Research on dynamic adjustment of cooperation in price duopoly game

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Abstract: The paper considers a Betrand model with incomplete information. Two strategies with price adjustment are mainly discussed: the tit-for-tat strategy and the tit-for-tat strategy with cooperative intention and their dynamic systems respectively.

Introduction

Duopoly is refer to a market situation in which the actions of several firms affect supply and price of same or homogeneous products. It is this characteristic of interdependence that makes oligopolist consider reactions of the other competitors.Under perfect competition all firm are output-takers or price-takers in oligopolistic competition [1]. Two of the most famous and important oligopoly models are Cournot model and Bertrand model. In Cournot model, each oligopolistic firm assumes that other firms hold their outputs constant. While Bertrand model is based upon the premise that each oligopolistic firm assumes other competitors hold their price constant. For maximizing profits, all firms select a quantity to produce by other's outputs in Cournot model [2][3]. Whereas, in Bertrand model each firm maximizes profits by setting a price that undercuts competitors' prices when competitors' prices exceed cost. In the above models, each firm's motion is independent. However, the firms is very limited in the duopoly market and they can realize the interdependence between them, and that can easy to cause the cooperation between them.

In game theory, Nash equilibrium is the basic concept which refers to competitive equilibrium, and it reflects individual rationality but it violates collective rationality— Nash equilibrium of the duopoly game is not Pareto optimal. The prisoners' dilemma shows that, there is a contradiction between individual rationality and collective rationality, and the correct choice based on individual rationality will reduce everybody's welfare. The main question which the prisoners' dilemma poses is whether a cooperative behaviour can emerge among rational and self-interested players whenever there is no formal agreement [4]. Theoretical and experimental studies have indicated several ways by which the cooperative solution can emerge [5][6]. For example, admitting an infinite number of interactions, the so-called "folk theorem" shows that cooperation can be established through a system of punish ments and rewards, although not payoff-maximizing in any single stage. Axelorod has also demonstrated that the best behaviour allowing the achievement of cooperation in repeated games is the "tit-for-tat" conduct, consisting in doing what the opponent did in the previous move.

In this paper, we study that how firms get bigger profits by adjusting their own price without the information of the competitor's output and profit, and consider the cooperative behaviour in duopoly competition with the "tit-for-tat" conduct.

The model

In Betrand game, firms choose the prices of their products instead of the quantities they will produce as in Cournot game. We consider two firms producing similar products in a oligopoly market. Let $p_{i,t}$, $i = 1,2$ represent the price of ith firm at discrete time periods $t = 1,2,3\ldots$. The quantity each firm sells q_i, a linear inverse demand function, is determined by the following equations:

$$\begin{aligned} q_1 &= a - bp_1 + p_2, \\ q_2 &= a - bp_2 + p_1, \end{aligned} \tag{1}$$

where a and b are positive constants. The cost function has the linear form

$$C_i = c_i q_i, \quad i = 1,2. \tag{2}$$

where c_i, the positive parameters, are marginal costs of the firm i, $i = 1,2$, respectively.

With above assumptions the profit of the firm at time t is given by

$$\begin{aligned} p_{1,t} &= \left(p_{1,t} - c_1\right) q_{1,t} = \left(p_{1,t} - c_1\right)\left(a - bp_{1,t} + p_{2,t}\right), \\ p_{2,t} &= \left(p_{2,t} - c_2\right) q_{2,t} = \left(p_{2,t} - c_2\right)\left(a - bp_{2,t} + p_{1,t}\right), \end{aligned} \tag{3}$$

This paper is about cooperation under the incomplete information, and the following models are based on which the firms compare their own profits with the cooperative profit. The solving of the cooperative profit has been introduced in duopoly game theory. The cooperative profit means the profit which is solved by maximizing the sum of all firms' profit. We consider the symmetrical: $c_1 = c_2 = c$, and get the cooperative profit, $p_c = \dfrac{\left[a - (b-1)c\right]^2}{4(b-1)}$ and the cooperative price $p_c = \dfrac{a + (b-1)c}{2(b-1)}$.

The tit-for-tat dynamic strategy

The tit-for-tat strategy is the best behaviour allowing the achievement of cooperation in repeated games [8]. Its characteristic is that every player consists in doing what the opponent did in previous move.

In this paper, the Betrand model is studied with the tit-for-tat conducting, and the dynamic equations are based on the incomplete information. Although each producer cannot obtain the competitor's complete information, he completely knows about his own price and profit. The firm i can compare his profit $p_{i,t}$ at time t with the cooperative profit p_c which is Pareto optimal. While his own profit is more than the cooperative profit ($p_{i,t} - p_c > 0$), he extrapolates that the competitor is cooperative, then he will properly raises his price in order to continue the cooperation as a "reward"[①]; Otherwise, if $p_{i,t} - p_c < 0$, the firm i cannot realize the cooperative profit, and

① Under the condition that the market demand is constant, raising the price can reduces the sales volume, and the profit decreases.

extrapolates that the competitor is not cooperative, then he will reduces his price as "penalty"[②]. Based on these thoughts, a dynamic equation with the price's adjustment is built as follows:

$$\begin{aligned} p_{i,t+1} &= p_{i,t} + u_i\left(p_{i,t} - p_c\right) \\ &= p_{i,t} + u_i\left[\left(p_{i,t} - c\right)\left(a - bp_{i,t} + p_{j,t}\right) - p_c\right] \end{aligned}, \quad i, j = 1, 2 \ \text{ and } \ i \neq j \tag{4}$$

where u_i ($i = 1, 2$) is a adjusting parameter, and $u_i > 0$.

With above assumptions, the duopoly game with heterogeneous players is described by a two-dimensional nonlinear map $T\left(p_1, p_2\right) \to \left(p_1^{'}, p_2^{'}\right)$ defined as

$$T : \begin{cases} p_1^{'} = p_1 - u_1\left[\left(p_1 - c\right)\left(a - bp_1 + p_2\right) - p_c\right] \\ p_2^{'} = p_2 - u_2\left[\left(p_2 - c\right)\left(a - bp_2 + p_1\right) - p_c\right] \end{cases} \tag{5}$$

where "'" denotes the unit-time advancement, that is if the right-hand side variables are price of period t, then the left-hand ones represent price of period $t+1$.

In this paper, we are considering a economic model where only non-negative equilibrium points are meaningful. So that we only pay attention to the nonnegative fixed points of the map (5), i.e. the solution of the nonlinear algebraic system as

$$\begin{cases} \left(p_1 - c\right)\left(a - bp_1 + p_2\right) - p_c = 0 \\ \left(p_2 - c\right)\left(a - bp_2 + p_1\right) - p_c = 0 \end{cases} \tag{6}$$

which is obtained by setting $p_i^{'} = p_i$, $i = 1, 2$ in system (5). We have an only fixed point of system (6) $E = (p_1^*, p_2^*)$ where

$$p_1^* = p_2^* = p_c = \frac{a + (b-1)c}{2(b-1)} \tag{7}$$

The study of the local stability of the fixed point of the two-dimensional system (5) depends on the eigenvalues of the Jacobian matrix of (5). The Jacobian matrix J at the point (p_1, p_2) has the form

$$J\left(p_1, p_2\right) = \begin{pmatrix} 1 + u_1\left(a - 2bp_1 + p_2 + bc\right) & u_1\left(p_1 - c\right) \\ u_2\left(p_2 - c\right) & 1 + u_2\left(a - 2bp_2 + p_1 + bc\right) \end{pmatrix} \tag{8}$$

We estimate the Jacobian matrix J at E, which is

$$J\left(p_1^*, p_2^*\right) = \begin{pmatrix} 1 - \frac{u_1\left[a - (b-1)c\right]}{2(b-1)} & \frac{u_1\left[a - (b-1)c\right]}{2(b-1)} \\ \frac{u_2\left[a - (b-1)c\right]}{2(b-1)} & 1 - \frac{u_2\left[a - (b-1)c\right]}{2(b-1)} \end{pmatrix}.$$

The characteristic equation is

[②] Under the condition that the market demand is constant, reducing the price can increase the sales volume, and the profit increases.

$$p(l) = l^2 - Tr\,l + Det = 0$$

where Tr is the trace and Det is the determinant of the Jacobian matrix $J(p_1^*, p_2^*)$.

$$Tr = 2 - \frac{u_1\left[a-(b-1)c\right]}{2(b-1)} - \frac{u_2\left[a-(b-1)c\right]}{2(b-1)} \quad Det = 1 - \frac{u_1\left[a-(b-1)c\right]}{2(b-1)} - \frac{u_2\left[a-(b-1)c\right]}{2(b-1)}$$

Then we have two eigenvalues of matrix $J(p_1^*, p_2^*)$, $l_1 = 1$ and $l_2 = 1 - \frac{u_1\left[a-(b-1)c\right]}{2(b-1)} - \frac{u_2\left[a-(b-1)c\right]}{2(b-1)}$. From the condition that u_i $(i=1,2)$ is very small, we have that $|l_2| < 1$. Since $l_1 = 1$ is special, we can't know the stability of the system (5). But the following numerical experiments show that its stability is sensitive to the parameters.

The tit-for-tat dynamic strategy with cooperative intention

For further studies with the cooperative behaviour, we improve the equation (4). Through adding feedback control, we have the form

$$\begin{aligned} p_{i,t+1} &= p_{i,t} - u_i\left(p_{i,t} - p_c\right) - v\left(p_{i,t} - p_c\right) \\ &= p_{i,t} - u_i\left[\left(p_{i,t} - c\right)\left(a - bp_{i,t} + p_{j,t}\right) - p_c\right] - v\left(p_{i,t} - p_c\right) \end{aligned} \tag{9}$$

where u_i $(i=1,2)$ is an adjustment parameter, and $u_i > 0$, $v > 0$, $-v\left(p_{i,t} - p_c\right)$ is the feedback control of the system.

In the equation (9), the first item ($-u_i\left[\left(p_{i,t} - c\right)\left(a - bp_{i,t} + p_{j,t}\right) - p_c\right]$) corresponds with the tit-for-tat strategy, and the second item ($-v\left(p_{i,t} - p_c\right)$) shows that the firm will reduce the price of this period while the previous one surpasses the cooperative price ($p_{i,t} - p_c > 0$), otherwise ($p_{i,t} - p_c < 0$), the firm will properly increase the price of this period. This shows that the firms have certain cooperative consciousness. Because of the feedback control, the firms have the spontaneity to the cooperative behaviour. That is to say, the firms both have the cooperative intention from their own angles.

In this system (9), $E = (p_1^*, p_2^*)$ ($p_1^* = p_2^* = p_c = \frac{a+(b-1)c}{2(b-1)}$) is also its fixed point. The Jacobian matrix J of (9) at the point (p_1, p_2) has the form

$$J(p_1, p_2) = \begin{pmatrix} 1 + u_1\left(a - 2bp_1 + p_2 + bc\right) - v & u_1\left(p_1 - c\right) \\ u_2\left(p_2 - c\right) & 1 + u_2\left(a - 2bp_2 + p_1 + bc\right) - v \end{pmatrix} \tag{10}$$

We can get the Jacobian matrix J at E, which is

$$J(E)=\begin{pmatrix} 1-\frac{u_1\left[a-(b-1)c\right]}{2(b-1)}-v & \frac{u_1\left[a-(b-1)c\right]}{2(b-1)} \\ \frac{u_2\left[a-(b-1)c\right]}{2(b-1)} & 1-\frac{u_2\left[a-(b-1)c\right]}{2(b-1)}-v \end{pmatrix},$$

and we have two eigenvalues of matrix $J(E)$, $l_1=1-v$ and $l_2=1-v-\frac{u_1\left[a-(b-1)c\right]}{2(b-1)}-\frac{u_2\left[a-(b-1)c\right]}{2(b-1)}$. So long as $0<v<2$ and $0<u_1+u_2<\frac{2(b-1)(2-v)}{a-(b-1)c}$, the absolute value of l_1 and l_2 is smaller than 1, and hence the equilibrium is local stable.

Conclusion

The paper considers a Betrand model with incomplete information. Two strategies with price adjustment are mainly discussed: the tit-for-tat strategy and the tit-for-tat strategy with cooperative intention and their dynamic systems respectively.Analysis results indicate that the firms both have the cooperative intention from their own angles in the tit - for - tat strategy.But the following numerical experiments show that its stability is sensitive to the parameters.Because of the feedback control, the firms have the spontaneity to the cooperative behaviour. the system stabilize in a certain range.

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