Fractal Analysis on the Stock Price of C to C Electronic Commerce Enterprise

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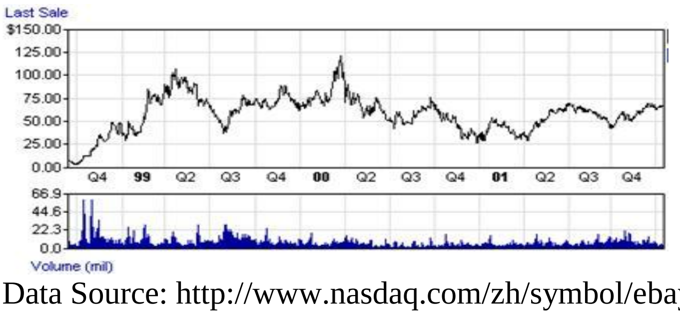
Abstract. To avoid the status that the stock value of C to C electronic commerce enterprises change radically, we have induced the fractal method to make an empirical analysis on the typical case of listed C to C e-commerce enterprises, and studied the structural factors which behind them and influence the market pricing of them. Based on the fractal analysis of EBAY, our study has found that the stock price volatility of C to C e-commerce enterprise appears as a weak trend with cyclical fluctuations. And furthermore, we have provided a new method for this separation.

Introduction

In the most influential listed C to C electronic commerce enterprises worldwide, EBAY is a typical one. We choose EBAY as the typical case. Using its historical stock price data, carry through demonstration analysis by the fractal method, to inquire about the structural factors which influence its stock price movements, and get a deepen understanding of the pricing of C to C e-commerce enterprise.

EBAY Inc. was founded in 1995. It is one of the world famous Internet intermediary market and the world's leading online auction site. Its trading varieties are involving C to C antiques and trading, books, audio and video, coins, stamps, collectibles, toys, jewelry, computer, electronic products, ceramics, toys and other items. EBAY is listed on NASDAQ National Market; its stock code is EBAY.

EBAY was listed on NASDAQ in September 24, 1998. Its daily stock closing price volatility from September 24, 1998 is shown in Fig. 1.



Data Source: http://www.nasdaq.com/zh/symbol/ebay

Figure 1. Volatility of EBAY Daily Stock Closing Price

The Biased Random Walk and the Fractal Analysis

Many empirical studies find that the distribution of stock return deviates significantly from the Gauss distribution, present a kind of "fat tail" characteristic [1]. The probability in the region of high yield and high loss is greater than the probability of Gauss distribution [2]. It reflects the herd behavior in financial markets, and the herd behavior leads to the "fat tail" characteristic of stock return distribution. The unexpected events of financial market not only has multi factor, nonlinear, but also has uncertainty [3]. Some studies show, in the financial markets, people are not equal to digest and confirm information. It often leads to the biased random walk of stock price, or called

Fractional Brown Motion (FBM) [4]. So, we have no reason to believe that markets must be balanced and effective [5]. Financial market may perfectly possible come to the fractal structure.

The word "Fractal" is proposed for the first time in 1975 by Benoti B. Mandelbrot [6]. Its original meaning is "no rules, score, reduced to fragments". The mathematical definition of fractal is: If a set in Euclidean space whose Hausdorff dimension (DU) [7] [8] is always greater than its topological dimension (DT), i.e. DU＞DT, we call the set as a "fractal set", and "fractal" for short. Common ground says, thing's components similar to the whole body in a certain way called fractal.

Contrary to the efficient market hypothesis, the financial market is not linear, balanced or orderly, but non equilibrium, nonlinear, and disordered. It is chaotic, and obey fractal Brown motion [9]. Rely on the fractal geometry language; we can get a more in-depth understanding of financial market.

The fractal theory of stock market suggests, the stock market is not, at least not always a general equilibrium efficient market. The changes of stock price are with the characteristics of long-term memory of past history. It is not a completely random walk, but a "biased random walk"[10] .This process is highly dependent on the initial conditions. Its direction of change is uncertain. The final price is not a unique price which stay in general equilibrium state, but a stochastic equilibrium price which comes from the polymorphic equilibrium market determined by multi-attractors.

Fractal Statistics Analysis of EBAY Stock Price

We now analyze the stock price volatility of EBAY by the R/S method (i.e. The Rescaled Range Analysis Method) founded by H.E. Hurst.

Assume a time series {xi}, the number of observations is M. The basic idea of R/S analysis method is: divide the time series {xi} whose length is M into An adjacent subintervals whose length are N (2≤N≤L). L is the length of maximum subinterval, AN=M. Each subinterval is named Ia, (a=1, 2,…, A). Each xi of Each Ia is named xi, a, (k=1, 2,…,N). Assume the means of {xi} of Ia is ea, We have:

$$x_{k,a} = \sum_{i=1}^{k} (x_{i,a} - e_a), \quad k = 1,2,...,N$$
$$R_{I_a} = \max\{x_{k,a}\}_{1\le k\le N} - \min\{x_{k,a}\}_{1\le k\le N}$$
$$S_{I_a} = \sqrt{\frac{\sum_{k=1}^{N}(x_{k,a} - e_a)^2}{N}} \tag{1}$$

Where N is the length of subinterval, xk,a is the accumulation deviation of subinterval, SIa is the standard deviation of the subinterval Ia, RIa is the range of the subinterval Ia, RIa become big with increasing N.

In order to compare different time series, Hurst used the standard deviation of the observed value divided by its range, that is: R divided by S. He has the following equation:

$$(R/S)_N = \frac{1}{A}\sum_{a=1}^{A}(R_{I_a}/S_{I_a}) = (aN)^H \tag{2}$$

In equation (2), A is the constant; H is the Hurst Index, and 0≤H≤1, 2≤N≤L.

There are 3 different types of Hurst Index: (1)H＝0.5, (2) 0≤H≤0.5, (3) 0.5≤H≤1. According to the statistic knowledge, when H=0.5, the original series is a standard random walk. The present will not affect the future, namely, the original series is lack of statistical correlation in long-term. When 0≤H≤0.5, the original series is with anti-persistence. It is often called the mean-reverting, that is, the past increment is negative correlation with the increment of the future. If a series is go up in the earlier period, then, it will probably go down in the next period. On the other hand, if it is go down, then in the next period, it will probably go up. When 0.5≤H≤1, the original series is with persistence, that is, its past increment is positively related to the incremental of the future. If a series is go up (down) in the earlier period, then, it will continue to go on (down) in the next period. The larger value of H, the less of noise in this series, the series has stronger persistence and more obvious trend. The persistence level can be measured through H minus 0.5. There are many of persistent series in nature; the stock market is one of them.

Take the logarithm on both ends of the equation (3):

$$\ln(R/S)_N = H\ln(N) + \ln a \qquad \mathrm{N} = 2,3,...,\mathrm{L} \tag{3}$$

The calculation of Hurst index generally has two methods, both based on equation (3). The first method is taking the length of each subinterval in turn based on the length of the sample, for example, N=50,100,…,M. Performing the regression by the OLS method according to equation (3), we can get the values of a plurality of H. These estimates are associated with L: the length of maximum subinterval, denoted as HL. Using {HL} series, the maximum value corresponding to L is the average cycle length. If HLM=MaxHL, then the average cycle length is LM, the Hurst index is HLM.

The second method is taking L = M, then compute HM by the first method. The value of HM is the Hurst index. But using this method we are unable to obtain the average cycle length.

For a series, in order to describe the present effects on the future, B.B.Mendelbrot[11] has introduced a correlation measurement index:

$$C_M = 2^{(2H-1)} \tag{4}$$

In equation (4), CM expresses the correlation during interval M. Thus, independent series (H =0.5) has no correlation; Series with persistence (H>0.5) has positive correlation, and series with anti-persistence (H <0.5) has negative correlation.

In this study, we use the first method to calculate the Hurst index. In the estimation and regression process, we use the logarithmic rate of stock return of EBAY. That is, make:

$$S_t = \ln(p_t / p_{t-1}) \tag{5}$$

Where pt is the stock price at time T; St is the logarithmic stock return rate at time T, For R/S analysis, Logarithmic rate of return is more suitable than the percentage change in price. In R/S method, the range is the cumulative deviation to the average value. The sum of logarithmic return rate is equal to the cumulative return rate, while the percentage change is not. In order to remove the linear correlation of St Series, before the R/S analysis, we first conduct a first-order regression to St, to obtain the residual series:

$$x_t = S_t - (a + bt) \tag{6}$$

Where a and b are linear regression coefficients of time series St, {xt} is the residual series, as show in Fig. 2.

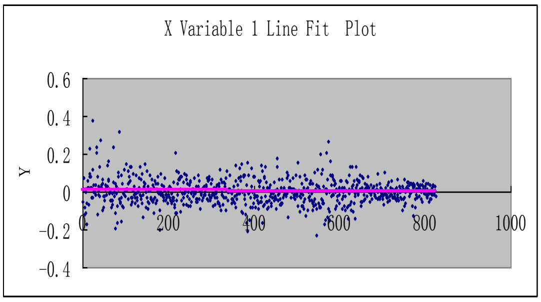


Figure 2. First-order Regression Residual Series of St

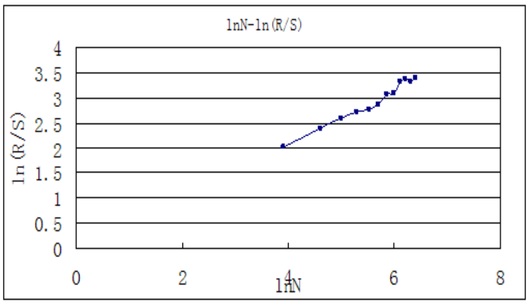


Figure 3. lnN ---ln (R/S) Line Graph

Carries on R/S analysis to {xt} series, respectively taking t=50,100,150,... ,600 trading days, Calculating according to the above formulas, We got the following N, lnN, R/S, ln (R/S) statistic table:

Table 1 N, lnN, R/S, ln (R/S) Statistic Table

N	**lnN**	**R/S**	**ln(R/S)**
50	3.912023	7.41277	2.003204
100	4.60517	10.78582	2.378232
150	5.010635	13.362	2.592415
200	5.298317	14.98825	2.707267
250	5.521461	15.91809	2.767456
300	5.703782	17.5242	2.863583
350	5.857933	21.4435	3.065421
400	5.991465	21.86063	3.084687
450	6.109248	27.47322	3.313212
500	6.214608	28.96521	3.366096
550	6.309918	27.6055	3.318015
600	6.39693	29.4969	3.384285

The lnN ---ln (R/S) line graph, as shown in Fig. 3. We can see from Figure. 3, the line shows an almost linear growth when lnN≤6.2, while it shows an obvious turning point when lnN>6.2, lnN= 6.2 corresponds to N=500. Therefore, we can draw a conclusion that the effective "long-term memory" cycle of EBAY's stock price volatility is about 500 trading days.

Then, we perform OLS regression use the data 50≤N≤500 to estimate the value of H, the result is:

$$\ln(R/S) = -0.25633 + 0.566253 \ln \mathrm{N}$$
$$(-1.2752) \quad (15.40016)$$
$$[0.201013] \; [0.036769] \tag{7}$$

The regression parameters and variance analy-sis are shown in Table 2:

Table 2 The Regression Parameters and Variance Analysis of R/S Analysis

Regression Statistics		Variance Analysis					
Mul	0.983549		df	SS	MS	F	P-Value
R^2	0.967369	Regression Aanalysis	1	1.550602	1.550602	237.1651	3.14E-07
AdjR	0.96329	Residual	8	0.052305	0.006538		
S	0.080858	Total	9	1.602906			

Its linear fitting chart is shown in Fig. 4:

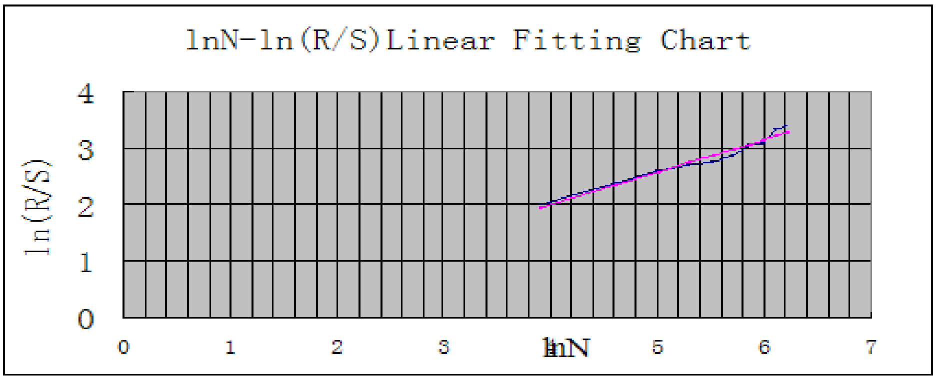


Figure 4. lnN---ln(R/S) Linear Fitting Chart

Regress results shows, H＝0.5663. The higher R2 (0.97), higher F value (237.2), closing to 2 of DW value(1.95) and lower standard deviation (0.08) show a good fitting degree of this regression. Through the value of H, we can calculate the value of correlation metric index CM is 1.096, it indicates that the volatility of EBAY stock price has fractal structure and persistence to a certain degree. It is not a random walk process, but a biased random walk process. However, the H value is not deviate greatly from 0.5, also show that there exist large noise components. Different from the ordinary random walk process, the stock return series is a biased random walk process, because it has a "long-term memory" in play. The long-term memory is not infinite, but is limited. According to the calculation above, the average long-term memory cycle of stock price volatility of EBAY is about 500 trading days. R/S analysis shows that, in the long-term memory cycle, the volatility of stock return rate of EBAY is a persistent series with fractal probability distribution. It follows a biased random walk process. The market shows some tendency strengthens behavior, rather than the mean reverting behavior, but there are still large noise components in it. When exceeding the long-term memory cycle, the volatility of stock return rate is no longer reflected the tendency strengthens behavior.

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