Study on the Vibration Characteristics of a Complex Multiple Launch Rocket System

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Abstract. Transfer matrix method for multibody system (MSTMM) is a new and efficient method for multibody system dynamics (MSD) developed in recent 20 years. Based on many advantages of MSTMM in studying MSD, the rapid and accurate computation problem of vibration characteristics of a complex multiple launch rocket system (MLRS) are solved in this paper. The dynamics model, the transfer equations of elements, the overall transfer equation and the eigenfrequency equation are established. The results got by MSTMM were validated by modal tests, and compared with ordinary dynamics method, the computational speed has been increased 65 times.

Introduction

With the development of modern industry, lots of complex mechanical systems have appeared in the fields of weaponry, aeronautics, astronautics, vehicle, robot and precision machinery, which can be considered as multibody system composed of many rigid and flexible bodies connected with hinges. The methods for multibody system dynamics (MSD), such as Wittenburg method [1], Schiehlen method [2], Kane method [3] and the others [4-6], have developed rapidly in the last 40 years and provided a powerful tool to study mechanical system dynamics. Generally speaking, almost all ordinary methods for MSD have the same characteristics as follows: it is necessary and very complicated to develop the global dynamics equations of the system; the order of system matrix depends on the number of degrees of freedom of the system hence is rather high for complex multibody system.

For a complex multibody system like a complex multiple launch rocket system (MLRS), the problem is, if using ordinary dynamics methods to compute the vibration characteristics, it would face not only the difficulties of high order of system matrix and slow computational speed that can not meet engineering requirements, but also the computation failure due to unavoidable computational ill-condition of eigenvalue problem. And because of the coupling of rigid and flexible bodies, the eigenvalue problem of multibody system is not self-adjoint.

As a brand new method for MSD gradually developed in recent 20 years, transfer matrix method for multibody system (MSTMM) [7-10] has greatly simplified the solving process of complex MSD, highly improved the computational efficiency, and provided a powerful tool for MSD study for its features as follows: without global dynamics equations of the system, high programming, low order of system matrix and high computational speed. MSTMM avoids computational ill-condition of eigenvalue problem of complex linear multiple system for its low order of system matrix. By presenting automatic deduction theorem of overall transfer equation of multibody system [10], the overall transfer equation of multibody system can be deduced automatically. MSTMM provides the theoretical basis for studying and solving the dynamics problem of complex mechanical system thus has been widely used in engineering.

Dynamics Model of the MLRS

The dynamics model of MLRS is shown in Fig.1, the body elements and hinge elements are numbered uniformly. The elements 1, 3, 4, 6, 7, 9, 10, 12, 13, 15, 16, 18, 20, 22, $24+5l$, $25+5l$ are considered as elastic hinges, 2, 5, 8, 11, 14, 17, 19, 21, 23, $26+5l$ are considered as rigid bodies, and $27+5l$, $28+5l$ are considered as elastic beams transversely vibrating in space. $l(l=1,2,\cdots,18)$ is the sequence number of the launch tube loading the newly launching rocket. There're 8 boundary ends in the system, which are numbered as 0. The dynamics model of MLRS is a multi-rigid-flexible system, which is composed of 28 rigid bodies and 18 flexible bodies connected with various linear springs, rotary springs and dampers respectively.

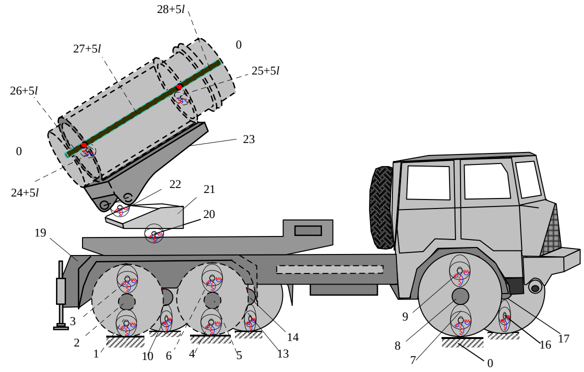

Figure 1. Dynamics model of the MLRS

The Eigenfrequency Equation of the MLRS

State Vectors. According to MSTMM, the state vector of each connection point or boundary end is defined as

$$\boldsymbol{Z}_{i,j}=[X \quad Y \quad Z \quad \Theta_x \quad \Theta_y \quad \Theta_z \quad M_x \quad M_y \quad M_z \quad Q_x \quad Q_y \quad Q_z]^{\mathrm{T}} \tag{1}$$

where, $[X,Y,Z]^{\mathrm{T}}$, $[\Theta_x,\Theta_y,\Theta_z]^{\mathrm{T}}$, $[M_x,M_y,M_z]^{\mathrm{T}}$, $[Q_x,Q_y,Q_z]^{\mathrm{T}}$ are the modal coordinates of linear displacements, angular displacements, internal torques and internal forces of a connection point relative to the equilibrium position along x,y,z directions in the inertia coordinate system $oxyz$ respectively.

The Transfer Equations of Elements. According to MSTMM, the transfer equations of any element j (body or hinge) with single input end and single output end vibrating in space can be expressed as

$$\boldsymbol{Z}_{j,O}=\boldsymbol{U}_j\boldsymbol{Z}_{j,I} \tag{2}$$

where, $\boldsymbol{Z}_{j,I}$ and $\boldsymbol{Z}_{j,O}$ are respectively the state vectors of the input end and the output end, $\boldsymbol{U}_j$ is the transfer matrix of element j .

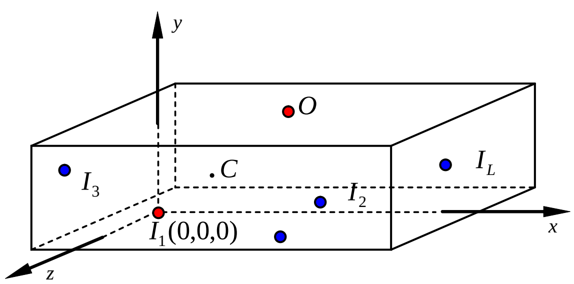

Figure 2. A rigid body with multiple input ends and single output end vibrating in space

For a body element with more than two connection ends, only one of the ends is considered as output end, and all the other ends are considered as input ends. The transfer equations should cover the geometrical relationship between its first input end I_1 and output end O, and the mechanical principle for the forces and moments acting on the element. Moreover, geometrical equations, which describe the geometrical relationship between the first input end I_1 and $k^{\text{th}}(k=2,3,\cdots L)$ input end I_k of the body, should be introduced. As shown in Fig.2, for a rigid body element j with $L(L \geq 2)$ input ends and single output end, the transfer equations and geometrical equations of the body can be written in the following forms

$$\boldsymbol{Z}_{j,O} = \boldsymbol{U}_j \boldsymbol{Z}_{j,I_1} + \boldsymbol{U}_{j,I_2} \boldsymbol{Z}_{j,I_2} + \cdots + \boldsymbol{U}_{j,I_L} \boldsymbol{Z}_{j,I_L} \tag{3}$$

$$\boldsymbol{H}_j \boldsymbol{Z}_{j,I_1} = \boldsymbol{H}_{j,I_k} \boldsymbol{Z}_{j,I_k} \quad (k=2,3,\cdots,L) \tag{4}$$

where

$$\boldsymbol{U}_{j,I_k} = \begin{bmatrix} \boldsymbol{O}_{3\times3} & \boldsymbol{O}_{3\times3} & \boldsymbol{O}_{3\times3} & \boldsymbol{O}_{3\times3} \\ \boldsymbol{O}_{3\times3} & \boldsymbol{O}_{3\times3} & \boldsymbol{O}_{3\times3} & \boldsymbol{O}_{3\times3} \\ \boldsymbol{O}_{3\times3} & \boldsymbol{O}_{3\times3} & \boldsymbol{I}_{3\times3} & \tilde{\boldsymbol{l}}_{I_kO} \\ \boldsymbol{O}_{3\times3} & \boldsymbol{O}_{3\times3} & \boldsymbol{O}_{3\times3} & \boldsymbol{I}_{3\times3} \end{bmatrix}$$

$$\boldsymbol{H}_j = \begin{bmatrix} \boldsymbol{I}_{3\times3} & \boldsymbol{O}_{3\times3} & \boldsymbol{O}_{3\times3} & \boldsymbol{O}_{3\times3} \\ \boldsymbol{O}_{3\times3} & \boldsymbol{I}_{3\times3} & \boldsymbol{O}_{3\times3} & \boldsymbol{O}_{3\times3} \end{bmatrix}, \quad \boldsymbol{H}_{j,I_k} = \begin{bmatrix} \boldsymbol{I}_{3\times3} & \tilde{\boldsymbol{l}}_{I_1I_k} & \boldsymbol{O}_{3\times3} & \boldsymbol{O}_{3\times3} \\ \boldsymbol{O}_{3\times3} & \boldsymbol{I}_{3\times3} & \boldsymbol{O}_{3\times3} & \boldsymbol{O}_{3\times3} \end{bmatrix}$$

and $\boldsymbol{Z}_{j,O}$ and $\boldsymbol{Z}_{j,I_k}(k=1,2,3,\cdots,L)$ are the state vectors of the output end and the kth input end of the body element respectively. $\boldsymbol{U}_j$ is the right transfer matrix of element j when I_1 and O are considered as the only input end and output end of the element; $\boldsymbol{U}_{j,I_k}$ is the corresponding transfer matrix which only extracts the force variables (including internal forcce and internal moment) from the state vector when multiplying $\boldsymbol{Z}_{j,I_k}$, considering the force terms in the dynamics equations of the element. And $\boldsymbol{H}_j$ is a constant matrix extracting displacement variables (including position coordinates and orientation angles) from a state vector when multiplying $\boldsymbol{Z}_{j,I_1}$, $\boldsymbol{H}_{j,I_k}$ is related to the relative position of the first input end I_1 and k^{th} input end I_k of element. The number of the geometrical equations is ($L-1$). For more details about the matrices mentioned above, please check reference [7].

The Overall Transfer Matrix. According to automatic deduction theorem of overall transfer equation of multibody system [10], the overall transfer matrix of the MLRS can be deduced automatically as

$$\boldsymbol{U}_{\text{all}} \boldsymbol{Z}_{\text{all}} = \boldsymbol{0} \tag{5}$$

where

$$\boldsymbol{Z}_{\text{all}} = [\boldsymbol{Z}_{28+5l,0}^{\text{T}}, \boldsymbol{Z}_{1,0}^{\text{T}}, \boldsymbol{Z}_{4,0}^{\text{T}}, \boldsymbol{Z}_{7,0}^{\text{T}}, \boldsymbol{Z}_{10,0}^{\text{T}}, \boldsymbol{Z}_{13,0}^{\text{T}}, \boldsymbol{Z}_{16,0}^{\text{T}}, \boldsymbol{Z}_{26+5l,0}^{\text{T}}, \boldsymbol{Z}_{23,25+5l}^{\text{T}}]^{\text{T}}$$

$$U_{\text{all}}=\begin{bmatrix} -I_{12} & T_{1-28+5l} & T_{4-28+5l} & T_{7-28+5l} & T_{10-28+5l} & T_{13-28+5l} & T_{16-28+5l} & T_{26+5l-28+5l} & T_{23-28+5l}C+T_{25+5l-28+5l} \\ O & G_{1-19} & G_{4-19} & O & O & O & O & O & O \\ O & G_{1-19} & O & G_{7-19} & O & O & O & O & O \\ O & G_{1-19} & O & O & G_{10-19} & O & O & O & O \\ O & G_{1-19} & O & O & O & G_{13-19} & O & O & O \\ O & G_{1-19} & O & O & O & O & G_{16-19} & O & O \\ O & G_{1-23} & G_{4-23} & G_{7-23} & G_{10-23} & G_{13-23} & G_{16-23} & O & G_{23-23}C \\ O & G_{1-27+5l} & G_{4-27+5l} & G_{7-27+5l} & G_{10-27+5l} & G_{13-27+5l} & G_{16-27+5l} & G_{26+5l-27+5l} & G_{23-27+5l}C \\ O & G_{1-28+5l} & G_{4-28+5l} & G_{7-28+5l} & G_{10-28+5l} & G_{13-28+5l} & G_{16-28+5l} & G_{26+5l-28+5l} & G_{23-28+5l}C+G_{25+5l-28+5l} \end{bmatrix}$$

U_{all} is the overall transfer matrix, $Z_{i,0}$ denote the state vectors of boundary ends, $Z_{23,25+5l}$ denotes the state vector of the connection cpoint between body element 23 and hinge element $25+5l$ in the closed loop subsystem. $T_{i-28+5i}$ denotes successive multiplication of the transfer matrices of all elements in the transfer path from the tip to the root (the state vector is $Z_{28+5l,0}$). G_{i-j} denotes successive multiplication of the transfer matrix of all elements in the transfer path from body element j to element i, and premultiplicate $(-H_j)$ (if $k=1$) or premultiplicate H_{j,I_k} (if $k=2,3,\cdots,L$). O is zero matrix, I_{12} is a 12×12 identity matrix, C is a constant matrix describing the relationship between the state vectors in the closed loop subsystem.

Applying boundary conditions above, getting rid of all zero elements from Z_{all}, then $\bar{Z}_{\text{all}}$ is obtained, getting rid of columns in the overall transfer matrix U_{all} that correspond to zero elements in Z_{all}, then a 60×60 square matrix $\bar{U}_{\text{all}}$ is obtained. We get

$$\bar{U}_{\text{all}}\bar{Z}_{\text{all}}=\mathbf{0} \tag{6}$$

Then the eigenfrequency equation of the MLRS is

$$\det\bar{U}_{\text{all}}=\mathbf{0} \tag{7}$$

Solving the equation above, the eigenfrequencies $\omega_k (k=1,2,3\cdots n)$ can be obtained.

Vibration Characteristics of the MLRS

Modal Tests. Modal tests are carried out using the modal measuring devices and modal analysis software. The flow chart of test procedure and the distribution of test positions of MLRS are shown in Fig.3 and Fig.4 respectively. The first and the third order modal shapes of the MLRS got by test are shown in Fig.5 and Fig.6 respectively. The eigenfrequencies of the MLRS got by MSTMM and by modal test are shown in Table 1. It can be seen clearly, the results get by MSTMM and by modal test have good agreements, which validating the dynamics model, numerical simulation and the proposed method used in this paper.

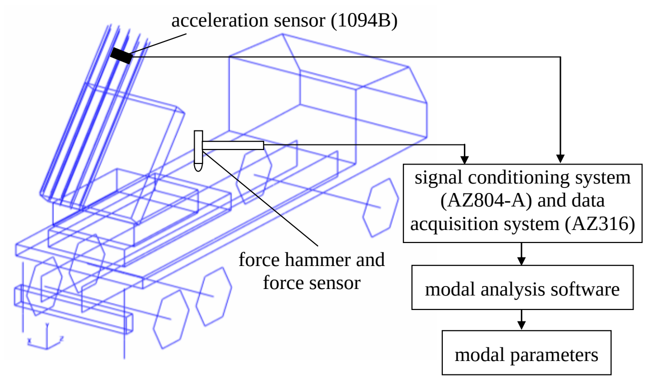

Figure 3. The flow chart of modal test

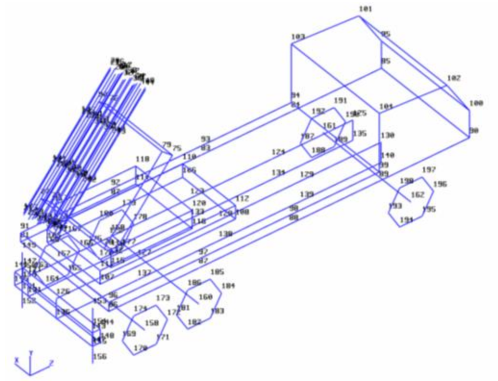

Figure 4. Distribution of test positions

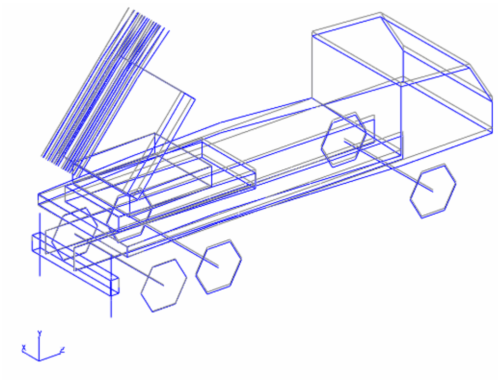

Figure 5. The first order modal shape

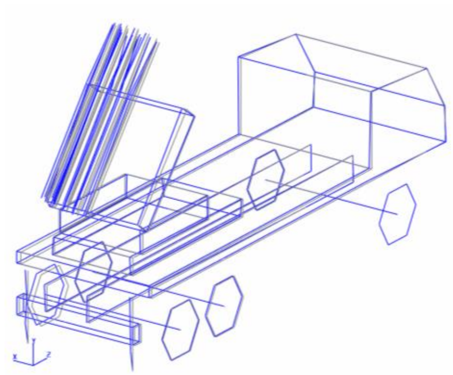

Figure 6. The third order modal shape

Table 1 The eigenfrequencies of the MLRS

Modal	MSTMM [Hz]	Test [Hz]	Relative error [%]
1	2.76	2.77	-0.36
2	2.99	2.99	0.00
3	3.96	3.94	0.51
4	4.32	4.25	1.60
5	——	5.01	——
6	5.33	5.23	1.90

Computational Time. For the same dynamics model of the MLRS in this paper, MSTMM and ordinary dynamics methods (using ANSYS software) are used to compute the vibration characteristics. Under the same solution accuracy, the computational time of vibration characteristics are shown in table 2. It is clear that, comparing with ordinary dynamics methods, the computational speed has been increased more than 1000 times by using MSTMM.

Table 2 Computational time of vibration characteristics

Dynamics methods	Computational time [s]	Computational time ratio
ANSYS	11.06	65:1
MSTMM	0.17	

Conclusions

Benefiting from its features as follows: without global dynamics equations of the system, high programming, low order of system matrix and high computational speed, MSTMM provides the theoretical basis for studying and solving the dynamics design problem of a complex MLRS, its high efficiency and validity have been verified. Under the same solution accuracy and compared with ordinary dynamics methods, the computational speed has been increased more than 65 times by using MSTMM.

Acknowledgements

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References

[1] J. Wittenburg, Dynamics of Systems of Rigid Bodies, B.G. Teubner, Stuttgart, 1977.

[2] W. Schiehlen, Multibody Systems Handbook, Springer, Berlin, 1990.

[3] T.R. Kane, P.W. Likins and D.A. Levinson, Spacecraft Dynamics, McGraw-Hill Book Company, New York, 1983.

[4] A. Shabana, Dynamics of Multibody Systems, Cambridge University Press, New York, 2009.

[5] O.A. Bauchau, Flexible Multibody Dynamics, Springer, Dodrecht, The Netherlands, 2011.

[6] K. Thoma, F. Schäfer, S. Hiermaier, et al., An approach to achieve progress in spacecraft shielding, Advances in Space Research 34 (5) (2004) 1063-1075.

[7] X.T. Rui, L.F. Yun, Y.Q. Lu, et al., Transfer Matrix Method of Multibody System and Its Applications, Science Press, Beijing, 2008.

[8] X.T. Rui, L.F. Yun, Y.Q. Lu. Augmented eigenvector of multibody system coupled with rigid and elastic bodies and its orthogonality, Acta ArmamentarⅡ 28(5) (2007) 581-586.

[9] X.T. Rui, G.P. Wang, Y.Q. Lu, et al., Transfer matrix method for linear multibody system, Multibody System Dynamics 19(3) (2008) 179-207.

[10] X.T. Rui, J.S. Zhang, Q.B. Zhou. Automatic deduction theorem of overall transfer equation of multibody system, Advance in Mechanical Engineering 2014 (2014) 378047 (12pp.).