Collective Action and Equilibrium Solution on Social Network

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Abstract. Based on the collective action model of social networks, this paper analyzes all the Nash equilibrium of them on two typical social networks, i.e. complete graph, star graph, and compares the relationships between the cooperation level of collective action and social control and network size under different influences between nodes and different networks as well.

Introduction

The game among individuals in collective action is embedded in the individual's social networks, so the network structure has a very important influence on collective action (that is, the level of collective cooperation). Research on collective action in the network dates from PRAHL R [1], the density of application network and centrality of network degree, and other social network parameters of collective action; Yann Bramoullé [2] presents a game model of public goods on social networks, discussing the equilibrium relationship between individual effort and collective welfare in the complete graph, star graph and circle graph; Michael Suk-Young Chwe [3] considered the impact of the strength or the weakness of the network connection on collective action; on this basis, David A. Siege[4] studied the characteristics of individuals with heterogeneous motivation involved in collective action, and network simulation results show that the scale of the network, the large number of weak connections, and the possibility of the individual to play elite, will have an important impact on collective action; finally, Károly Takács [5, 6] brings the synergistic evolution of social network and collective action into the research on collective action[7, 8].

In this context, to compare the direct and indirect effects between nodes on the impact of collective action cooperation level on social networks, this paper introduces a collective action model and compares and analyzes the nodes' characteristics of equilibrium and the level of cooperation of the collective action under different types of impacts.

Collective Action Model on Social Network

Consider that the collective of n, denoted by $N = \{1,2,...,n\}$, in which $n > 2$; $g_{ij}(i, j \in N, i \neq j)$ represents the social relationships among collective members, $g_{ij} = 1$ represents some sides directly connected between i and j, which means social interaction exists among individuals, $g_{ij} = 0$ represents no side directly connected between i and j which means no social interaction exists among individuals; $N_i(g) = \{ j | g_{ij} = 1, j \in N, j \neq i\}$ is neighbor aggregate of node i, $k_i(g) = |N_i(g)|$ is neighbor number of the node i, $S = \{0,1\}$ is strategy aggregate of the individual i, $s_i \in S, s_i = 1$, is the individual who joins the collective action and cooperates in the action, $s_i = 0$ is the individual who does not join the collective action and takes non-cooperation in the action. The benefit of the individual i under the strategy of $\Omega = (s_i, s_{-i})$ marks $\pi_i(s_i, s_{-i})$, in which s_{-i} is the strategy of the other individuals, $r = (\sum_{j=1}^{n} s_j)/n$ is the ratio of individual that taking the strategy of $s_i = 1$.

Each individual provides certain public goods, all members of the collective can use them for free and for the provision of public goods shall pay a certain cost. Under these circumstances, the cost provided by the individual shall be more than the benefit that a single member in the collective who provided public goods, but if all members in the collective provide a certain amount of public goods, its total benefit shall be more than the individual's cost [9].

Based on the above assumptions, the profits function of individuals participating in the collective action consists of the following components; $k_i s$, selective incentives from all the neighbors; $k_{iC} b$, affirmative action incentives from neighbors adopting a cooperative strategy C, and benefit of collective action, and c, cost of participating in collective action. The profits function of individuals not participating in the collective action only include $k_{iD} b$, affirmative action incentives from a neighbor adopting non-cooperation strategy D, and profits of collective action. Therefore, the individual profit function is defined as follows:

$$\pi_i(s_i = 1) = k_i s + k_{iC} b + \sum_{j \in N, j \neq i} s_j \alpha + \alpha - c$$

$$\pi_i(s_i = 0) = k_{iD} b + \sum_{j \in N, j \neq i} s_j \alpha$$

Game rules for individual participating in collective action: ① individuals only play game with their directly connected neighbors; ②Individuals make decisions at same time, and they do not make clear the strategy information of the neighboring nodes in the choice of strategy.

For ease of discussion, this paper only takes into account the situation that the individuals using strategy. As Jackson-Wolinsky [10] research points out, the star graph and complete graph are the main network structures with effectiveness and stability, so many social networks appear as star graph and complete graph form. The loop graph is also a very specialized social networks structure, so this paper will consider the three special networks of collective action, shown in Fig. 1.

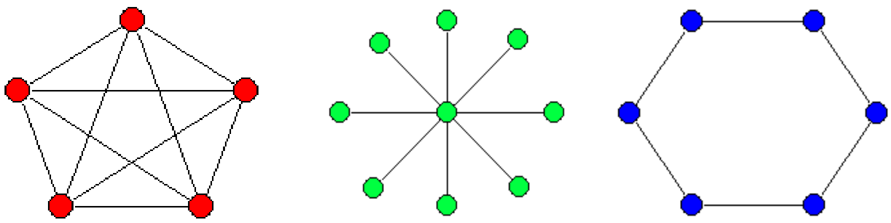

Figure 1. Complete graph, star and loop graph of the network structure

In the collective action model, the conditions of individuals choosing strategy of $s_i = 1: \pi(s_i = 1) - \pi(s_i = 0) > 0$, that is $\Delta = k_i s + (k_{iC} - k_{iD}) b + \alpha - c > 0$. That is to say, only when the compensation that social control brought in is sufficient to offset the costs of public goods provided for the participation in collective action, individuals will adopt a cooperative strategy to participate in collective action [5].

The Equilibrium Solution of Collective Action on the Complete Graph

In the complete graph, any node can be $k_i = n-1$, that is to say, each node has $n-1$ neighbor(s).

Proposition 1: The equilibrium solution of collective action on the complete graph

(1) If $(n-1)(s+b) + \alpha - c < 0$, so $r^* = 0$ is the only Nash equilibrium solution of network.

(2) If $(n-1)(s-b) + \alpha - c < 0$ and $(n-1)(s+b) + \alpha - c > 0$, so $r^* = 0$ and $r^* = 1$ are the Nash

equilibrium solution of network.

(3) If $(n-1)(s-b)+\alpha-c>0$, so $r^*=0$ is the only Nash equilibrium solution of network.

Proof:

If $\Delta=(n-1)(s-b)+\alpha-c>0$, $s_i=1$ is the dominant strategy, any one node in the network will choose strategy $s_i=1$, so $r^*=1$ is the only Nash equilibrium solution. For the same reason, if $\Delta=(n-1)(s-b)+\alpha-c<0$, so $r^*=0$ is the only Nash equilibrium solution of the network.

If $\Delta=(n-1)(s-b)+\alpha-c<0$ and $\Delta=(n-1)(s+b)+\alpha-c>0$, when other individuals select the strategy of $s_i=1$, the individual's profit will be $c-\alpha-(n-1)(s+b)<0$ if the individual changes his strategy, so individuals will not change their strategy, so $r^*=1$ is the Nash Equilibrium solution of the network. For the same reason, $r^*=0$ also is the Nash Equilibrium solution; if there are other r^* solutions, the network can be divided into two collectives, one part takes the strategy of $s_i=1$, marked as C, the other part takes $s_i=0$, marked as D. If r^* is the Nash Equilibrium solution of the network, the return is $\Delta_D=(k_{iD}-k_{iC})b-(n-1)s-\alpha+c=(n-1-2nr^*)b-(n-1)s-\alpha+c$ when changing the strategy.

For any $j\in D$, the profit variation of changing strategy is

$\Delta_C=(n-1)s+(k_{jC}-k_{jD})b+\alpha-c=(n-1)s+(nr^*-(n-1-nr^*))b+\alpha-c$ 。

Due to $\Delta_D*\Delta_C<0$, one side at least will change his strategy, so $r*$ is not the Nash equilibrium solution of the network, therefore, $r\neq 1$ or $r\neq 0$ are not the Nash equilibrium solution of the network, that is to say, $r^*=1$, $r^*=0$ are the Nash equilibrium solution of the network. This concludes the proof.

The following proposition can similarly be proved.

The Equilibrium Solution of Collective Action on the Star Graph

In the star figure, the central node $k_i=n-1$, external nodes $k_i=1$, that is the central nodes have $n-1$ neighbor(s), the external nodes have only one neighbor.

Proposition 2: The equilibrium solution of collective action on the star graph

Under the condition of $\frac{s}{b}>\frac{n}{n-2}$:

(1) If $(n-1)(s-b)+\alpha-c<0$, $r^*=0$ is the only Nash equilibrium solution of the network.

(2) If $s+b+\alpha-c<0$ and $(n-1)(s-b)+\alpha-c>0$, the central node choosing cooperative strategy and the external nodes taking non-cooperation strategy is the only Nash equilibrium solution.

(3) If $s+b+\alpha-c>0$, $r^*=1$ is the only Nash equilibrium solution of the network.

Under the condition of $1<\frac{s}{b}<\frac{n}{n-2}$:

(1) If $s+b+\alpha-c<0$, $r^*=0$ is the only Nash equilibrium solution of the network.

(2) If $s+b+\alpha-c>0$ and $(n-1)(s-b)+\alpha-c<0$, $r^*=0$, $r^*=1$ are the Nash equilibrium solution of the network.

(3) If $(n-1)(s-b)+\alpha-c>0$, $r^*=1$ is the only Nash equilibrium solution of the network.

Under the condition of $\frac{s}{b}<1$:

(1) If $s+b+\alpha-c<0$, $r^*=0$ is the only Nash equilibrium solution of the network.

(2) If $s+b+\alpha-c>0$, $r^*=0$, $r^*=1$ are the Nash equilibrium solution of the network.

Conclusions

This paper, in consideration of direct and indirect effects among the nodes, proposes the equilibrium solution of an extended model on collective action in three main social networks. Theoretical analysis shows that only the conditions of the direct effects among the nodes has been considered, the collective action model in a social network introduced in social control mechanisms emerges as a state of collective cooperation, with selective incentives as the key to the emergence and maintaining of collective cooperation, and social control mechanisms plays different roles in the different network structures.

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