The Prediction of Temporal-Spatial Distribution of Space Debris

XiaoRui Li[1,a]

[1]North China Electric Power University, Baoding, China

[a]584825295@qq.com

Keywords: Space Debris, Distribution, Prediction, Diameters, Orbits

Abstract. In this paper, we establish a Temporal-Spatial Distribution of Space Debris Prediction Model based on *Kessler model*. We deduce debris' flux and the probability that space objects be hit by debris in certain space-time. Next, we make predictions about debris' distribution with different diameters, in different orbits and during discrete years. Besides, we put forward the probability in different orbits.

Introduction

It is estimated that more than 500,000 pieces of orbital debris are being tracked to space craft. There is a wide discussion on the issue. It is important to research the distribution of space debris. The space around the earth can be divided into four orbits. The tracked space objects are mainly in LEO, followed by GEO and MEO.

1. Low earth orbit (LEO): Heights below 5500km.
2. Middle earth orbit (MEO): Heights between the LEO's and the GEO's.
3. Geosynchronous orbit (GEO): Heights around 36000km.
4. Other orbit: Include the high-eccentricity orbit and transfer orbit from low earth orbit to higher orbit.

Analysis on the space debris

Going through the relevant literature we get some clues. In 1981, *DonaldJ. Kessler* established an algorithm about space density[1]:

$$S(r,\phi)=\frac{1}{2\pi^3 ra\left[\left(\sin^2 i-\sin^2\phi\right)(r-q)(q'-r)\right]^{\frac{1}{2}}} \quad (1)$$

In the algorithm, S is the density at ϕ degrees latitude in r height. It is influenced by the inclination i, perigee height q and apogee height q'. Besides, $a=(q+q')/2$, a is the symbol of the semi major axis. Density of space debris $\rho(h,\phi)$ at ϕ degrees latitude in h height is related to the N_Σ, $p(h_\rho)$, $p(e)$, $p(i)$.

$$\rho(h,\phi)=\frac{N_\Sigma}{2\pi^2(h+R)^2\Delta h}\cdot F(\phi)\cdot\int_{h_p}\int_{e}\Delta\tau(h,h+\Delta h)\cdot\Phi(h_p,e,h)\cdot p_{h_p}(h_p)\cdot p_e(e)\cdot dh_p\cdot de \quad (2)$$

As the equation shows, R stands for the radius of the Earth, Δh is the calibration of space grid along the height. $\Delta\tau$ stands for the probability that space debris drop in the range $(h,h+\Delta h)$,

$$F(\varphi)=\int_i\frac{p(i)\cdot di}{\sqrt{\sin^2 i-\sin^2\varphi}},\sin i\geq\sin\varphi \quad (3)$$

$$\Phi(h_p, e, h) = \frac{(1-e)^2}{\sqrt{1-e^2}} g \left(\frac{h+R}{h_p+R} \right)^2 \quad (4)$$

Know then, every set $\left(N_{\Sigma}, p(h_p), p(e), p(i)\right)$ corresponds an environment of space debris, but not vice versa certainly. According to the large amounts of data, accumulated flux F of space debris can be fitted by Kessler model**[1]**. It is expressed as follows:

$$F\left(d,h,i,t,S\right) = H(d)\Phi(h,s)\Psi(i)\left[F_1(d)g_1(t) + F_2(d)g_2(t)\right] \quad (5)$$

Where l is the diameter of space debris, t means time. h is height which ranges from 350 to 2000. S is the solar flux on 10.7cm wavelengths in the year of t-1, i is inclination.

Where q is the designed to be the annual growth rate of debris, q=0.4. p is the designed to be the annual growth rate of the total mass of debris. The flux can deduce other quantity.

$$N = \int_0^l F\left(d,h,i,t,S\right)dl \quad (6)$$

Where N is the amount of space debris. The equation express the amount in unit time and physical volume.

Prediction about the amount of space debris

Different diameters: While other quantities are fixed, the diameter of space debris varies from 0.001 to 0.8. To realize the purpose, we use *MATLAB* to simulate the problem. The table 1 is shown as follows:

Table 1: The density of debris with different diameters

Diameter(cm)	0.001	0.002	0.003	0.004	…	0.799	0.8
Average amount(pcs/m^2)	4.57E-07	1.19E-06	1.89E-06	2.49E-06	…	4.46E-07	4.45E-07

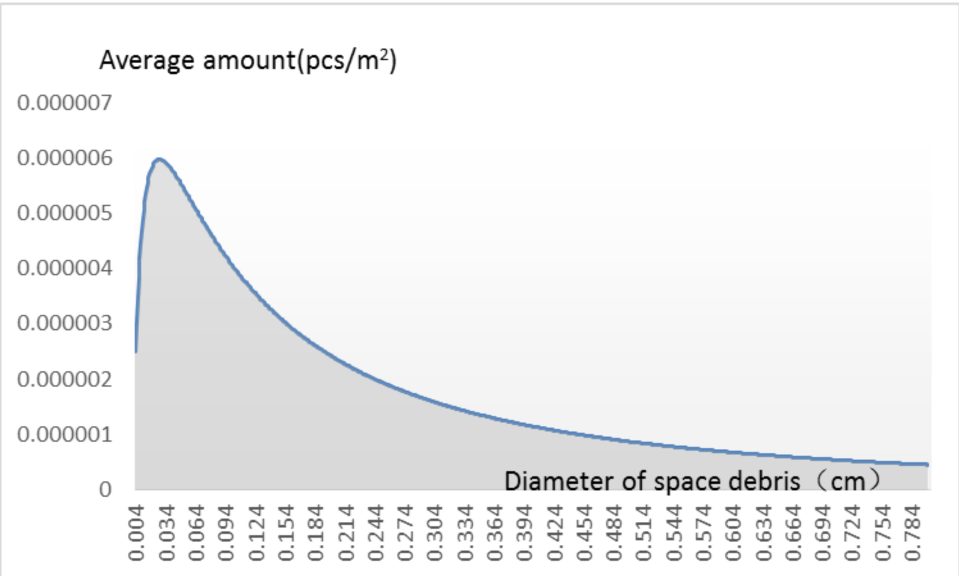

Figure 1:the density of space debris in different size

Different orbits: Different orbits has various height. The height is defined as the difference between orbit’s radius and the Earth’s radius. If the size of debris is fixed, We get the simulations:

Table 2: The relationship of Flux and height

Height(km)	1	5	9	13	…	3561	3565
Flux	4.35E-07	4.4E-07	4.45E-07	4.51E-07	…	7.05E-05	7.05E-05

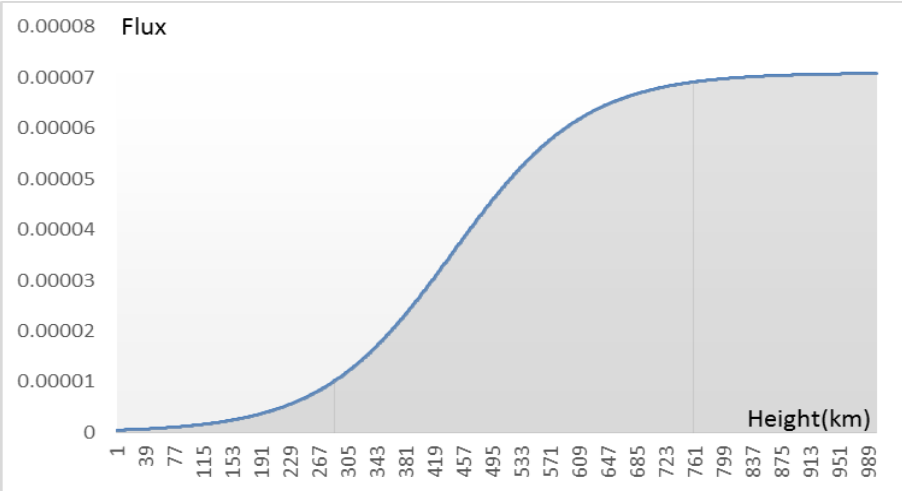

Figure 2: The function of flux and height

As indicated in the figure and table, the flux start to raise up when the height is about 300km. After 730km, the flux begin to flatten.

Conclusion

By the research of the Space Debris, the distribution of the debris can be measured by the physical expression such as S, F and N. They express the debris’ amount in unit time and physical volume. Based on the distribution, we dividedly make the prediction about the amount of space debris while other quantities are fixed. When the debris have different diameters, The prediction shows the debris are mainly concentrated in the range of 0 to 1. When the debris are in different orbits, the amount starts to raise up when the height is about 300km. After 730km, the amount begins to flatten.

References

[1]Peng keke, “research on space debris environment exploration data processing measure and engineering modeling method”, Harbin Institute of Technology, July, 2010

[2]Fukushige S,Akahoshi Y,Kitazawa Y.Comparison of debris environment models:ORDEM2000,MASTER2001 and MASTER2005[J].IHI Engineering Review,2007,40(1):31-41

Click to display the text

[3]Liou J C.The new NASA orbital debris engineering model [R].ORDEM2000,NASA/TP-2002-210780,2002

Click to display the text

[4]Liou J C.NASA's ORDEM2010 status[C]//28th Inter-agency Debris Coordination Committee Meeting(IADC).Trivandrum,India:NASA,2010:1-20

Click to display the text

[5]Wang Haifu,Yu Qingbo,Liu Youying.Orbital debris risk assessment system[J].Transactions of Beijing Institute of Technology,2008,28(12):1039-1042(in Chinese)

[6]Xu X L,Xiong Y Q.A research on the collision probability calculation of space debris for

nonlinear relative motions[J].Chinese Astronomy and Astrophysics,2011(35):304-317