

Comparative Analysis on Performance Evaluation of Naval Minesweeping

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Abstract. The estimation of number of remaining mines is an important basis for the performance evaluation of naval minesweeping, and the accuracy of the estimation is related to the mastery of prior information and the selection of evaluation model. In this paper, we build the evaluation models by employing Markov process and Bayesian estimation theory to estimate the number of remaining mines. Then simulation experiments based on the above models were carried out, using the operational data which come from prior information of the minefield and information of swept mines. The results show that the Bayesian estimation is more reliable than Markov process in the case of sufficient prior information.

Introduction

The main purpose of mine operational evaluation is to quantify the risk of minefield after minesweeping, while the number of remaining mine is one of the key links for that. How to estimate the number of remaining mines according to sampling information has become the main problem which is most concerned about by various models. We can build a model which meets the following requirements: strong description ability which is related to mine array state, information retrieval ability, practical value for dealing with various problems, different processing methods to choose according to the different conditions [1-5]. In this paper, two typical models, Markov process and Bayesian estimation [4-7] are analyzed and compared. The estimation of remaining mine number is studied for different prior information, starting from the samples of information related the mines which have been swept.

Establishment of the Model of Performance Evaluation

Mathematical Model Based on Markov process.

Markov process is a random process which can describe many practical problems whose coming state is related only to current state. In order to describe it, we define some symbols as follows: DC - a mine will explode when the sweeper affected it DC times, Q - maximal value of DC , N - sum of mine sweeping times, x_q^n - number of mine which DC is q , swept for the n th time,

$\mathbf{X}_n = (x_1^n, \mathbf{L}, x_{Q-1}^n, x_Q^n)^T$ - vector made up of variables x_i^n , p_0 - mine-sweeping probability, y_n - sum of the mine which have been swept, \mathbf{Y}_n - vector made up of variables y_n , $\mathbf{Y}_n = (y_1, y_2, \mathbf{L}, y_n)^T$, w_q^n - variable which denotes error of x_q^{n+1} , \mathbf{W}_n - vector made up of variables w_q^n , e_n - variable which denotes error of $y(n)$, \mathbf{E}_n - vector made up of variables e_n .

According to Markov process theory [3, 4, 8], when the minefield has been swept $n+1$ times, the number of mines whose DC is q only relates to the mines whose DC are q and $q+1$. Then the equation can be obtained as follows:

$$x_q^{n+1} = (1-p_0)x_q^n + p_0x_{q+1}^n + w_q^n, n \in [1, N], q \in [1, Q] \quad (1)$$

Suppose

$$A = \begin{bmatrix} 1-p_0 & p_0 & 0 & \mathbf{L} & 0 & 0 \\ 0 & 1-p_0 & p_0 & \mathbf{L} & 0 & 0 \\ \mathbf{L} & \mathbf{L} & \mathbf{L} & \mathbf{L} & \mathbf{L} & \mathbf{L} \\ 0 & 0 & 0 & \mathbf{L} & p_0 & 0 \\ 0 & 0 & 0 & \mathbf{L} & 1-p_0 & p_0 \\ 0 & 0 & 0 & \mathbf{L} & 0 & 1-p_0 \end{bmatrix}, C = \begin{bmatrix} p_0 \\ 0 \\ \mathbf{M} \\ \mathbf{M} \\ 0 \\ 0 \end{bmatrix}$$

The equation (1) can be rewritten as follows:

$$\mathbf{X}_{n+1} = A\mathbf{X}_n + \mathbf{W}_n = \mathbf{L} = A^{n+1}\mathbf{X}_0 + \mathbf{W}_n \quad (2)$$

When the minefield has been swept $n+1$ times and the number of mines which have been swept out is y_{n+1} , it is only related to mines whose DC is 1:

$$y_{n+1} = p_0x_1^n + e_{n+1} = C^T \mathbf{X}_n + e_{n+1} \quad (3)$$

The above equation can be rewritten as follows:

$$y_n = C^T \mathbf{X}_{n-1} + e_n = C^T (A^{n-1} \mathbf{X}_0 + \mathbf{W}_n) + e_n \quad (4)$$

If

$$\mathbf{H}_n = \begin{bmatrix} (C^T)^T (A)^1 \\ (C^T)^T (A)^2 \\ \mathbf{M} \\ (C^T)^T (A)^{n-1} \end{bmatrix}, \mathbf{Y}_n = \begin{bmatrix} y_1 \\ y_2 \\ \mathbf{M} \\ y_n \end{bmatrix}, \mathbf{E}_n = C^T \cdot \mathbf{W}_n + \begin{bmatrix} e_1 \\ e_2 \\ \mathbf{M} \\ e_n \end{bmatrix}$$

Then according to the equation (4), the following expression can be obtained:

$$\mathbf{Y}_n = \mathbf{H}_n \mathbf{X}_0 + \mathbf{E}_n \quad (5)$$

Mathematical Model based on Bayesian estimation theory

Some symbols are defined as follows: N -the total number of mine, $p(m)$ -the prior distribution of mine, $\mathbf{m}_0 = [m_{10}, m_{20}, \mathbf{L}, m_{n0}]$ - the initial allocation proportion of the mine number when the minefield is divided into n regions, $\mathbf{p}_j = [p_{1j}, p_{2j}, \mathbf{L}, p_{nj}]^T$ -the mine sweeping probability of the single mine in the sub area when the j th estimation is done, $\mathbf{x}_j = [x_{1j}, x_{2j}, \mathbf{L}, x_{nj}]$ -the number of mines in each sub region between j and $j-1$, R - the remaining number of mines of the current status, X - the sample of swept mines in the current status, N_f is the total number of swept mines..

Suppose $\mathbf{X}_j = [\mathbf{x}_1, \mathbf{x}_2, \mathbf{L}, \mathbf{x}_j]^T$, $N_j = \sum_{k=1}^j \sum_{i=1}^n x_{ij}$, $\Delta N_j = N_j - N_{j-1}$, $P_j = \mathbf{m}_0 \times \mathbf{p}_j$

If the remaining number of mine is estimated for the first time, according to Bayesian statistical theory, the joint distribution of the sample (X_1) and the number of remaining mines of the whole area (R_1) is:

$$h(R_1, \mathbf{X}_1) = p(R_1 + N_1) \binom{R_1 + N_1}{N_1} P_1^{N_1} [1 - P_1]^{R_1} \quad (6)$$

The conditional distribution of R_1 is:

$$p(R_1 | \mathbf{X}_1) = p(R_1 + N_1) \frac{(R_1 + N_1)!}{R_1!} [1 - P_1]^{R_1} / \left[\sum_{R_1=0}^{N-N_1} p(R_1 + N_1) \frac{(R_1 + N_1)!}{R_1!} [1 - P_1]^{R_1} \right] \quad (7)$$

For sub area i , the distribution proportion of the number of remaining mines is:

$$m_{i1} = m_{i0} \frac{1 - p_{i1}}{1 - P_1} \quad (8)$$

The posterior distribution of the number of remaining mines can be obtained:

$$p(r_{i1} | \mathbf{X}_1) = \sum_{R_1=r_{i1}}^{N-N_1} p(R_1 | \mathbf{X}_1) \binom{R_1}{r_{i1}} (m_{i1})^{r_{i1}} [1 - m_{i1}]^{R_1 - r_{i1}} \quad (9)$$

Set $G_1 = \sum_{R_1=0}^{N-N_1} p(R_1 + N_1) \frac{(R_1 + N_1)!}{N_1!} [1 - P_1]^{R_1}$, we can get the following expressions:

$$\begin{aligned} p(r_{i1} | \mathbf{X}_1) &= \frac{1}{G_1} \sum_{R_1=r_{i1}}^{N-N_1} \left\{ p(R_1 + N_1) \frac{(R_1 + N_1)!}{R_1!} \frac{R_1!}{(R_1 - r_{i1})! r_{i1}!} [1 - P_1]^{R_1} (m_{i0} \frac{1 - p_{i1}}{1 - P_1})^{r_{i1}} \left[1 - m_{i0} \frac{1 - p_{i1}}{1 - P_1} \right]^{R_1 - r_{i1}} \right\} \\ &= \frac{1}{G_1} \sum_{R_1=r_{i1}}^{N-N_1} p(R_1 + N_1) \frac{(R_1 + N_1)!}{(R_1 - r_{i1})! r_{i1}!} \cdot [m_{i0}(1 - p_{i1})]^{r_{i1}} [1 - P_1 - m_{i0}(1 - p_{i1})]^{R_1 - r_{i1}} \end{aligned} \quad (10)$$

We can see $p(R_{j-1} | \mathbf{X}_{j-1})$ as the prior distribution in the j th estimation, and obtain the distribution of number of remaining mines:

$$\begin{cases} p(R_j | \mathbf{X}_j) = \frac{1}{G_j} p(R_j + N_j) \frac{(R_j + N_j)!}{R_j!} [1 - P_j]^{R_j} \\ p(r_{ij} | \mathbf{X}_j) = \frac{1}{G_1} \sum_{R_j=r_{ij}}^{N-N_j} p(R_j + N_j) \frac{(R_j + N_j)!}{(R_j - r_{ij})! r_{ij}!} [m_{i0}(1 - p_{ij})]^{r_{ij}} [1 - P_j - m_{i0}(1 - p_{ij})]^{R_j - r_{ij}} \\ G_j = \sum_{R_j=0}^{N-N_j} p(R_j + N_j) \frac{(R_j + N_j)!}{N_j!} [1 - P_j]^{R_j} \end{cases} \quad (11)$$

By analysis of equation (11), we find that the distribution of remaining number of mines has nothing to do with previous estimation. So it can be rewritten as follows:

$$\begin{cases} p(R | \mathbf{X}) = \frac{1}{G} p(R + N) \frac{(R + N)!}{R!} [1 - P]^R \\ p(r_i | \mathbf{X}) = \frac{1}{G} \sum_{R=r_i}^{N-N_f} p(R + N) \frac{(R + N)!}{(R - r_i)! r_i!} [m_{i0}(1 - p_i)]^{r_i} [1 - P - m_{i0}(1 - p_i)]^{R - r_i} \\ G = \sum_{R=0}^{N-N_j} p(R + N) \frac{(R + N)!}{N!} [1 - P]^R \end{cases} \quad (12)$$

Then we can obtain the mean and variance of remaining mines as follows:

$$\begin{cases} E_R = \sum_{R=0}^{N-N_f} p(R | N_f) \cdot R \\ V_R = \sum_{R=0}^{N-N_f} p(R | N_f) \cdot (R - E_R)^2 \end{cases} \quad (13)$$

Sample Data Acquisition

We can obtain the sample data from Monte Carlo stochastic simulation algorithm or the results of minesweeping training and see it as the observation sequence value of the swept mines. Here we take the result of Monte Carlo method simulation as the sample data:

Generate a random number $p_i, p_i \in [0, 1]$ for each mine. Define p'_0 as the threshold valve. If $p_i < p'_0$, the mine can be considered swept mine and taken as the observation sequence value.

Emulation and results

Assume that there are 40 mines in one minefield and the maximum value of DC is 4. Its distribution is $[15;15;5;5]$. The minesweeping probability is 0.6 and 0.9 in Markov process model. If the quality of prior information is high, the estimation result which comes from binomial distribution is better than others in Bayesian estimation model. So we adopt binomial distribution: $B(50,0.8)$ and $B(80,0.5)$. Take the emulation 30 rounds ($N = 4$ times per round). The estimated values of the remaining mines in above mine field are plotted in Fig1, Fig2.

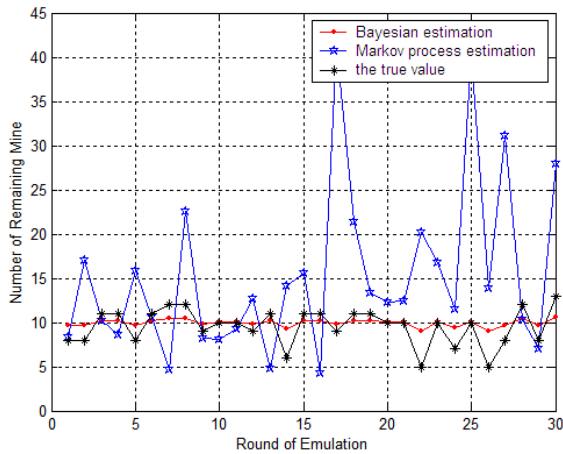


Fig1.a number of remaining mines
when $B(80, 0.5)$, $P_0 = 0.9$

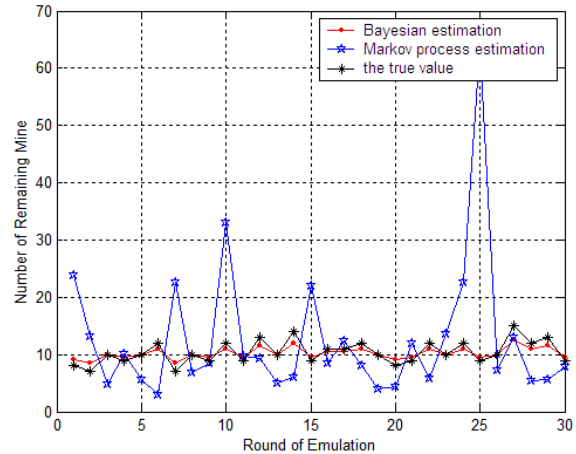


Fig1.b number of remaining mines
when $B(50, 0.8)$, $P_0 = 0.9$

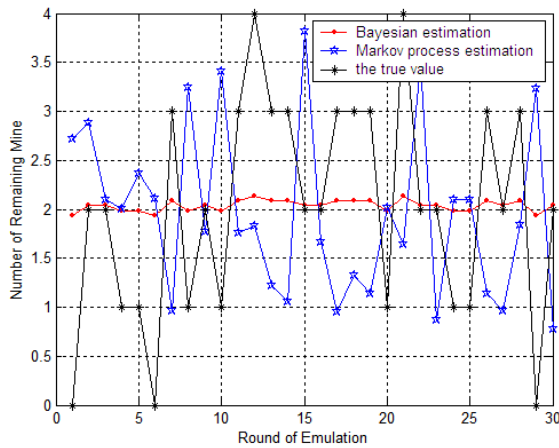


Fig2.a number of remaining mines
when $B(80, 0.5)$, $P_0 = 0.6$

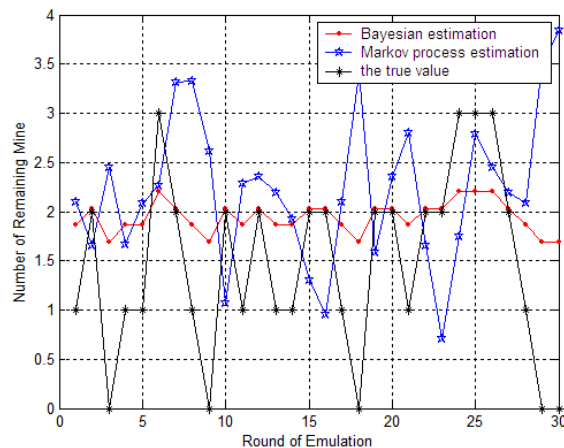


Fig2.b number of remaining mines
when $B(50, 0.8)$, $P_0 = 0.6$

In Fig1.a and Fig2.a, the minesweeping probability is 0.9 and 0.6, and the prior distribution is binomial distribution $B(80,0.5)$. In order to make the prior information closer to the truth, we change the parameters and take $B(50,0.8)$ as the prior distribution. Then the estimation results can be obtained as Fig2.a and Fig2.b.

Discussions and Conclusions

The result shows that Bayesian estimation model is better than Markov process model. Although Markov process model can reflect the statistical results, for some round of simulations, the deviation will appear between the estimated result and the true value. On the contrary, the trend of the curve is

consistent with the true value in Bayesian estimation model, although there are different amplitudes with the different prior information. The results are achieved because the Bayesian estimation model can exploit the prior information fully which can be obtained accurately in many practical situations.

Comparing fig.1 with fig.2, we find that the probability has a direct influence on the assessment. The high sweeping probability can obtain satisfactory results because the sampling data is affected to a lesser degree by the "noise". In both fig.1b and fig.2a, we can find that the amplitude of the posterior estimate has been greatly improved compared with the situation shown in Fig.1a and fig.2b because the parameter $P_b = (0.8, 0.5)$ in the latter is closer to weeping probability $P_0 = (0.9, 0.6)$. This shows that when we use the Bayesian method to estimate the number of remaining mines, the accuracy of prior information will play a crucial role.

Of course, it is feasible to adopt Markov process model only if we cannot get prior information, moreover, Markov process model can process more complicated situation. So in practice, the choice of what kind of model depends on the availability of information on the situation.

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