The Optimal Search Region in the Sea

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Abstract. The major assumption is that the main body of the crashed plane is complete and don't drift with flow. All the work units are not affected by weather conditions and obey the unified dispatch.

The model, Determining search region, is based on the theory that the crashed plane obeys normal distribution on both sides of the baseline which we determine according to the predetermined route and other information about the crashed plane. After the baseline determined, we use the work units' affordability, the probability density of the target's distribution, the probable error and the crashed plane's distance of unpowered gliders as constraints to find the optimal search width.

Introduction

In recent years the frequency of aircraft accidents has risen rapidly such as MH370. Due to the lack of relevant search experience and in the absence of good cooperation between the countries, finally the progress is a very slow search and the search result is far from meeting expectations. How to search and rescue better at sea with limited time and resource is a problem worth considering.

Our mathematical model is based on the establishment of the above-described problems. Within a defined area, we developed an optimal search plan. The point is that by understanding the relevant search criteria and specific scenarios we set one of the best search region.

Table 1. Terminology and definitions		
A	Search region	
	Alternative search region	
R	The best search width[1]	
L	Reference baseline [2]	
l	The length of reference baseline	
V_i	The flying speed of search aircraft	
T_i	Work unit's maximum flight time	
W_i	Work unit's detection width	
Z_i	Work unit's detection ability[3]	
${\mathcal Y}_1$	The crashed plane's last contact point	
y_2	The crashed plane's maximum range along the route	
r_i	The affordable search width of work unit	
ε	Probable error	
σ	Mean square deviation	
E	The total probable error of position	

The Model to Determine Search Region

Terminology and Definition.

X	Initial target position error
Y	Search facility position error
$C_{_{y}}$	Crashed plane's lift coefficient
ρ	Air density
S	Crashed plane wing area
E'	The distance of crashed plane unpowered glider

Establish a Baseline. Regard the point where the console contract with the crashed plane last time as a starting point y_1 . End of the maximum range is the end point y_2 . Set the aircraft's predetermined routes as a baseline *I* and its length is *L*.

Determine the Best Search width R. Ignoring the effect of weather conditions, aircraft search capabilities can be deployed from a search speed V, search duration T, and scan width W to determine. The maximum width each search aircraft can withstand:

$$r_i = \frac{W_i L}{V_i T_i} \tag{1}$$

The maximum width search teams can withstand:

$$r = \sum r_i \tag{2}$$

In the case that some of the government provide resources. The maximum width search teams can withstand become kr, k is determined by the maximum resources government can provide. So the first constraint is:

$$R \le kr \tag{3}$$

Search target is evenly distributed along the reference baseline and it obeys normal distribution along the direction perpendicular to the reference baseline. Generally speaking, left and right sides of the reference baseline is the same probability distribution. Baseline *l* is used as the axis *y* of a Coordinate system. Start and end points are respectively $(0, y_1)$ and $(0, y_2)$. Make (x, y) represents the target true position. Then when the *x* and *y* are independent, the *x* obeys normal distribution and the symmetry axis is *l*, just as the Figure 2 shows. The probability density function is:

$$f(x) = \frac{1}{2\pi\sigma^2} \exp(-\frac{x^2}{2\sigma^2}), \qquad -\infty < x < +\infty$$
(4)

$$f(y) = \begin{cases} \frac{1}{y_2 - y_1}, & y_1 < y < y_2 \\ 0, & \text{others} \end{cases}$$
(5)



Figure 1.The probability density distribution of the search target

In the direction of the shaft x, the longer from the baseline l (i.e. Axis y), the smaller the target's probability of containing is. The target's probability of containing on the both sides of baseline is expressed as:

$$POC = 1 - \exp(-\frac{x_r^2}{2})$$
 (6)

 x_r is the vertical distance from baseline.

The standard variance σ is used to measure the target position offsetting reference points of the size of the error. But in the maritime search and rescue, we commonly use probable error ε to measure the size of the target position error. σ usually makes x_r whose constraint is that *POC* is 50% cumulative. So we can get :

$$\varepsilon = \sigma \sqrt{-2\ln(1 - 0.5)} = 1.1774\sigma \approx 1.18\sigma \tag{7}$$

So probable error is 1.8 times of standard variance. Usually ε is associated with navigational fix error and can be obtained by navigation device query literature [4] schedule 14-1 to schedule 14-3. According to the above conditions, we can get the second constraint conditions:

 $R \ge \varepsilon$ (8)

The total probable error of position, E, (or the total probable error for short) is the comprehensive standard to measure uncertainty of search target's position (benchmark calculation error) and the work units' ability for accurate navigation, which is also decisive factor to determine specific search region. The plane glide distance E' can be worked out under the help of the glide model.

We can figure out *E*:

$$E = \sqrt{X^2 + Y^2 + D_{\varepsilon}^2} \tag{9}$$

Here we ignore the drift error D_{ε} . So E can be simplified as:

$$E = \sqrt{X^2 + Y^2} \tag{10}$$

The error of the search targets *X* is:

$$X = Fix_e + DR_e \tag{11}$$

Here Fix_{ε} is the search target's positioning error and DR_{ε} is the error of its estimated path. Fix_{ε} value can refer to Table 2. DR_{ε} is proportional to the voyage. Take crashed aircraft's dead reckoning DR_{ε} as 2% of the estimated voyage.

The error of the work units is *Y*:

$$Y_i = Fix_e + DR_e \tag{12}$$

$$Y = \sum Y_i \tag{13}$$

The Fix_{ε} is the work units' positioning error and DR_{ε} is the error of its estimated path. Fix_{ε} value can refer to Table 2. DR_{ε} is proportional to the voyage. Take crashed aircraft's dead reckoning DR_{ε} as 2% of the estimated voyage.

Table 2. Fix_{ε} value			
Positioning method	The positioning error (Fix_{ε})		
GPS	0.1NM		
Landmark(there lines of position)	1NM		
Astronomical positioning(there lines of	2NM		
position)			
Radio direction finding (there lines of	4NM		
position)			
LORANC	1NM		
Radar	1NM		

When the plane lost power, the plane glide distance E' can be worked out under the help of the glide model.

Integrated afore-mentioned conditions, finally the value range of R width of the optimal search is:

$$\begin{cases}
R \le kr \\
R \ge \varepsilon \\
R \ge E + E'
\end{cases}$$
(14)

Conclusion

Now that we have get the baseline and the best search width R, we can work out an optimal search region just as the Figure 2 show.



References

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