Low-angle estimation method for meter-wave radar via matrix completion

Qichao Ge[1,a] , Yongshun Zhang[2,b] and Shanshan Ding[3,c]

[1] Air Force Engineering University, Xi'an, Shaanxi, China

[2] Collaborative Innovation Center of Information Sensing and Understanding, Xi'an, Shaanxi, China

[3] Air Force Engineering University, Xi'an, Shaanxi, China

[a]geqichao927@163.com, [b]zysgcdx@sina.com, [c]dingshanshan001@hotmail.com

Keywords: Low-angle estimation; matrix completion; Hankel matrix; inexact augmented Lagrange multiplier method;

Abstract. The deficiencies of array elements causes the decrease of angle survey precision when detect low-angle estimation. In order to solve this problem, this paper transform the data missing matrix which isn't meeting the condition of the SIP into appropriate one based on the MC theory by using the Hankel matrix's properties. Then according to the ILAM algorithm and the generalized MUSIC algorithm, the method can recover the complete data matrix and get the angle messages in the reality. The simulated experiment verifies the method effectiveness.

Introduction

In the modern battlefield, the meter wave radar more and more apply in the covered target detection and track because of its unique effect. Generally the beam width of the meter band radar is board, so the reflection signal echoes of low altitude target and the multi path echoes appear in the same detection beam. It's hard to distinguish the angle of the low altitude goals effectively by using traditional angle-measure, such as mono pulse angle-measure and so on. According to long-time research, many effectual algorithms has been established, such as the generalized MUSIC algorithm and the low-altitude angle-measure based on the Compressive Sensing [1]. But the algorithms above need the precision data matrix to achieve the right angle estimation of goals. In the fact, it's infeasible to receive the precision data matrix due to the damage of data store and other influence. So the urgent problem is to estimate the right angle of the targets under the loss of the message in the data matrix.

The Matrix Completion method which is established by Candès can preciously reconstitute the original matrix according the low rank constraints of several elements [2]. Naive MC algorithm require the sampling matrix which each line each column exists a nonzero element at least in order to allow perfect recovery. But the damage of the elements leads to the deficiency of the relevant data matrix. Therefore Dong Yang [3] reshape the signal vector in a single snap short into an equivalent low-rank matrix and then recover the original data by using the MC method effectively. However a signal snap short causes the loss of precision and the element number limits the freedom of system and applications in the reality.

To address this problem, a new data matrix is proposed in this paper. Each line of the received data matrix is transformed into a Hankel matrix and then made up a two-fold Hankel matrix. The matrix above, which is completed and recovered, can achieve the perfect received data matrix, according to the inverse transformation. Simulation results validate the effectiveness of the proposed method and improve the precision under the loss of elements.

Signal model

It is necessary to take the multipath signal into account when detecting the low altitude targets, because of the meter wave radar wide beam and the serious multipath effect. The multipath reflection signal is defined as the diffuse and the mirror reflection based on the differences of the ground. The reflector is defined the smooth one if the Rayleigh criterion would be satisfied in the first Fresnel zone [4]. In the fact, the wave length of meter wave radar is large, so the reflected signal always are regard as the mirror reflection and the diffuse reflection is used as the noise generally. The model of the mirror reflection is shown in Fig. 1.

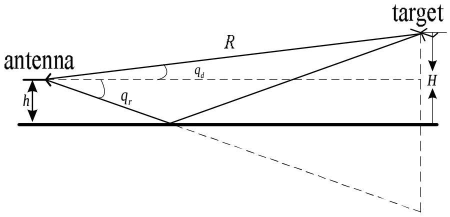

Fig. 1. Mirror reflection model

In the Fig. 1, h is the height of radar, $\boldsymbol{H}$ is the height of the targets, $\boldsymbol{R}$ denotes the straight distance between the radar and the goal, q_d is the incidence angle of direct wave and the q_r is the incidence angle of the mirror reflection.

Assuming antenna as half a wavelength spacing uniform linear array with L elements and define the source as the far field narrow targets. So the data of antenna element receipt is shown as $\boldsymbol{y}(t)$

$$\boldsymbol{y}(t) = \boldsymbol{a}_d s_d(t) + \boldsymbol{a}_r s_r(t) + \boldsymbol{n} . \tag{1}$$

$\boldsymbol{a}_d = \left[1, \exp\left(-jp\sin(q_d)\right), \mathbf{L}, \exp\left(-j(L-1)p\sin(q_d)\right)\right]^{\mathrm{T}}$ can be considered as the steering vector of the direct signal. $\boldsymbol{a}_r = \left[1, \exp\left(-jp\sin(q_r)\right), \mathbf{L}, \exp\left(-j(L-1)p\sin(q_r)\right)\right]^{\mathrm{T}}$ can be considered as the steering vector of the multipath reflection signal. And the s_d , s_r denote the range of the direct and the multipath reflection signal; $\boldsymbol{n}$ is defined as statistically independent white noise.

According to the Fig. 1, the connection between the direct signal and the mirror reflection signal can be formed as follows

$$s_r = b s_d . \tag{2}$$

where β represents the complex multipath coefficient.

Hence the data of antenna element receipt can be transformed into

$$\boldsymbol{y}(t) = \boldsymbol{A}_{q_d} s_d(t) + \boldsymbol{n} . \tag{3}$$

$\boldsymbol{A}_{q_d} = [\boldsymbol{a}_d, \boldsymbol{a}_r]\,\boldsymbol{p}$, ($\boldsymbol{p} = [1, b]^{\mathrm{T}}$) represents the manifold matrix of q_d ,q_r .

Consider the multiple snapshots with M snaps, data matrix is shown as $\boldsymbol{Y}$

$$\boldsymbol{Y} = \boldsymbol{A}_{q_d}\boldsymbol{S} + \boldsymbol{N} . \tag{4}$$

Where $\boldsymbol{Y}$, $\boldsymbol{S}$ are defined as follows : $\boldsymbol{Y} = [\boldsymbol{y}_1, \boldsymbol{y}_2, \mathbf{K}, \boldsymbol{y}_M]$, $\boldsymbol{S} = [\boldsymbol{s}_1, \boldsymbol{s}_2, \mathbf{L}, \boldsymbol{s}_M]$; $\boldsymbol{N}$ denotes statistically independent white noise.

Process of angle estimation

Due to the loss of the antenna element array, the matrix $\boldsymbol{Y}$ with appearance of the zero column doesn't meet the SIP requires of the MC algorithms[2,5]and cannot recover original data directly according to the MC algorithms. Therefore the $\boldsymbol{Y}$ should be reshaped to meet the requires of the SIP.

Firstly transpose the $\boldsymbol{Y}$ [6], we get

$$\hat{\boldsymbol{Y}} = \boldsymbol{Y}^{\mathrm{T}} \cdot \tag{5}$$

Define a two-fold Hankel structure as follows

$$\boldsymbol{Y}_e = \begin{bmatrix} \hat{\boldsymbol{Y}}_1 & \hat{\boldsymbol{Y}}_2 & \cdots & \hat{\boldsymbol{Y}}_{M-k_1+1} \\ \hat{\boldsymbol{Y}}_2 & \hat{\boldsymbol{Y}}_3 & \cdots & \hat{\boldsymbol{Y}}_{M-k_1+2} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{\boldsymbol{Y}}_{k_1} & \hat{\boldsymbol{Y}}_{k_1+1} & \cdots & \hat{\boldsymbol{Y}}_M \end{bmatrix}. \tag{6}$$

Based on the formula above, the k_1 （$1 \le k_1 \le M$ ） is the structure parameters of $\boldsymbol{Y}_e$, meanwhile each element belongs to $\boldsymbol{Y}_e$ is the $k_2 \times (L - k_2 + 1)$ Hankel matrix, the detail is shown as follows

$$\hat{\boldsymbol{Y}}_l = \begin{bmatrix} \hat{Y}_{l,1} & \hat{Y}_{l,2} & \cdots & \hat{Y}_{l,L-k_2+1} \\ \hat{Y}_{l,2} & \hat{Y}_{l,3} & \cdots & \hat{Y}_{l,L-k_2+2} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{Y}_{l,k_2} & \hat{Y}_{l,k_2+1} & \cdots & \hat{Y}_{l,L} \end{bmatrix}. \tag{7}$$

The range of parameter l is $1 \le l \le M$. The parameter $k_2 (1 \le k_2 \le L)$ is another structure parameter of the matrix above.

Based on the certification of the paper[6], $\boldsymbol{Y}_e (rank(\boldsymbol{Y}_e) \le K)$ satisfies the requires of the SIP. The complete data matrix can be recovered and realize the precise estimation according to reshape the $\boldsymbol{Y}_e$ by using the MC algorithms and inverse transform it.

Define the parameter χ as

$$c = \max\left\{ \frac{ML}{k_1 k_2}, \frac{ML}{(M - k_1 + 2)(L - k_2 + 1)} \right\}. \tag{8}$$

The least work to recover matrix can be realized when the parameter χ get the minimum. Hence establish $\boldsymbol{Y}_e$ as a matrix ease to calculate.

Above all, the algorithm proposed in this paper is summarized as follows

a. Reshape the data matrix $\boldsymbol{Y}$ into the matrix $\boldsymbol{Y}_e$ which is shown in the Eq. 6;

b. Realize the precise reconstitute of $\boldsymbol{Y}_e$ by used of the ILAM algorithm;

c. Get the complete receipt data matrix according to the inverse transformation;

d. Estimate the angle of source based on the generalized MUSIC algorithm.

Simulations

Assume that the radar is a uniform linear array with 15 elements, 1 meter wave length, 55 snaps. The interval between every two elements is half a wave length. The height of target is 400 meters and the radar is 10 meters high. The straight distance between radar and target is 20 kilometers. The angle of direct wave is 1.1° and the Monte Carlo experiment runs 100.

The spatial spectrum of angle estimation when there are 5 elements damage randomly is shown in. Fig. 2. Through the Fig. 2, we can find that the spatial spectrum by using the algorithm in this paper is close to the original data matrix by using the generalized MUSIC algorithm. The two line upper the figure denotes two methods, the direct generalized MUSIC algorithm and the direct generalized MUSIC algorithm based on the ILAM, under the loss of element's receive data. Two lines are alike, but totally lower than the measure proposed above. The simulation testifies the effectiveness of the algorithm in this paper.

The Fig. 3 shows the effect of algorithm property due to the change of the SNR. In Fig. 3, the property of the algorithm improves with SNR increasing . The improvement is close to the one of the original data matrix and is more higher than the property without the matrix completion.

The effect of algorithm property due to the damage of the array elements is shown in Fig. 4. Fig. 4 shows that, by decreasing the number of elements, the property of algorithm using in this paper declines but still better than using the generalized MUSIC algorithm, which deals with the data missing matrix directly. The superiority is obvious when the damage of elements is serious.

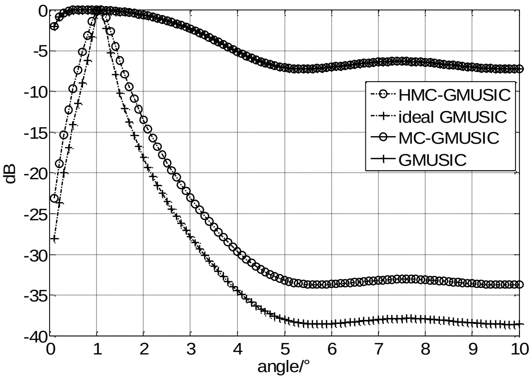

Fig. 2. Spatial spectrum

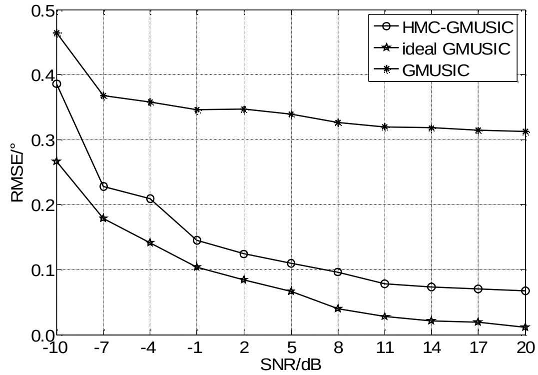

Fig. 3. The effect of the change of SNR

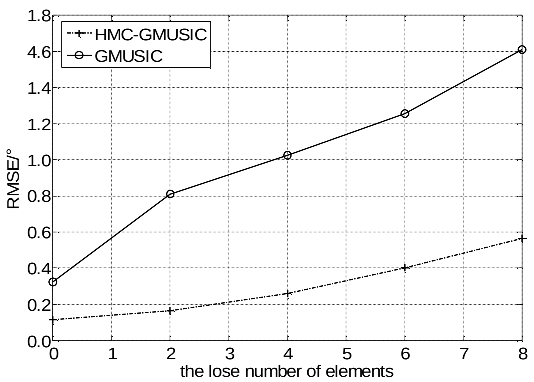

Fig. 4 The influence of the loss number of elements

Conclusions

According reshape the receive data matrix of the damage array element based on the transformation of the Hankel matrix, this paper proposed a low-altitude detection for meter wave radar based on the matrix completion. The method can reshape the matrix which is not suitable for the SIP requires of the MC algorithm into a new matrix. Then recover a precise and complete receive data matrix according to the MC algorithm. Finally, the precise estimation of the angle can be realized under the low-altitude and the loss of elements. The method in this paper is suitable for kinds of signal models and the existence of the diffuse reflection and the applications of multi targets will be the next problem to resolve.

Acknowledgements

This work was financially supported by the Natural Science Foundation (61372033) and Natural Science Foundation (61501501).

References

[1]Yuanyuan Wang, Zheng Liu and Yunhe Cao: Systems Engineering and Electronics Vol.36(2014), p. 667-671 (In Chinese)
[2] E. J. Candès, B. Recht: Foundations of Computational Mathematics Vol.9(2008), p. 717-772
[3] Dong Yang, Guisheng Liao, Shengqi Zhu and Kai Wang: Journal of Xidian University (Natural Science Edition) Vol.41(2014) p.30-35 (In Chinese)
[4] EM. AI-Ard, RM. Shubair and ME. AI-Mualla: Electronics Letters Vol.40(2004), p.908-910
[5] E. J. Candès, Y. Plan: Proceedings of the IEEE Vol. 98 (2010), p. 925-936
[6] Y. Chen, Y. Chi: IEEE Trans. on Information Theory Vol.60(2014) p.6576- 6601

