Fractal characteristic of virtual fibrous porous media

Fu Hai-ming[1], Fu yu[1]

[1]State Environmental Protection Engineering Center for Pollution Treatment and Control in Textile Industry, School of Environmental Science and Engineering, Donghua University, Shanghai 201620, China

[2]Wescon Controls (Shanghai),Inc.Building 6,471 Guiping Road, Shanghai 200233, P,R,China

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Abstract: Fractal theory is a very effective method for investigating pore-structure of fibrous porous materials, which build a bridge between micro-morphology and macro-performance. Fractal dimension was calculated based on the fractal theory and combined with computer virtual building model technology, which was explored to describe the pore structure of virtual fibrous porous materials. The geometric model construction program was completed to generate virtual filter media which has straight fiber and bending fiber. These fibers are randomly distributed in two directions or three directions. A variety of virtual fibrous media were used to calculate packing density or porosity and fractal dimension. The fractal dimension of 2D or 3D pore structure was calculated by the soft and the effects of packing density or porosity on fractal dimension were also discussed. Fractal dimension of the two-dimensional cross-sectional view, plan view or section view has the same form of expression, their coefficients was different with the variation in position observations. The results show that the fractal dimension decreases continuously with the increase of fiber diameter, but it increases gradually with the packing density increasing. Furthermore, the relationships between the fractal dimension and the fiber diameter and packing density were obtained.

Introduction

Computer simulations have great potential in the development process of the non-woven industry and it can help us to optimize the geometry parameters of non-woven. Non-woven parameters such as porosity, fiber thickness distribution, fiber orientation and porosity gradients are largely controlled by experienced non-woven designer and manufacturer. Trying out new parameter or structural parameter optimization is usually difficult. Lab scale production often does not completely agree with production scale non-woven, and stopping the production lines for parameter studies is very costly. Computer simulations can reduce the number of trial runs in producing new material prototypes significantly. They help understanding the production processes as well as the functionality of the non-woven and may point out trends or improvements ways which would be costly to find using traditional methods. Thus, much study work has gone into building virtual material using computer models [1, 2].

For the past 10 years, many scientists have been trying to research fractal characteristic of fibrous porous media via SEM technology[3], but it was very difficult to exactly measure packing density of media, the relationship between fractal dimension and fiber diameter and packing density cannot be obtained via SEM technology. So that, we developed a VBA computer program to produce a 3-D virtual non-woven material with three directions or two direction randomly distributed straight and curved fiber. Geometry of virtual media would be exported by CAD software. Fractal dimension of virtual fibrous porous media was obtained based on those virtual media.

Method of Virtual Materials Generating

Mathematical methodologies from stochastic geometry allow creating very realistic computer models of non-woven. Which is called as virtual filtration media. Two kinds of virtual filtration media were built. A one-parameter model for the anisotropy is developed that generates anisotropic virtually non-woven that are isotropic in the plane of the media, but has a controlled anisotropy in the through direction of the media. This model was applied for the layered structures of typical filter media. Each individual layer can be modeled as a non-woven, and the layers can then be stacked. This model has two directions randomly distributed fiber. Another model for non-woven based on the three parameters such as porosity, fiber diameters and fiber directions has three direction changes.

To generate 3-D random fibrous geometries, a VBA computer program was developed to produce fibrous structures of different porosities. The virtual materials generation process was explained in the flow chart shown in Fig. 1. Fibers are treated as curly or straight cylinder randomly placed in a cubic domain. The virtual materials generation starts by sequentially adding the fibers into a cubic domain with a given size. The desired solid volume fraction (SVF) is achieved by randomly placing fibers and adding up their contribution. This is done by counting the number of fibers that are set to a cubic domain when placing the fiber. In typical media, the fibers are oriented randomly yet preferably perpendicular to the flow. This procedure continues until a desired porosity is reached.

Both porosity and fiber thickness or fiber shapes are comparatively easy to be determined. Much harder is the estimation of fiber orientation. In [2], a simple no woven model is developed based on the assumptions that the portion of the non-woven that can be represented in the computer memory is small compared to fiber length and fiber crimp. Thus, fibers are viewed as infinitely long lines, with appropriate thickening and orientation. The porosity is governed by the number of fibers,

In our computer models of virtual material, a non-woven model with curly fiber is promoted on the assumption that each straight cylinder was taken place of Sinusoidal one. The curly cylinder could be dived into several aliquot segments shown in Fig.2. The vertex of the curve was named as Fitting Point and central line of Sine function was called as baseline. The number of segments and the vertical distance from f fitting point to baseline, which defined as Curvature and Symbol are expressed as S, could be set in computer models. The straight cylinder model would be produced if the curvature is equal to zero. In the virtual material, fibers are allowed to overlap. Possibly modeling a thermal bonding process. To reproduce existing media, the grammage and specific weight of the fiber material can be used to infer the porosity from the thickness of the non-woven.

Results

The geometric model construction program was expected to be completed to generate virtual filter media which has straight fiber or bending fiber. Visualization of virtual fiber media which has straight or curved fibers will be constructed, and these fibers are randomly distributed in two directions or three directions. A variety of fibrous material models can be generated which have a variety of different structural forms. This three-dimensional solid geometry model can be input into fluent software to simulate or be exported through the 3D printing technology. It provides the premise and the condition for the fibrous medium model experimental measure and simulation calculation [11, 12].

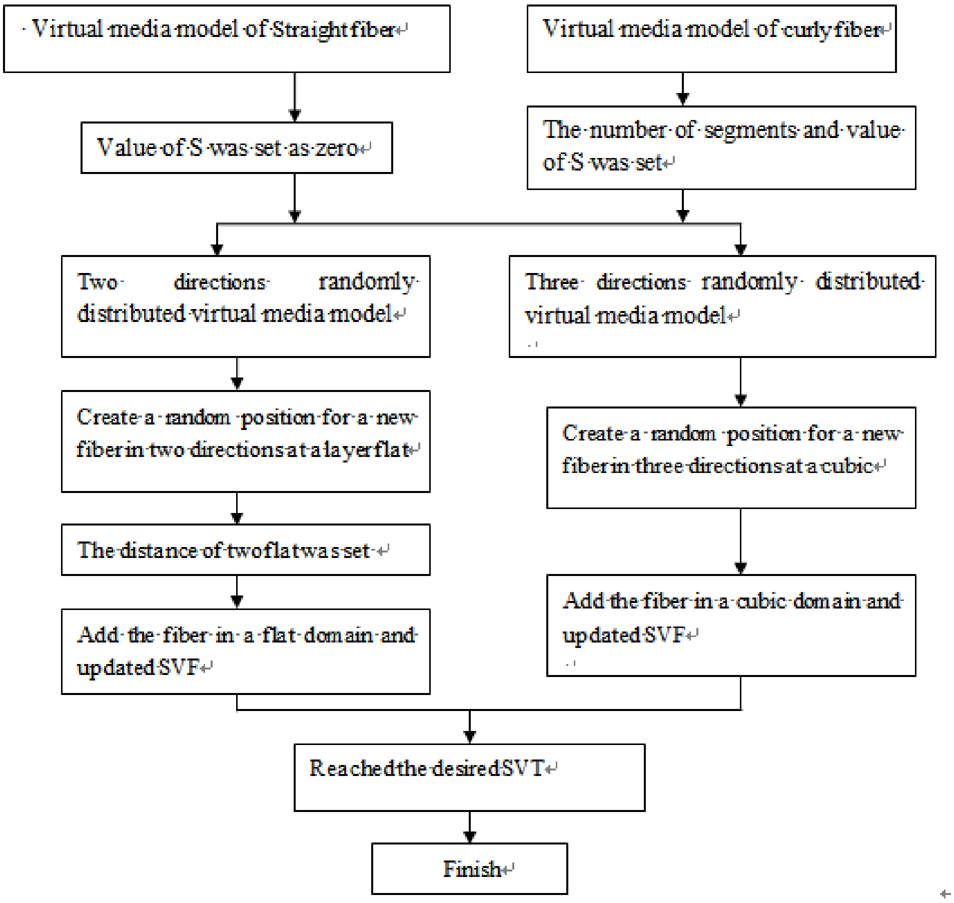


Fig. 1. Flow chart of our disordered 3-D virtual media generation algorithm

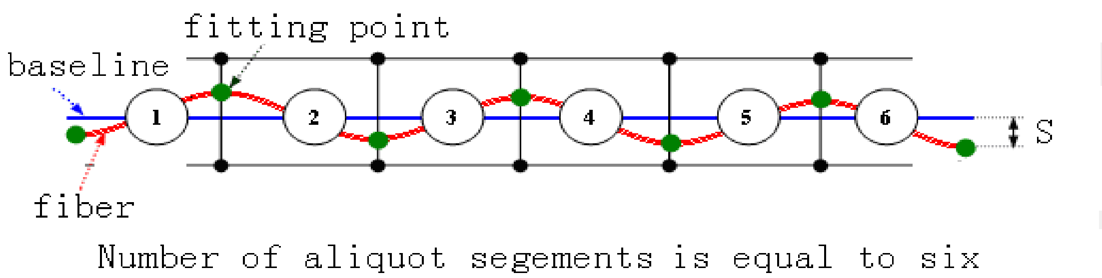
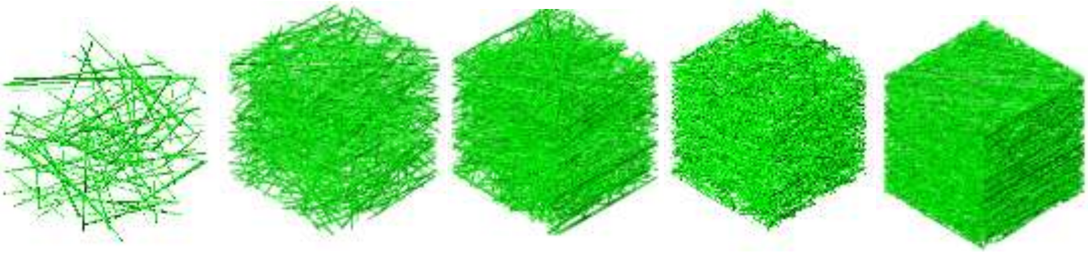


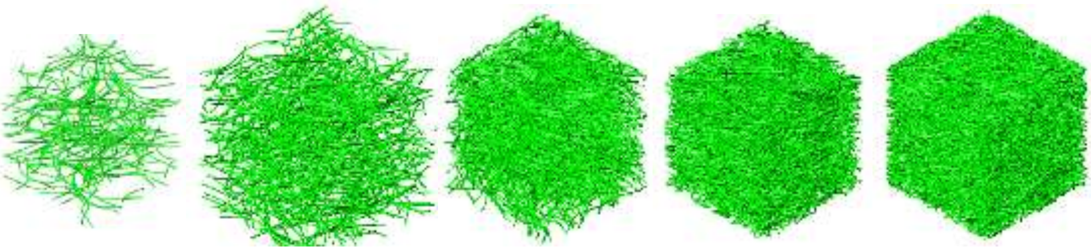
Fig. 2. chart of curly fiber model

Two-dimensional Directions Randomly Distributed Virtual Fiber Porous Media. Straight fiber medium randomly allocated in both directions was generated, it was shown in Fig. 3. Bending fiber medium was randomly distributed in two directions was generated, which was shown in Fig.4.



(a)c=0.67 (b)c=3.4% (c)c=6.72% (d)c=10.04% (e)c=20.28%

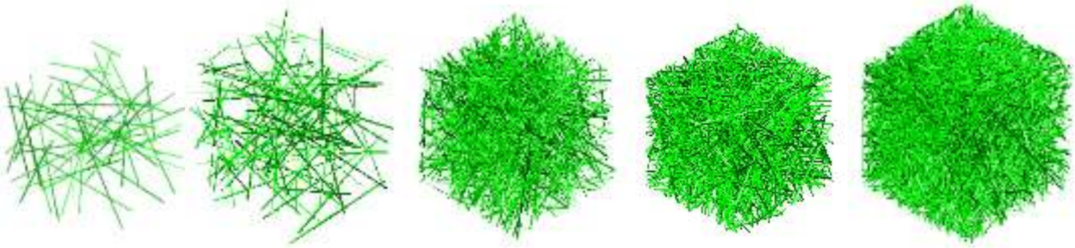
Fig. 3. Straight fiber and bending offset value s = 0 (d_f = 1um)



(a)c=0.735% (b)c=2.06% (c)c=4.95% (d)c=10.2% (e)c=20.86%

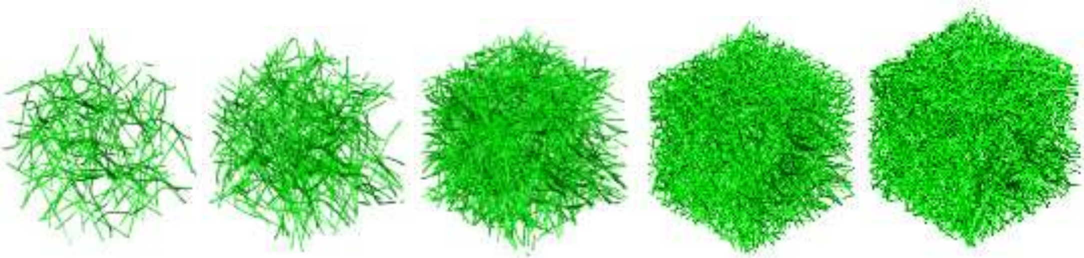
Fig. 4. Curved fiber and bending offset value s = 5 (d_f = 1um)

Three-dimensional Directions Randomly Distributed Virtual Fiber Porous Media. Fiber straight randomly distributed in three directions were generated, which was shown in Fig.5. Curved fiber medium randomly distributed in three directions was generated, it was shown in Fig. 6.



(a)c=0.36% (b)c=0.69% (c)c=3.60% (d)c=7.18% (e)c=10.63%

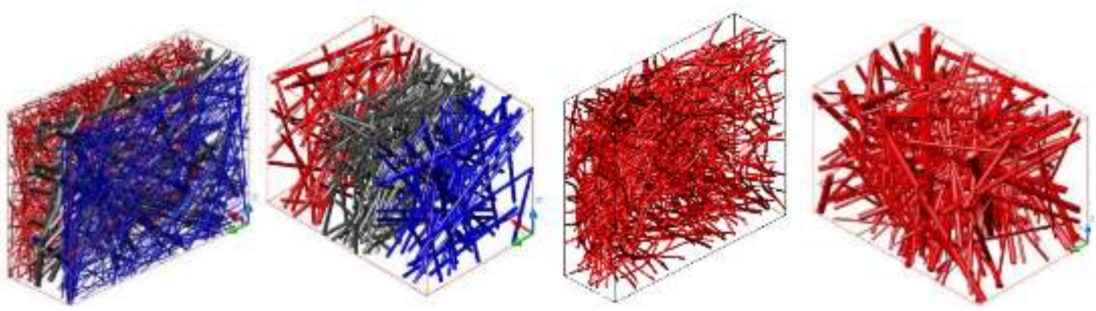
Fig. 5. Straight fiber and bending offset s = 0 (d_f = 1um)



(a)c=0.76% (b)c=1.47% (c)c=3.67% (d)c=7.74% (e)c=11.03%

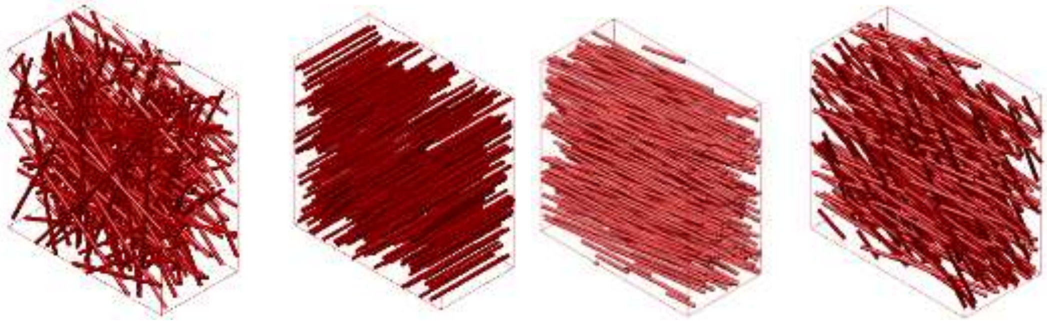
Fig. 6. Curved fiber and bending offset s = 5 (d_f = 1um)

Filter Media Construction of Various Arrangements. Various virtual fiber filter media model was generated. It was shown as Fig.7. For calculated fractal dimension, filter media construction of various arrangements was generated by the VBA. It was shown as Fig.8



(a)Coated fiber filter (b)Multi-fiber filter c)Crimped fiber filter (d)Multi-dispersion fiber filter

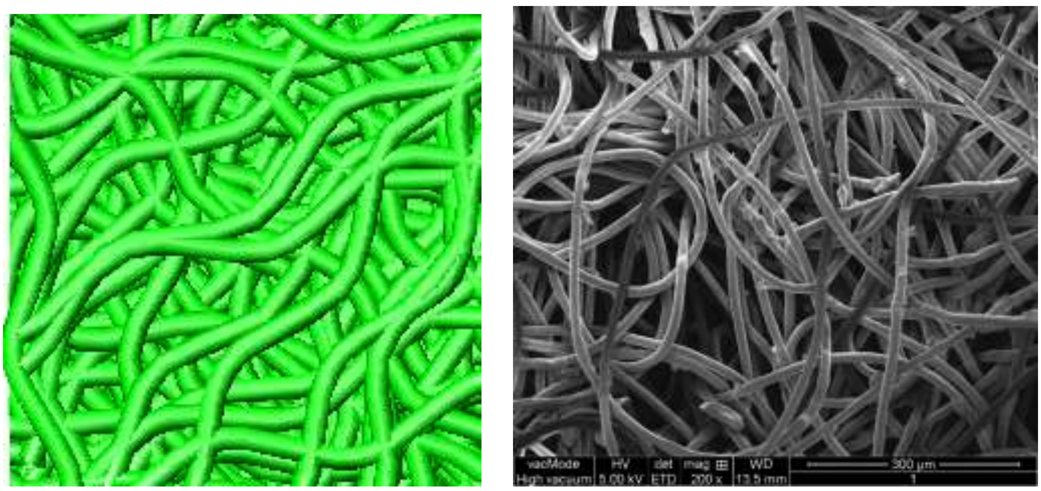
Fig.7. Various virtual fiber filter media model



(a)2D Random (b)2D Parrel (c)2D lean fiber (d)2D Staggered

Fig. 8. Virtual fiber media fibers which having different fiber arrangement

Geometry of Virtual Media and Real Filtration Media. The structure of filtration medium is very complex, the scale magnify photo of virtual medium internal structure be showed in Fig. 9(a), the real structure of filtration medium be showed in Fig. 9(b), View of real filtration media can be known that geometry of virtual media is very similar with that of real filtration media.



(a) Virtual media (b)real media

Fig. 9. Microscopy image of two kind of media comparison

By simulation calculation, the relationship between SVF and fiber diameter and number of fiber could be described as:

$$\alpha = 0.58 \cdot d_f^2 \cdot n + 0.0078;\quad \varepsilon = 1 - \alpha;\quad R = 0.97 \qquad (1)$$

Here, α -SVF of virtual media with randomly distributed fiber in three direction; ε -porosity d_f --fiber diameter; n -number of fiber; R --Regression coefficient

Fractal Dimension. Three-dimensional fractal dimension and two-dimensional fractal dimension were taken for virtual fiber medium. Three-dimensional fractal dimension computing results existed some problems. Correlation coefficient between fractal dimension and its structural parameter was small, probable cause was calculation program errors, and this issue requires further study.

For fiber medium randomly distributed in two-dimensional directions, their cross-section fractal dimension and the surface view fractal dimension were calculated. For virtual fiber medium randomly distributed in three-dimensional directions, they sectional fractal dimension was computed.

Of 2D random model, it was shown as Fig.8 (a). Its fractal dimension can be described as:

$$\mathrm{D} = 2.0 - 0.943\exp(-32.04\frac{\alpha}{d_f}) \qquad (2)$$

Two-dimensional section fractal dimension was calculated and its shape was shown as Fig.8(b), its fractal dimension can be described as:

$$\mathrm{D} = 1.764 - 0.323\exp(-195.3\frac{\alpha}{d_f}) \qquad (3)$$

Section fractal dimension was calculated and its shape was shown as Fig.10 (a), its fractal dimension can be described as:

$$\mathrm{D} = 1.67 - 0.75\exp(-18.18\frac{\alpha}{d_f}) \qquad (4)$$

Research indicates that fibrous porous media has fractal characteristics and it is a function of fiber diameter and packing density. Fractal dimension of the two-dimensional cross-sectional view, plan view or section view has the same form of expression, their coefficients was different with the variation in position observations. The relationship between fractal dimension and fiber diameter and packing density can be described as:

$$\mathrm{D} = K_1 - K_2\exp(-K_3\frac{C}{D_f}) \qquad (5)$$

Discussion and Conclusion

The cumulative size distribution of pores in porous media follows the fractal scaling law [8]:

$$N(L \geq \lambda) = (\frac{\lambda_{max}}{\lambda})^{D_f} \qquad (6)$$

Where, λ is the diameter of pores, $\lambda_{max}$is the maximum diameter of pores, N is the cumulative population of pores. Whose sizes are greater than or equal to λ , and D_f is the fractal dimension for pores, with $1 < D_f < 2$ in two dimensions and $2 < D_f < 3$ in three dimensions. λ_{max} and λ are related with fiber diameter and packing density. In general, $\frac{\lambda_{min}}{\lambda_{max}} \ll 10^{-2}$ in porous media, the fractal geometry theory and technique can be used to analyze properties of porous media, fractal dimension D_f is given by [9]

$$D_f = d_E - \frac{ln\varepsilon}{\ln(\frac{\lambda_{min}}{\lambda_{max}})} \qquad (7)$$

Where, d_E is the Euclidean dimension, and d_E= 2 (3) in two (three) dimensions.

Table 1. The fractal dimension calculation coefficient table

coefficient	Two-dimensional section fractal dimension	Two-dimensional surface fractal dimension	Three-dimensional section fractal dimension
K_1	1.76398	2.0	1.67
K_2	0.32329	0.9427	0.75
K_3	195.3125	32.04	18.18

Note that the definition of the fractal dimension of literature was said for the porosity, the bigger porosity was , the greater fractal dimension was, when porosity reaches the limit value $\varepsilon = 1$, the fractal dimension was equal to it limit value $D = 2$. AT this time the geometry images was blank, but definition of fractal dimension of our model was said for packing density, the bigger packing density was, the greater fractal dimension was, when packing density reached its limit value $\alpha = 1$, its fractal dimension was equal to its limit value $D = 2$, then the geometry images was entity.

If we the analysis self-similarity of packing geometric entity, according to box count method to calculate the fractal dimension of the packing density, we can deduce the following equation.

$$D = 2 - \frac{ln\alpha}{\ln(\lambda_{min/}\lambda_{max}} \qquad (8)$$

When we selected fiber diameter was equal to $5\mu m$ and $\frac{\lambda_{min}}{\lambda_{max}} = 0.0001$, the curve of fractal dimension was plotted in Fig.10.Literature calculated results also were plotted in Fig.10. Because the mean of fractal dimension of literature was meant for porosity and mean of our fractal dimension was for packing density, so that it's varied trend was contrary. If we selected equation (8) compared with two-dimensional section fractal dimension, we can find that its curve varied tendency was good agreement.

The research results demonstrate that the fractal dimension decreases continuously with the increase of fiber diameter, but it increases gradually with the packing density increasing. We can give the conclusion that the relationships between the fractal dimension and the fiber diameter and packing density can be expressed by the model of $D = K_1 - K_2\exp(-K_3\frac{C}{D_f})$.for 2D,$K_1 \rightarrow 2$,for 3D, $K_1 \rightarrow 3$.K_2varied from 0.32 to 0.94,K_3 varied from 18.2 to 195.3.

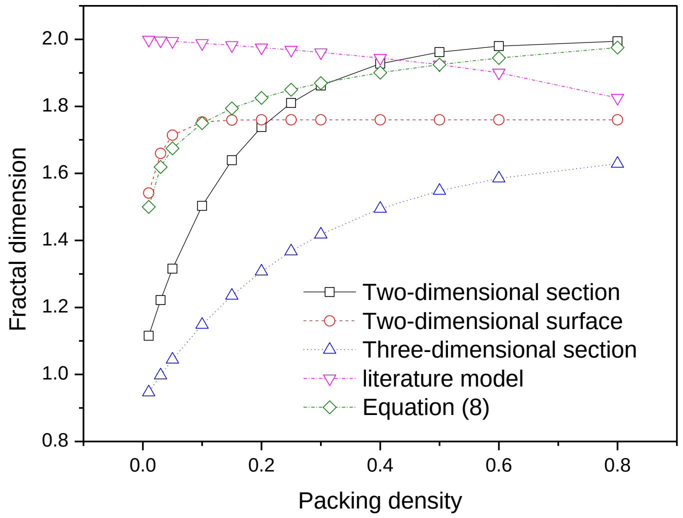


Fig.10. Fractal dimension compared with literature

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