# Simulation and analysis of the change of shadow length based on MATLAB for a period of time <br> Yanbo Wang <br> North China Electric Power University, Huadian-Lu 689, 071000 Baoding,China 15733259935@163.com 

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#### Abstract

We can get the relation of the solar altitude and latitude, the angel of equator and latitude and the solar hour angle according to the knowledge of astronomy. Then establish a mathematical model for the length of the shadow of straight bar, in accordance with the relation of height, the length of shadow of straight bar, and the solar elevation angle; Using the control variable method, the variation law of the length of the shadow is analyzed. Then take the value of each parameter into the model, and draw the length of straight bar between 9:00-15:00 Beijing Time by taking help from MATLAB , the length of shadow firstly short then become longer, minimum is 3.8411 m , maximum is 8.7288 m .


## 1. Introduction

We shall firstly confirm some basic parameters before we establish a model for solving the length of the shadow of straight bars under the sun. We can come to a mathematical expression about length of straight bar shadow by understanding the relations between solar altitude with latitude, and angel of equator with solar hour angle, then according to the geometry relation between length of straight bar, the length of straight bar shadow, and the solar altitude; Because of the large number of parameters, we can use the control variable method to analyze the variation law of the shadow length, and control the remaining parameters to study the relationship between shadow length and Beijing time. Finally, the values of the corresponding parameters shall be brought into the model ${ }^{[1]}$. With the aid of MATLAB, the curve of the shadow length can be drawn under the corresponding conditions.

## 2. Model assumptions

1. Assuming that the sun's rays on the earth's surface are parallel to the light.
2. Assuming that the straight bar is located in the ground plane。
3. Assuming that the length of the straight rod is not affected by the external factors such as temperature, pressure and so on.
4. Assuming that the earth is a ball, and place on earth level ground is the section of surface sphere to the plane

## 3. Symbol Description

| Symbol | Significance | Symbol | significance |
| :---: | :---: | :---: | :---: |
| $\theta$ | solar elevation | $\psi$ | longitude |
| $\square a$ | solar altitude | $h$ | time difference |
| $\square \delta$ | solar azimuth | $E$ | Beijing time |
| $\square \omega$ | angel of equator and latitude | $L$ | Straight rod length |
| $\varphi$ | solar hour angle | $l$ | Length of straight bar |
| $\square\left(\mathrm{x}_{\mathrm{i}}, \mathrm{y}_{\mathrm{i}}\right)$ | latitude | $\left(\mathrm{x}_{\text {yesi }}, \mathrm{y}_{\text {yesi }}\right)$ | Length of the shadow of straight bar |

## 4. Establishment and Solution Model

### 4.1 Establishment of model.

Calculate the length of the straight bar shadow, we need to determine some basic parameters, including the solar altitude, the angel of equator and latitude and the solar hour angle.

## 4.2 (1) solar hour angle:

The angle of the earth rotation of the unit time is defined as the hour angle $\omega$.The noon hour is 0 , the morning is negative, and the afternoon is positive. The rotation of the earth is 360 degrees, the corresponding time is 24 hours, that is, the corresponding hourly angle is 15 degrees. The need to take into account the influence of time difference on the corner ${ }^{[2]}$. There is a formula

$$
\begin{aligned}
& \omega=(h+E) \times 15+\psi-300 \\
& E=\frac{\psi-120^{\circ}}{15^{\circ}}
\end{aligned}
$$

Therefore, based on the time of Beijing time, the formula for calculating time:
$\omega=15 h+2 \psi-420$
In this formula:
$\omega$-Solar hour angle;
$h$-Beijing Time;

$\psi$-Local longitude;
$E$-time difference
(2) the angel of equator and latitude: The angle between the earth's equatorial plane and the connection line which between the sun and the center of the earth. It take a year as cycle, moving between $+2326^{\prime}$ and- $2326^{\prime}$, therefore it is also the symbol of seasons ${ }^{[3]}$. Because of a very small change, this angle can be calculated in accordance to the Cooper equation as below:

$$
\delta=23.45 \sin \frac{360(284+d)}{365}
$$

In this formula:
$\delta$-the angel of equator and latitude;
$d$-day of year. 1st January is 1, common year is 365 at the 31st December, and leap year shall be 366 at 31st December.
(3) solar altitude:

The angle between the sun's rays and the earth's surface that is connected with the inner earth through the earth's surface, which is at the same latitude degree, the degree of solar altitude is along with the change of solar hour angle and the angel of equator and latitude. The formula for calculating the solar altitude angle is:
$\sin \theta=\sin \varphi \sin \delta+\cos \varphi \cos \delta \cos \omega$
In this formula:
$\theta$-solar altitude;
$\varphi$-latitude;
$\delta$-the angel of equator and latitude;
$\omega$-solar hour angle
Assuming the height of straight bar is $L$, The length of the straight bar of the sun is $l$, the solar altitude in Beijing $\theta$ impossible is obtuse.


Fig. 1 Height model of the sun in Beijing
According to the tangent function, it is easy to get the length of straight bar shadow under the sun, the 1 shall be :
$l=\frac{L}{\tan \theta}=\frac{L}{\arcsin [\arcsin (\sin \varphi \sin \delta+\cos \varphi \cos \delta \cos \omega)]}$,
In this formula:
$l$-length of straight bar;
$L$-height of straight bar;
$\varphi$-latitude;
$\psi$-longitude;
$\omega$-Solar hour angle.

### 4.3Analysis of parameters in the model

When the days $d$ and latitude $\varphi$ is stable, so does the angel of equator and latitude, l, the length of straight bar shadow only relative with the solar hour angle $\omega$. There is a formula between Solar hour angle $\omega$, Beijing Time $h$ and the longitude $\psi^{[4]}$ :

$$
\omega=15 h+2 \psi-420 .
$$

Therefore, even for the are with same longitude, $\omega$ it is increasing with the increase of $h$. Besides, the formula of $l$ is using the $\cos \omega$, therefore the condition of $l$ is along with the change of $h$.

For regions right on the prime meridian, with longitude of 180 degree, the trend of change of $l$ with $h$ is increasing, decreasing, increasing, then become shorter again ${ }^{[5]}$.

## 5. Summary

2015 is common year, assuming d= 295, $\varphi=3954^{\prime} 26^{\prime \prime}, \psi=11623^{\prime} 29 ", \mathrm{~L}=3, \mathrm{~h} \in[9,15]$, With the aid of MATLAB, the changing curve of solar attitude of a 3m high straight bar in Tiananmen Square from Beijing Time 9:00-15:00 in 22rd October 2015 can be shown as Figure2, the curve of shadow length is shown in Figure 3.


Fig. 2 The change curve of solar altitude angle

Shadow length/m


Fig3 Change curve of shadow length

It is shown in the above figure, between 9am to 12:30, the length of straight bar shadow is decreasing, at the point of 9am, the length is maximum 8.7288 m , the minimum of length is at the point of $12: 30,3.8411 \mathrm{~m}$. Then it came to 15:00, the length of straight bar is increasing again, and it reached 5.818 m at 15:00.

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