Near Optimal Low-hit-zone Frequency Hopping Sequence Set with Partial Hamming Correlation Property

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***Abstract*—In this paper, we present a class of low-hit-zone (LHZ) frequency hopping sequence set (LHZ FHS set) with periodic partial Hamming correlation property from the Cartesian product of known FHS sets. It is shown that the sequence set is near optimal with respect to the partial Hamming correlation bound of the LHZ FHS set.**

Keywords-frequency hopping sequence set; periodic partial Hamming correlation; low hit zone; periodic Hamming correlation

I. INTRODUCTION

Frequency hopping spread spectrum and direct sequence spread spectrum are two main spread coding technologies in wireless communication systems. Being anti-jamming, secure, and multiple access properties, Frequency hopping multiple access (FHMA) spread spectrum systems have been found many applications in cellular mobile communication, short-range wireless communication, personal communication, and so on [1-2]. In practical application systems, frequency hopping sequences (FHSs) are used to specify which frequency will be used for transmission in each time slot. And in multiple access environments, it is important to keep the mutual interference between transmitters at a level as low as possible. When two or more sources transmit the same frequency slot at the same time, mutual interference occurs. The degree of the mutual interferences closely related to the Hamming crosscorrelation properties of the FHSs ([1-3]). So, it is desirable to employ FHSs with low Hamming correlation to reduce the multiple-access interference of frequencies [4]. Thus, it is an important problem that design FHS set with good Hamming correlation properties.

In quasi-synchronous FHMA systems, different from the conventional FHSs, relative delays between district users are restricted within a zone around the origin, so, FHS set with low Hamming correlation within the fixed zone is more useful, this class of sequence set is called LHZ FHS set. The significance of LHZ FHS set is that, even there are relative delays between the transmitted FHSs, the number of hits will be kept at a very low level between different sequences as long as the relative delays do not exceed certain limit, thus reducing or eliminating the mutual interference. Now, several optimal LHZ FHS sets meeting the Peng-Fan-Lee bounds [5] have been covered in the literature [5-11].

Practically, because the limited synchronization time or hardware complexity, so that one often pay more important attentions to the periodic partial Hamming correlation properties (PPHCP) of FHSs, where the Hamming correlation is computed over only subsequences of FHSs. And correlation window length is shorter than the period of the chosen FHSs. Moreover, the window length may vary from time to time depending on the channel conditions. So, FHSs with PPHCP are important for certain application scenarios than the full period Hamming correlation sequences. Up to now, the constructions of this class of FHSs have not yet been popularly reported [12-14].

Furthermore, a LHZ FHS set with PPHCP is called LHZ-PPHC FHS set. In wireless communication systems, where possess the properties of quasisynchronous FHMA systems and limited synchronization time or hardware complexity, it is necessary to construct LHZ-PPHC FHS set. It is a challenging and interesting problem to study this class of FHS set. Now, very few of LHZ-PPHC FHS sets were covered in the literature. In 2013, Xing Liu et al. [15] constructed a class of LHZ-PPHC FHS set based on interleaving. In 2015, Ch. Y. Wang et al. [16] constructed two classes of LHZ-PPHC FHS set based on the Cartesian product of known FHS sets.

In this paper, we pay particular attentions to the construction of the LHZ-PPHC FHS set. The new construction is based upon the Cartesian product. The rest of this paper is organized in the following manners: In section 2, we review some preliminaries on FHSs, and summarize the key terminologies and notations; In section 3, we construct a new class of LHZ-PPHC FHS set, and verify that the FHS set is near optimal with respect to the Niu-Peng bound [12].

II. PRELIMINARIES

Let $F=\{f_1, f_2,\ldots, f_q\}$ be a frequency slot set with size q, S is a set of M FHSs of length N. For any two frequency slots f_1, $f_2\in F$, let

$$h(f_1, f_2)=\begin{cases}1 & \text{if } f_1=f_2,\\ 0 & \text{otherwise.}\end{cases}$$

For any two FHSs $x=(x_0, x_1,\ldots,x_{N-1})$, $y=(y_0, y_1,\ldots,y_{N-1})\in S$, the periodic Hamming correlation $H(x,y;\tau)$ of x and y at time delay τ is defined as follow

$$H_{x,y}(\tau)=\sum_{k=0}^{N-1} h(x_k, y_{k+\tau}) \quad 0\le\tau<N \tag{1}$$

where all operations among the position indices are performed modulo N. The maximum periodic Hamming autocorrelation $H_a(S)$ and the maximum periodic Hamming crosscorrelation $H_c(S)$ are defined as follows, respectively:

$$H_a(S) = \max_{1\le\tau<N}\left\{H_{x,x}(\tau) \mid x \in S\right\},$$
$$H_c(S) = \max_{0\le\tau<N}\left\{H_{x,y}(\tau) \mid x, y \in S, x \ne y\right\}.$$

The maximum periodic Hamming correlation $H_m(S)$ is defined as follow

$$H_m(S) = \max\left\{H_a(S), H_c(S)\right\}.$$

Let integers $H_{La} \ge 0$, $H_{Lc} \ge 0$.Then the low hit zone L_{HZ}, the autocorrelation low hit zone L_{AHZ} and the crosscorrelation low hit zone L_{CHZ} of S are defined as follows, respectively:

$$L_{HZ} = \min\left\{L_{AHZ}, L_{CHZ}\right\},$$
$$L_{AHZ} = \max_{0<\tau\le T}\left\{T \mid H_{x,x}(\tau) \le H_{La}, \forall x \in S\right\},$$
$$L_{CHZ} = \max_{0\le\tau\le T}\left\{T \mid H_{x,y}(\tau) \le H_{Lc}, \forall x, y \in S, x \ne y\right\}.$$

An FHS set S with $L_{AHZ} > 0$ is called LHZ FHS set.

The periodic partial Hamming correlation between x and y, for the correlation window length L starting at j, is defined by

$$H_{x,y}(j \mid L;\tau) = \sum_{i=j}^{j+L-1} h\left(x_i, y_{i+\tau}\right), 0 \le \tau < N, 1 \le L < N.$$

where all operations among the position indices are performed modulo N. Moreover, $H_{x,y}(j \mid L;\tau)$ is called the periodic partial Hamming autocorrelation when x = y, and the periodic partial Hamming crosscorrelation when $x \ne y$. The maximum periodic partial Hamming autocorrelation $P_{S,a}(L)$, the maximum periodic partial Hamming crosscorrelation $P_{S,c}(L)$ and the maximum periodic partial Hamming correlation PS, m (L) are defined by

$$P_{S,a}(L) = \max_{0<\tau<N, 0\le j<N}\left\{H_{x,x}(j \mid L;\tau) \mid x \in S\right\},$$
$$P_{S,c}(L) = \max_{0\le\tau<N, 0\le j<N}\left\{H_{x,y}(j \mid L;\tau) \mid x \in S, y \in S, x \ne y\right\},$$
$$P_{S,m}(L) = \max\left\{P_{S,a}(L), P_{S,c}(L)\right\}.$$

For any correlation window length L, $1 \le L < N$,let positive integers $l_1 \ge 0$, $l_1 \ge 0$.Then, the LHZ $L_{\text{PAHZ}}(L)$, the LHZ $L_{\text{PCHZ}}(L)$ and the LHZ $L_{\text{PHZ}}(L)$ are defined as follows:

$$L_{\text{PAHZ}}(L) = \max_{\substack{0<\tau\le T\\0\le j<N}}\left\{T \mid H_{x,y}(j \mid L;\tau) \le l_1, \forall x \in S\right\},$$
$$L_{\text{PCHZ}}(L) = \max_{\substack{0\le\tau\le T,\\0\le j<N}}\left\{T \mid H_{x,y}(j \mid L;\tau) \le l_2, \forall x, y \in S, x \ne y\right\},$$

$$L_{\text{PHZ}}(L) = \min\left\{L_{\text{PAHZ}}(L), L_{\text{PCHZ}}(L)\right\}.$$

In 2010, Niu et al. [17] obtained the lower bound on the periodic partial Hamming correlation properties within the LHZ of LHZ FHS set.

Lemma 3. For any FHS set of M FHSs of length N and correlation window length L over a frequency slot set F with size q, and LP the low hit zone of S with respect to the constant P_{Lm}. For any integer Z, $1 \le Z \le L_P$, we have

$$P_{Lm} \ge \left\lceil \frac{(MZ + M - q)L}{(MZ + M - 1)q} \right\rceil. \tag{2}$$

We call this bound as Niu-Peng bound.

Throughout this paper, we use (N, M, q, h) to denote a set of M FHSs of length N over an alphabet of size q, and maximum periodic Hamming correlation h, and use (N, M, q, L_H, h) to denote a LHZ FHS set with periodic partial Hamming correlation properties, whose LHZ is L_H and maximum periodic Hamming correlation is h within LHZ.

Definition 2. Let S be a LHZ FHS set with parameters (N,M,q,L_H,h), S is said to be near optimal if h−1 reach the lower bound in (2).

III. Near Optimal LHZ-PPHC FHS Set

Proposition 1 ([16]). For $l = 1,2$ and $i = 1, 2, \cdots, k$, let $S_l^{(i)} = \left\{S_l^{(i)}(t)\right\}_{t=0}^{N_i-1}$ be an FHS over F_i, and let $R_l = \left\{R_l(t)\right\}_{t=0}^{N-1}$ be an FHS over $F_1 \times \cdots \times F_k$, defined by

$$R_l(t) = \left(S_l^{(1)}(t), \ldots, S_l^{(k)}(t)\right)$$

where $N = \operatorname{lcm}(N_1, \cdots, N_k)$, then

$$H_{R_1,R_2}(\tau) \le \prod_{i=1}^{k} H_{S_1^{(i)},S_2^{(i)}}(\tau).$$

For any $0 \le \tau \le N-1$. In particular, for any $1 \le i_1 \ne i_2 \le k$, $\gcd\left(N_{i_1}, N_{i_2}\right) = 1$, we have

$$H_{R_1,R_2}(\tau) = \prod_{i=1}^{k} H_{S_1^{(i)},S_2^{(i)}}(\tau).$$

Theorem 1 ([16]). For $1 \le i \le k$, let $X_i = \{x_{i,l} \mid 0 \le l \le L_i - 1\}$ be an FHS set $(N_i, L_i, M_i; \lambda_i)$ over $\mathrm{F_i}$ satisfying

1). For $0 \le l \ne m \le L_i - 1$, $H_{x_{i,l},x_{i,m}}(0) = 0$.

2). $N_1 \ne \gcd\left(lcm\left(N_2, \cdots, N_k\right), N_1\right)$.

Let $N = \mathrm{lcm}\left(N_1, \ldots, N_k\right)$. And an FHS set over $F_1 \times \cdots \times F_k$ is defined by

$$\begin{cases} X = \left\{x_{i_1,\ldots,i_k}(t) \mid 0 \le i_1 \le L_1 - 1, \ldots, 0 \le i_k \le L_k - 1, 0 \le t \le N-1\right\}, \\ x_{i_1,\ldots,i_k}(t) = \left(x_{1,i_1}(t), \ldots, x_{k,i_k}(t)\right). \end{cases}$$

Let $L = L_1 \cdots L_k$, $M = M_1 \cdots M_k$, and $\lambda = \lambda_1 \cdots \lambda_k$. The set X is a LHZ FHS set with parameters $(N, L, M, N_1 - 1, \lambda)$.

Construction A: Step1: Let q be a prime power, $0 \le i < q$, and $X = \{x_i(t)\}_{t=0}^{q-2}$ be an FHS set over F_q with parameters $(q-1, q, q, 1)$. And satisfies

$$H_{x_{l_0},x_{l_1}}(0) = 0 \quad \text{for } 0 \le l_0 \ne l_1 < q.$$

Step2: Let p be a prime and $q < p-1$, $\gcd(q-1, p) = 1$, $0 \le j < p-1$, $Y = \{y_j(t)\}_{t=0}^{p-1}$ be an FHS set over F_p with parameters $(p, p-1, p, 1)$.

Step3: Defined the FHS set W over $F_q \times F_p$ as follow

$$\begin{cases} W = \left\{w_{i_0,i_1}(t) \mid 0 \le t < (q-1)p\right\}, \\ w_{i_0,i_1}(t) = \left(x_{i_0}(t), y_{i_1}(t)\right), \\ 0 \le i_0 < q, 0 \le i_1 < p-1. \end{cases}$$

we select q sequences from W to form an FHS set S, where the subscript i_1 isn't constant and the subscript i_0 take difference values between 0 and q-1,we have

Theorem 1. With respect to the Niu-Peng bound (2), the FHS set S is a near optimal LHZ FHS set with parameters $\left((q-1)p, q, pq, p-1, 1\right)$.

Proof: If i_1 isn't constant, by definition of the maximum periodic Hamming correlation, we have

$$H_{(i_0,i_1),(i'_0,i'_1)}(\tau) = \sum_{t_1=0}^{q-2} h\left(X_{i_0}(t_1), X_{i'_0}(t_1+\tau)\right) \cdot \sum_{t_2=0}^{p-1} h\left(Y_{i_1}(t_2), Y_{i'_1}(t_2+\tau)\right).$$

We obtain

$$H_a(S) = 0 \quad \text{if } i_1 = i'_1.$$

And

$$H_c(S) = \begin{cases} 0 & \text{if } \tau \equiv 0 \bmod (q-1), \\ p & \text{if } \tau \equiv 0 \bmod p, \tau \ne 0 \bmod (q-1) \text{ and } i_1 = i'_1, \\ 0 & \text{if } \tau \ne 0 \bmod p, i_1 = i'_1, \\ 1 & \text{otherwise.} \end{cases}$$

So, we have

$$H_m(S) = 1 \quad \text{if } \tau < p.$$

Furthermore, when $0 \le \tau < p$, we can obtain

$$P_m(L) = \left\lceil \frac{L}{(q-1)p} \right\rceil \quad \text{if } 1 \le L \le (q-1)p.$$

According to the Niu-Peng bound (2), we obtain

$$P_m(L) \ge \left\lceil \frac{(MZ+M-q)L}{(MZ+M-1)q} \right\rceil = \left\lceil \left(\frac{pq-pq}{pq-1}\right) \cdot \frac{L}{pq} \right\rceil = 0.$$

It is obvious that S is near optimal for any correlation window.

Lemma 7 (Solomon, [18]): Let p be a prime, $0 \ne \beta_r \in GF(p)$, $a_r = \left\{a_k^{(r)} = k\beta_r \mid k = 0, 1, \cdots, p-1\right\}$ be an FHS over $GF(p)$, $0 \le r < p$,the FHS set given by

$$A = \{a_r \mid 1 \le r \le p-1\}.$$

Then

$$H_{A_i,A_j}(\tau) = \begin{cases} p & \text{if } \tau \equiv 0 \bmod p \text{ and } i = j, \\ 0 & \text{if } \tau \ne 0 \bmod p \text{ and } i = j, \\ 1 & \text{otherwise.} \end{cases}$$

Lemma 6 (Titlebaum, [19]): Let q be a prime power and α a primitive element of F_q. Assume that $F_q = \{\zeta_i \mid 0 \le i < q\}$, for $0 \le i < q$, $X = \{X_i(t)\}_{t=0}^{q-2}$ be an FHS set over F_q, given by

$$X_i(t) = \alpha^t + \zeta_i.$$

Then

$$H_{X_i,X_j}(\tau) = \begin{cases} q-1 & \text{if } \tau \equiv 0 \bmod (q-1) \text{ and } i = j, \\ 0 & \text{if } \tau \ne 0 \bmod (q-1) \text{ and } i = j, \\ 0 & \text{if } \tau \equiv 0 \bmod (q-1) \text{ and } i \ne j, \\ 1 & \text{otherwise.} \end{cases}$$

The follow example comes from the product of Solomon's FHS set and Titlebaum's FHS set.

Example: Let p=31, we construct a Titlebaum's set as follows:

Z_0 = { 0,10,20,30,9,19,29,8,18,28,7,17…};

Z_1 = {0,11,22,2,13,24,4,15,26,6,17,28… };

Z_2 = {0,12,24,5,17,29,10,22,3,15,27,8… };

…

Z_{30} = {0,30,29,28,27,26,25,24,23,22,21… }

Let q=7, we construct a Solomon's set as follows:

Z_0={1,3,2,6,4,5}; Z_1={2,4,3,0,5,6};

Z_2={3,5,4,1,6,0}; Z_3={4,6,5,2,0,1};

…

By the Chinese remainder theorem, we denote the frequency slot taking an integer instead of order pair of Cartesian product. By applying Construction A, we obtain:

S={ s_0={32,157,189,159,160,37,193,101,133,103,104…}

s_1={187,95,127,97,98,192,131,39,71,41,42, 136,75…}

s_2={125,33,65,35,36,130,69,194,9,196,197,74,13…}

…

s_6={157,128,6,39,103,43,45,16,111,144,208,148…}}.

when $0 \le \tau \le 30$, the maximum periodic Hamming correlation $H_m(S) = 1$ .And for the correlation window length L, it can be verified that the maximum periodic partial Hamming correlation of S is given by

$$P_m(L) = \left\lceil \frac{L}{186} \right\rceil \; 1 \le L < 186.$$

One can easily check that the hopping sequence set S is near optimal with respect to the Niu-Peng bound (2).

IV. CONCLUSIONS

In some wireless communication system, the LHZ-PPHC FHS set is used to eliminate MA interference. It is important to construct the LHZ-PPHC FHS set. In this correspondence, a near optimal LHZ-PPHC FHS set is constructed based on the Cartesian product, it may be a challenging problem to find more classes of optimal LHZ-PPHC FHS sets, which are not involved in this paper.

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