Cost Optimization in a Warranty Service Contract with Asymmetric Actions

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Keywords: warranty, warranty period, warranty price, asymmetric actions, cost optimization

Abstract: A warranty is a service contract between a manufacturer and a customer which plays a vital role in many businesses and legal transactions. This paper focuses on the research on optimizing the cost in a service contract among one manufacturer and one agent with asymmetric actions. In order to obtain an optimal cost, we use the method of game theory to simulate the interaction among participants. Through setting the manufacturer's optimal sale price, warranty period and warranty price, and the agent's optimal maintenance price or repair price, we minimize the cost and maximize the profit. Finally, the paper presents several numerical examples and numerical analysis to illustrate the models, and put forward some interesting managerial insights.

Introduction

Warranty cost is an important problem what manufacturers concerned and one of the directions that the scholars widely research. Blischke and Murthy[1] defined several costs of interest to manufacturers and buyers, such as warranty cost per unit sale、warranty cost over the lifetime of an item and cost per unit time. Huang et al [2] analyzed repairable product and failure process in time and use rate based on weibull process model with double variables, and developed a two-dimensional periodic preventive maintenance warranty policy , minimizing the production cost.

The treatment of the warranty cost problem leads to the study of product maintenance(preventive and corrective). Kim et al[3] established a framework to study the preventive repair problems respectively from warranty products perspective of the customer and manufacturer. Djamaludin et al[4] found that whether the buyer or seller to implement preventive repair will have a great effect on the total cost of both sides. Chang et al [5] studied the optimal repair strategy and warranty length of the repairable products from the seller's point of view.

Nowadays many products, especially high-tech products, not only the manufacturer provides warranty, but also the agent provides the corresponding service. For example, some electronic products such as mobile phone, computer and so on, the manufacturer repairs or replaces the faulty products in the warranty period, and the agent provides the repair service after the warranty expires. Some big companies will get warranty service outsourcing to the third party Proxy Companies in order to focus on the development of their own advantages. Based on these realities, this paper studies a service contract between a manufacturer and an agent with asymmetric actions, and the agent provides product repair services.

Notation and Assumptions

The Introduction of the Notation of the Models. Decision variables：

P_{ip} sale price per unit item offered by the manufacturer under strategy $i(i=1,2)$

T_w warranty period offered by the manufacturer

P_w warranty price per unit item offered by the manufacturer

P_{ja} maintenance price offered by the agent under strategy $j(j=2,3)$

C_{jr} repair price offered by the agent under strategy $j(j=1,4)$

Input parameters：

L lifetime of the product

C_p production cost per unit item

P_s residual value of a failed product $(P_s < C_p)$

$C_r^{'}$ the agent's repair cost per unit item

N_1 number of failures of a product during the lifetime

N_2 number of failures of a product during the warranty period

N_3 number of failures of a product after the warranty has expired

$\lambda(t)$ failure rate

r The aging rate of product ($0<r<1$)

S_{Mi} sales volume of manufacturer under strategy i ($S_{Mi} \geq 0$)

K_0 The basic demand for products

a_i price elasticity coefficient of manufacturer under strategy i ($a_i > 0$)

b_j price elasticity coefficient of agent under strategy j ($b_j > 0$)

Π_{Mi} total profit of manufacturer under strategy $i(i=1,2)$

Π_{Aj} total profit of agent under strategy $j(j=1,2,3,4)$

Assumptions. The proposed models are based on the following assumptions:

(1) There is one agent and one manufacturer; before they conduct a contract, they cannot observe the other's action.

(2) The failure intensity is an increasing function of time,in this model, the failure hazard represents the failure intensity[6], is given by

$$\lambda(t)=\lambda_0+rt; 0\le\lambda_0\le 1 \tag{1}$$

Where λ_0 is the initial failure rate and t is the age of product.

The Establishment of Objective Function

In this paper, several scenarios will be proposed between the manufacturer and the agent. The manufacturer has two strategies: (1) free replacement per failure during the warranty period, (2) no warranty. On the other hand, the agent faces two strategies: (1) repairing any failure of the product for a fixed price per failure, (2) repairing all failures of the product for a fixed total maintenance price.

The optimal sale price, warranty price and warranty period for the manufacturer, and the optimal maintenance price or repair price for the agent is determined by maximizing their profits. The interaction between the manufacture and the agent under non-cooperative game is modeled using a game theory approach.

In the following sections, we establish the corresponding objective function for the manufacturer and agent under different strategies, and obtain the optimal revenue.

The Objective Function of the Manufacturer. Recall that the manufacturer offers the following two options to the customer:

M_1 : The failed item will be replaced free of charge during the warranty period.

M_2 : No warranty is offered.

Based on the two options, the optimal solutions of the manufacturer's profit functions will be obtained below.

The optimal decision of the manufacture under strategy $M1$. The total profit model of the manufacture under strategy M_1 is

$$\Pi_{M1}(P_{1p},T_w)=S_{M1}(P_{1p}+P_w-C_p-(C_p-P_s)E(N_2))$$

S.t.

$$P_w=\beta T_w$$

$$S_{M1}=K_0-a_1P_{1p}-t_1b_1C_r-t_2b_2P_a$$

$$t_1+t_2=1 \tag{3}$$

t_1 and t_2 are considered as the zero-one decision variables and are based on the choice of the agent. If the agent selects the strategy A_1, $t_1=1$, $t_2=0$, otherwise, $t_1=0$, $t_2=1$. According to assumption 4, the warranty price, P_w, is a linear function of warranty period, T_w. By assumption 6, the sales volume, S_{M1} depends on its price, P_{1p} and C_r or P_a. In this model, the customer pays the sale price and warranty price to the manufacturer for purchasing the product by free replacement warranty, and after its expiration, pays a repair cost to the agent for fixing each failure or pays a total maintenance price to the agent for fixing all failures. Besides, by assumption 3, the number of failures, N_2, is a Poisson process over the warranty period, therefore

$$E(N_2)=\int_0^{T_w}\lambda(t)dt=\lambda_0T_w+\frac{1}{2}rT_w^{\ 2} \tag{4}$$

Hence, (3) can be rewritten as

$$\Pi_{M1}(P_{1p},T_w)=\left(K_0-a_1P_{1p}-t_1b_1C_r-t_2b_2P_a\right)\left(P_{1p}+\beta T_w-C_p-(C_p-P_s)\right.$$

$$\left.\left(\lambda_0T_w+\frac{1}{2}rT_w^{\ 2}\right)\right) \tag{5}$$

the optimal warranty period and the sale price for the manufacturer are obtained by applying the first order deviation to (5) resulting in

$$T_w^*=\frac{\beta-(C_p-P_s)\lambda_0}{r(C_p-P_s)} \tag{6}$$

$$P_{1p}^*=\frac{K_0-t_1b_1C_r-t_2b_2P_a-a_1\left(\frac{(\beta-(C_p-P_s)\lambda_0)^2}{2r(C_p-P_s)}-C_{\mathrm{p}}\right)}{2a_1} \tag{7}$$

The optimal decision of the manufacture under strategy M_2**.** For the option of no warranty, M_2, the total profit model is

$$\Pi_{M2}(P_{2p}) = S_{M2}(P_{2p} - C_p)$$

S.t.

$$S_{M2} = K_0 - a_2 P_{2p} - t_1 B_1 C_r - t_2 B_2 P_a$$

$$t_1 + t_2 = 1 \tag{8}$$

By Assumption 6, the sales volume , S_{M2} depends on its price P_{2p}, P_a or C_r. As the profit function $\Pi_{M2}(P_{2p})$ is a concave function in P_{2p}, the optimal sale price for the manufacturer are obtained by applying the first order deviation to (8) resulting in

$$P_{2p}^* = \frac{K_0 - t_1 B_1 C_r - t_2 B_2 P_a + a_2 C_P}{2a_2} \tag{9}$$

The Objective Function of the Agent. The agent has two options：

A_1: To repair the failed product for a fixed cost per failure.

A_2: To repair all failures for a fixed total maintenance price.

Based on the two options of the agent the optimal solutions of profit functions are obtained as follows.

The optimal decision of the agent under strategy A_1**.** Under this strategy, the agent’s profit is

$$\Pi_{A1}(C_r) = y_1 S_{M1}(C_r - C_r^{'})E(N_3) + y_2 S_{M2}(C_r - C_r^{'})E(N_1)$$

S.t.

$$S_{M1} = K_0 - a_1 P_{1p} - t_1 b_1 C_r - t_2 b_2 P_a$$

$$S_{M2} = K_0 - a_2 P_{2p} - t_1 B_1 C_r - t_2 B_2 P_a$$

$$t_1 + t_2 = 1 \tag{10}$$

At this point, the agent has chosen strategy A_1, therefore $t_1 = 1, t_2 = 0$. y_1 and y_2 are considered as the zero-one decision variables and are based on the choice of the manufacture. If the agent selects the strategy M_1, $y_1 = 1$, $y_2 = 0$, otherwise, $y_1 = 0$, $y_2 = 1$.

where N_3 is the number of failures after the warranty period, and N_1 is the number of failures during the lifetime. Using assumptions 3 and 7,

$$E(N_3)=\int_{T_w}^{L+cT_w}\lambda(t)dt=\lambda_0(L+cT_w-T_w)+\frac{1}{2}r((L+cT_w)^2-T_w^2) \qquad (11)$$

$$E(N_1)=\int_0^{L}\lambda(t)dt=\lambda_0 L+\frac{1}{2}rL^2 \qquad (12)$$

Therefore, (10) can be rewritten as

$$\Pi_{A1}(C_r)=y_1\left(K_0-a_1P_{1p}-b_1C_r\right)(C_r-C_r^{'})\left(\lambda_0(L+cT_w-T_w)+\frac{1}{2}r((L+cT_w)^2-T_w^2)\right)$$

$$y_2\left(K_0-a_2P_{2p}-B_1C_r\right)(C_r-C_r^{'})\left(\lambda_0 L+\frac{1}{2}rL^2\right) \qquad (13)$$

The profit function $\Pi_{A1}(C_r)$ is a concave function in C_r (refer to Appendix(3)). Therefore, the optimal repair cost, C_r^*, is

$$C_r^*=\frac{y_1E(N_3)\left(K_0-a_1P_{1p}+b_1C_r^{'}\right)+y_2E(N_1)\left(K_0-a_2P_{2p}+B_1C_r^{'}\right)}{2b_1y_1E(N_3)+2B_1y_2E(N_1)} \qquad (14)$$

The optimal decision of the agent under strategy A_2. Under this strategy, the agent's profit is

$$\Pi_{A2}(P_a)=y_1S_{M1}(P_a-C_r^{'}E(N_3))+y_2S_{M2}(P_a-C_r^{'}E(N_1))$$

S.t.

$$S_{M1}=K_0-a_1P_{1p}-t_1b_1C_r-t_2b_2P_a$$

$$S_{M2}=K_0-a_2P_{2p}-t_1B_1C_r-t_2B_2P_a$$

$$t_1+t_2=1 \qquad (15)$$

Where the agent has chosen strategy A_2, therefore $t_1=1, t_2=0$.(15) can be rewritten as

$$\Pi_{A2}(P_a)=y_1\left(K_0-a_1P_{1p}-b_2P_a\right)\left(P_a-C_r^{'}\left(\lambda_0(L+cT_w-T_w)+\frac{1}{2}r((L+cT_w)^2-T_w^2)\right)\right)$$

$$y_2\left(K_0-a_2P_{2p}-B_2P_a\right)\left(P_a-C_r^{'}\left(\lambda_0 L+\frac{1}{2}rL^2\right)\right) \qquad (16)$$

The profit function $\Pi_{A2}(P_a)$ is a concave function in P_a (refer to Appendix(4)). Therefore, the optimal P_a^* is

$$P_a^* = \frac{y_1\left(K_0 - a_1P_{1p} + b_2C_r^{'}E(N_3)\right) + y_2\left(K_0 - a_2P_{2p} + B_2C_r^{'}E(N_1)\right)}{2b_2y_1 + 2B_2y_2} \tag{17}$$

The Mathmatical Model

In the following, we describe the components of the two-person game which will be used in the models . We have a set of players $N(N=2)$, and for each player, we specify the set of strategies and the corresponding profit models.

We use M and A to represent the manufacture and the agent, $N = \{M, A\}$.

Strategies for each participant are represented by S_i, $i = M, A$, is

$$S_M = \{M_1, M_2\}$$

$$S_A = \{A_1, A_2\}$$

The payoff of each player, $u(i)$, $i = M, A$, is

$$u(M) = \{\Pi_{M1}(P_{1p}, T_w), \Pi_{M2}(P_{2p})\}$$

$$u(A) = \{\Pi_{A1}(C_r), \Pi_{A2}(P_a)\}$$

In the static model as the non-cooperative game, the players obtain their best strategy simultaneously and separately. Nash equilibrium [错误!未找到引用源。]is a kind of strategy combination, which makes each participant's strategy is the best response to other participants' strategy.

In order to obtain the Nash equilibrium, we first get the optimal solution of the manufacturer and the agent under each strategy combination.

When the manufacturer chooses strategy M_1, the agent chooses strategy A_1, and $t_1 = 1, t_2 = 0$, $y_1 = 1$, $y_2 = 0$, from Eqs. (6)、(7) and (14), we have

$$T_w^* = \frac{\beta - (C_p - P_s)\lambda_0}{r(C_p - P_s)} \tag{18}$$

$$P_{1p}^* = \frac{K_0 - b_1C_r^{'} - 2a_1\left(\dfrac{(\beta - (C_p - P_s)\lambda_0)^2}{2r(C_p - P_s)} - C_{\mathrm{p}}\right)}{3a_1} \tag{19}$$

$$C_r^* = \frac{K_0 + 2b_1 C_r^{'} + a_1\left(\dfrac{(\beta-(C_p-P_s)\lambda_0)^2}{2r(C_p-P_s)} - C_{\mathrm{p}}\right)}{3b_1} \tag{20}$$

When the manufacturer chooses strategy M_1, the agent chooses strategy A_2, and $t_1 = 0, t_2 = 1, y_1 = 1, y_2 = 0$, from Eqs. (6)、(7) and (17), we have

$$T_w^* = \frac{\beta-(C_p-P_s)\lambda_0}{r(C_p-P_s)} \tag{21}$$

$$P_{1p}^* = \frac{K_0 - b_2 C_r^{'} D - 2a_1\left(\dfrac{(\beta-(C_p-P_s)\lambda_0)^2}{2r(C_p-P_s)} - C_{\mathrm{p}}\right)}{3a_1} \tag{22}$$

$$P_a^* = \frac{K_0 + 2b_2 C_r^{'} D + a_1\left(\dfrac{(\beta-(C_p-P_s)\lambda_0)^2}{2r(C_p-P_s)} - C_{\mathrm{p}}\right)}{3b_2} \tag{23}$$

$$D = \lambda_0(L + cT_w^* - T_w^*) + \frac{1}{2}r((L + cT_w^*)^2 - T_w^{*2}) \tag{24}$$

When the manufacturer chooses strategy M_2, the agent chooses strategy A_1, and $t_1 = 1, t_2 = 0, y_1 = 0, y_2 = 1$, from Eqs. (9) and (14), we have

$$P_{2p}^* = \frac{K_0 - B_1 C_r^{'} + 2a_2 C_P}{3a_2} \tag{25}$$

$$C_r^* = \frac{K_0 + 2B_1 C_r^{'} - a_2 C_P}{3B_1} \tag{26}$$

When the manufacturer chooses strategy M_2, the agent chooses strategy A_2, and $t_1 = 0, t_2 = 1, y_1 = 0, y_2 = 1$, from Eqs. (9) and (17), we have

$$P_{2p}^* = \frac{K_0 - B_2 C_r^{'}\left(\lambda_0 L + \dfrac{1}{2}rL^2\right) + 2a_2 C_P}{3a_2} \tag{27}$$

$$P_a^* = \frac{K_0 + 2B_2 C_r^{'}\left(\lambda_0 L + \dfrac{1}{2}rL^2\right) - a_2 C_P}{3B_2} \tag{28}$$

The obtained payoffs of each player under different strategies are summarized in Table 1.

Table 1 Profits of the manufacture and the agent under different strategies

	A_1	A_2
M_1	$\left(\Pi^*_{M_1(A_1)}, \Pi^*_{A_1(M_1)}\right)$	$\left(\Pi^*_{M_1(A_2)}, \Pi^*_{A_2(M_1)}\right)$
M_2	$\left(\Pi^*_{M_2(A_1)}, \Pi^*_{A_1(M_2)}\right)$	$\left(\Pi^*_{M_2(A_2)}, \Pi^*_{A_2(M_2)}\right)$

Computational Results

In this section, we give some numerical examples to illustrate the important features of the proposed models. We will also perform the managerial insights of two main parameters (lifetime and repair cost) of these models. We note that Examples 1-2 illustrate the non-cooperative static and Stackelberg models,

In all these examples, we set $C_r^{'} = 10\$$, $\lambda_0 = 0.3$, $r = 0.1$ (per year), $L = 4$ (years), $C_p = 90\$$, $P_s = 60\$$, $\beta = 15\$$, $K_0 = 240$, $c = 0.55$, $a_1 = 0.3$, $a_2 = 0.3$, $b_1 = 0.8$, $b_2 = 0.4$, $B_1 = 0.9$, $B_2 = 0.35$.

In the static game, the optimal profits of the manufacturer and the agent's models are presented in Table 2.

Table 2 Profits of the manufacture and the agent under different strategies

	A_1	A_2
M_1	**(15839,12061)**	(15820,11866)
M_2	(15412,10275)	(15717,13472)

Where the bold set indicates Nash equilibrium. The method we get this Nash equilibrium is iterated elimination of strictly dominated strategies. Firstly, for the manufacture, no matter what strategy the agent selects, strategy M_1 is strictly superior to strategy M_2, so eliminating strategy M_2. Then, for the agent, the strategy A_1 is better than strategy A_2 when the manufacture provides warranty, the iterated dominance equilibrium is (M_1, A_1). The result shows that the manufacturer selects option M_1 and the agent chooses option A_1. Therefore $P^*_{1p} = 313.78\$$, $\Pi^*_{M1} = 15839\$$, $C^*_r = 96.17\$$ and $\Pi^*_{A1} = 12061\$$.

Conclusion

In this paper, a warranty service contract has been presented among manufacturer and agent with asymmetric actions by using the game theory approach. The optimal sale price, warranty price and warranty period for the manufacturer, and the optimal maintenance price or repair price for the agent is determined by maximizing their profits. This paper considers the static model under non-cooperative game, and analyzes some important parameters, and obtains some managerial insights that for the agent, the strategy of taking a maintenance price for all failures is better when the product life is short, otherwise, the strategy of taking a fixed cost per failure is better, and for the manufacture, the strategy for providing warranty is better in most situation.

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