classic 6-SPS structure in aspect of simulating the driver's sense when the car especially heavy vehicle is turning a corner.

## Acknowledgements

This work is supported by the Key Project of Hubei Natural Science Foundation, China.
(No.2014CFB177).

## References

[1] D. Toffin, G. Reymond, A. Kemeny, et al. Role of steering wheel feedback on driver performance: Driving simulator and modeling analysis. Vehicle Syst. Dyn., 2007, 45(4):375-388.
[2] Same, Adam, Stipe, Alex, Grossman, David, et al. A study on optimization of hybrid drive train using Advanced Vehicle Simulator (ADVISOR). Journal of Power Sources, 2010,195(19):6954-6963.
[3] Stewart D., A platform with six degrees of freedom. proceedings of the IMechE, 1965, 180(15):371-385.
[4] Wang Xuanyin, Liu Rong, JiaGuangzheng, et al. RESEARCH ON COORDINATED CONTROL OF THE SIX DEGREES OF FREEDOM VEHICLE MOTION SIMULATOR. Chinese Journal of Mechanical Engineering,2004,40 (4):160-163.
[5] Bhaskar Dasgupta, T. S. Mruthyunjaya. The Stewart platform manipulator: a review. Mechanism and Machine Theory, 2000, 35: 15-40.
[6] Huang Zhen, kong Lingfu, Fang Yuefa. Theory and Control of Parallel Robot Mechanism. BeiJing: China Machine Press, 1997.
[7] Wohlhart, Karl. Mobile 6-SPS parallel manipulators. Journal of Robotic Systems, 2003, 20 (8):509-516.
[8] M. S. KIM, Y. G. MOON, G. D. KIM, et al. Partial Range Scaling Method Based Washout Algorithm for a Vehicle Driving Simulator and its Evaluation. International Journal of Automotive Technology, 2010, 11(2):269-275.
[9] Min Kyu Park, Min Cheol Lee, Ki Sung Yoo, et al. Development of the PNU vehicle driving simulator and its performance evaluation// Proceedings - IEEE International Conference on Robotics and Automation. 2001, 3: 2325-2330.
[10] Chin-I Huang, Human Visual-Vestibular Based(HVVB) Adaptive Washout Filter Design for VR-Based Motion Simulator//2010 International Symposium on Computer, Communication, Control and Automation. 2010:179-182.

