Computational Theory for Hooping Force of Concrete-filled Rectangular Steel Tube Column

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Abstract: One of the mechanical key problems on concrete-filled steel tube (CFT) structure is the confining effect between the steel tube and the core concrete. The key technology to solve the problem is to accurately describe the interfacial mechanics properties. Based on continuum mechanics theory, by means of the solving method on plane problem in elasticity and structural mechanics displacement solution, using the function expression to describe the interfacial normal force distribution, a small-deformation elasticity analysis theory on the interfacial mechanics problem is framed for the CFT structure. The results show: The hooping force distribution in the interface between the core concrete and the steel tube is not uniform. The hooping force in the long side of rectangle is smaller than one in the short side of rectangle.

Introduction

Based on the experiments, P.K.Gupta, S.M.Sarda[1] concluded the steel tube's hooping force on core concrete will increase as the axial force increases, and as the diameter-thickness ratio decreases, the hooping force will increase. Hsuan-The Hu[2], etc. did simulation of finite element analysis on different shape concrete-filled steel tubes. They found the hooping force becomes more significant as the axial force increases, and the hooping force of concrete-filled tube with round shape cross section is more significant comparing to tube with rectangle shape cross section. Scientists in China have conducted enormous amount of researches on the mechanical behaviors of the structure of concrete-filled steel tube through experiments and finite element analysis [3-8]. However, there are few research papers that describe the distribution rules of hooping force on CFT. In fact, the hooping force of steel tube on concrete is a key mechanical problem on CFT structure. By studying interfacial mechanics of CFT columns, it could be possible to add the calculation of the hooping force of steel tube in the calculation of the stiffness of CFT columns. So the stiffness superposition in the current specification [9] can be improved.

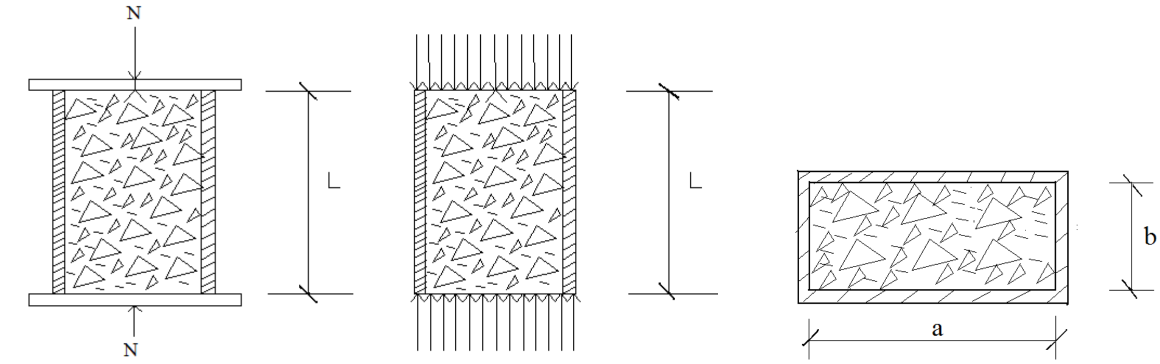
Computational Model

A computational model, shown in Fig. 1(b), can be set up to study the axial force effect on concrete-filled steel tube, shown in Fig. 1(a). Under small deformed condition, the stress distribution on the end of column will have the same vertical strain on steel tube and concrete. This will become a general plane strain question in elastic mechanics, the plane is shown in Fig.1(c) . For simplicity, first treat it as plane stress problem.

Solution of problem

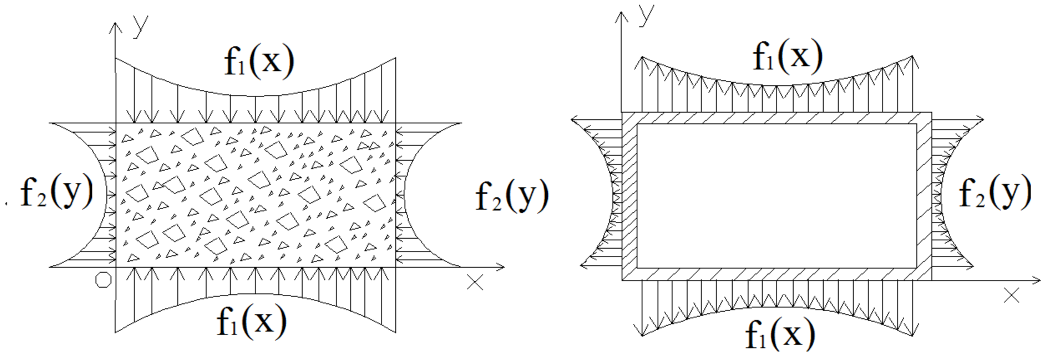
Assuming only consider the normal forces in the interface between the core concrete and the steel tube, denoting as $f_1(x)$ and $f_2(y)$. We separate the interface of the concrete-filled steel tube, shown in Fig. 2(a) and Fig. 2(b). In order to study the internal force state of steel tube and core concrete, the continuity conditions of the normal displacements on the interface have to be based on.

The solution to plane problem in elasticity shown in Fig. 2 (a).
Stress function $\varphi(x,y)$ and normal force $f_1(x)$ and $f_2(y)$ can be expressed in triangle series.



(a) Loading model (b) Calculation model (c) Composite section

Fig.1 Concrete-filled steel tube member



(a) Core concrete (b) Steel tube

Fig.2 Interaction between core concrete and steel tube

For Fig. 2 (a), just considering $f_1(x)$,

$$\varphi(x,y)=\sum_{n=1}^{\infty}\sin(n\pi x/a)[A_n ch(\kappa y)+B_n sh(\kappa y)+C_n y ch(\kappa y)+D_n y sh(\kappa y)],\kappa=n\pi/a\,. \tag{1}$$

Four parameters A_n,B_n,C_n,D_n can be estimated by using boundary conditions listed below,

$$y=0,b;\sigma_y=\frac{\partial^2\varphi}{\partial x^2}=-f_1(x),\tau_{xy}=-\frac{\partial^2\varphi}{\partial x\partial y}=0\ . \tag{2}$$

For Fig. 2 (a), just considering $f_2(y)$,

$$\varphi(x,y)=\sum_{n=1}^{\infty}\sin(n\pi y/b)[\overline{A}_n ch(\overline{\kappa}x)+\overline{B}_n sh(\overline{\kappa}x)+\overline{C}_n x ch(\overline{\kappa}x)+\overline{D}_n x sh(\overline{\kappa}x)],\overline{\kappa}=n\pi/b\ . \tag{3}$$

Four parameters $\overline{A}_n,\overline{B}_n,\overline{C}_n,\overline{D}_n$ can be estimated by using boundary conditions listed below,

$$x=0,a;\sigma_x=\frac{\partial^2\varphi}{\partial y^2}=-f_2(y),\tau_{xy}=-\frac{\partial^2\varphi}{\partial x\partial y}=0\ . \tag{4}$$

$f_1(x)$ and $f_2(y)$ can be expanded by using cosine series, i.e.

$$f_1(x)=\sum_{m=0}^{\infty}\xi_m\cos(m\pi x/a)\qquad f_2(y)=\sum_{m=0}^{\infty}\beta_m\cos(m\pi y/b)\quad m=2,4,...\ . \tag{5}$$

In which, ξ_m and β_m are the coefficients in the expanded series.

To conveniently analyze it, first solve the normal displacements around the core concrete, i.e. $\Delta_m^{(1)}$ and $\Delta_m^{(2)}$, caused respectively by single force $\cos(m\pi x/a)$ and $\cos(m\pi y/b)$, then using superposition method to obtain normal displacements around the core concrete, i.e. $\Delta_c^{(1)}$ and $\Delta_c^{(2)}$, caused by $f_1(x)$ and $f_2(y)$, respectively. There are

$$\Delta_c^{(1)}=\sum_{m=0}^{\infty}\xi_m\Delta_m^{(1)} \qquad \Delta_c^{(2)}=\sum_{m=0}^{\infty}\beta_m\Delta_m^{(2)} \quad . \tag{6}$$

Now discuss the solution for the displacements surrounding the rectangle concrete under force $\cos(m\pi x/a)$. According to the displacement – strain relationship

$$\varepsilon_x=\frac{\partial u}{\partial x}=\frac{1}{E_1}(\sigma_x-v_1\sigma_y)=\frac{1}{E_1}(\frac{\partial^2\varphi}{\partial y^2}-v_1\frac{\partial^2\varphi}{\partial x^2}) \ , \ \varepsilon_y=\frac{\partial v}{\partial y}=\frac{1}{E_1}(\sigma_y-v_1\sigma_x)=\frac{1}{E_1}(\frac{\partial^2\varphi}{\partial x^2}-v_1\frac{\partial^2\varphi}{\partial y^2}) \ ,$$

$$\gamma_{xy}=\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}=\frac{2(1+v_1)}{E_1}\tau_{xy}=-\frac{2(1+v_1)}{E_1}(\frac{\partial^2\varphi}{\partial x\partial y}) \ . \tag{7}$$

Here $E_1=E_c/(1-v_c^{\ 2})$, $v_1=v_c/(1-v_c)$, (the constant of material E_1, v_1 can be transferred according to the plane strain, E_c, v_c are denoted as the elasticity modulus and Poisson ratio of concrete)

According to the Eq. 7,

$$u(x,y)=\int\frac{1}{E_1}(\frac{\partial^2\varphi}{\partial y^2}-v_1\frac{\partial^2\varphi}{\partial x^2})dx+g_1(y) \ , \quad v(x,y)=\int\frac{1}{E_1}(\frac{\partial^2\varphi}{\partial x^2}-v_1\frac{\partial^2\varphi}{\partial y^2})dy+g_2(x)\cdot \tag{8}$$

Inserting Eq.8 into the third part of Eq.7,

$$\frac{\partial}{\partial y}\int\frac{1}{E_1}(\frac{\partial^2\varphi}{\partial y^2}-v_1\frac{\partial^2\varphi}{\partial x^2})dx+g_1'(y)+\frac{\partial}{\partial x}\int\frac{1}{E_1}(\frac{\partial^2\varphi}{\partial x^2}-v_1\frac{\partial^2\varphi}{\partial y^2})dy+g_2'(x)=-\frac{2(1+v_1)}{E_1}(\frac{\partial^2\varphi}{\partial x\partial y}) \ \cdot \tag{9}$$

So functions, $g_1(y)$ and $g_2(x)$, can be derived; it includes several constants of integration. These constants can be determinated by the marginal condition (displacement condition) in the questions. Since the cross-section of concrete column is rectangle, it has two symmetry axises, therefore:

$$x=a/2, u=0; \ \ y=b/2, v=0 \ . \tag{10}$$

Eq.10 can determine the constants of integration in the previous equation.

The similar method can be used to solve the displacement surrounding concrete under the force, $\cos(m\pi y/b)$; So far, $\Delta_m^{(1)}$, $\Delta_m^{(2)}$ on the right side of Eq.6 are solved. The expansion coefficients ξ_m, β_m of Eq.5 and Eq.6 must use the coordination relationship between the deformation of core concrete column and that of external steel tube to derive.

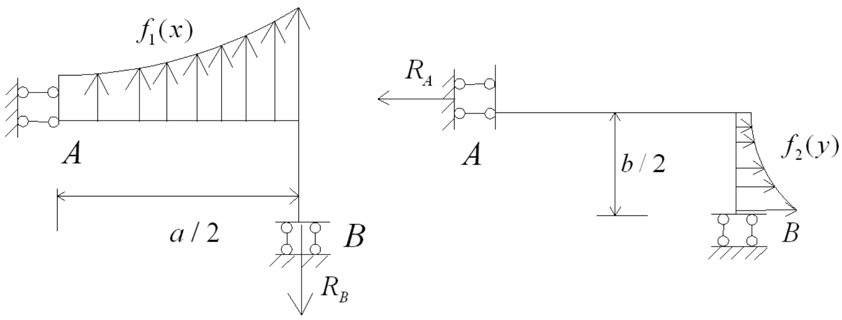
Mechanical analysis of the cross section of steel tube shown in Fig.2 (b)

Use the structural mechanical approach to solve the problem of hyperstatic closed rectangular structure in Fig. 2(b). Because of the symmetric, the closed structure shown in Fig. 2(b) is similar to that in Fig. 3. The supports on both ends are sliding supports with rotational restraints, $\theta_A=\theta_B=0$. And there is no shearing force, so $Q_A=Q_B=0$.

Applying the structural mechanics approach, to determine the displacements $\bar{\Delta}_m^{(1)}$, $\bar{\Delta}_m^{(2)}$ under forces $\cos(m\pi x/a)$ and $\cos(m\pi y/b)$ in the series expansion for the rectangular closed structure shown in Fig. 3(a) and 3 (b), the displacement surrounding rectangular closed structure under the forces $f_1(x)$ and $f_2(y)$ can be expressed in the series expansion .

$$\Delta_s^{(1)} = \sum_{m=0}^{\infty} \xi_m \overline{\Delta}_m^{(1)}; \quad \Delta_s^{(2)} = \sum_{m=0}^{\infty} \beta_m \overline{\Delta}_m^{(2)} . \tag{11}$$

In which, $\Delta_s^{(1)}$, $\Delta_s^{(2)}$ are denoted as the surrounding normal displacements of steel tube under the normal force $f_1(x)$ and $f_2(y)$.



(a) Effect of $f_1(x)$ (b) Effect of $f_2(y)$

Fig. 3 Equivalent analysis chart, taking 1/4 of the structure

Estimate coefficients ξ_m, β_m

According to the coordination condition of interface normal displacement between steel tube and concrete:

$$(\Delta_s^{(1)} + \Delta_s^{(2)}) + (\Delta_c^{(1)} + \Delta_c^{(2)}) = \Delta . \tag{12}$$

From Eq. (6), (11), and (12), derive:

$$\sum_{m=0}^{\infty} [\xi_m (\overline{\Delta}_m^{(1)} + \Delta_m^{(1)}) + \beta_m (\overline{\Delta}_m^{(2)} + \Delta_m^{(2)})] = \Delta . \tag{13}$$

In which, Δ is the difference of normal displacement surround core concrete and steel tube caused by the axial forces N_c and N_s ,which acts in the core concrete and steel tube respectively, with taking out constrain between core concrete and steel tube. For the long side, $\Delta = v_c N_c b / E_c A_c - v_s N_s b / E_s A_s$, and for the short side, $\Delta = v_c N_c a / E_c A_c - v_s N_s a / E_s A_s$, here, v is the Poisson ratio symbol, s and c stands for steel tube and concrete. E, A is the elastic modulus symbol and the area symbol respectively.

Additionally, following equation exists:

$$N_c + N_s = N \quad , \quad \varepsilon_z = \varepsilon_c = \varepsilon_s . \tag{14}$$

Connecting Eq.13, Eq.14, and having the surrounding least squares collocation on the equation derive ξ_m, β_m.

So far, based on the previous analysis, it can efficiently determine the mutual normal force $f_1(x)$ and $f_2(y)$ on the interface and achieves the more accurate analysis of the elastic mechanic behavior of concrete filled rectangular steel tube column under the axial force.

Example

Square concrete-filled steel tube column

Square concrete-filled steel tube column, length a=b=300mm; thickness of steel tube t=5mm, steel rate: ρ=0.067, concrete label is C30, the steel tube is made of Q235 steel.

Using Matlab program, setting n=10, m=4, satisfying the convergence of the calculation, N=1(unit force), obtains the normal forces $f_1(x)$ and $f_2(y)$ on the interface for the square CFT.

$$f_1(x) = 0.0252 + 0.01139 \cos(20\pi x / 3) + 0.00147 \cos(40\pi x / 3) . \tag{15}$$

$$f_2(y) = 0.0252 + 0.01139\cos(20\pi y/3) + 0.00147\cos(40\pi y/3). \quad (16)$$

The plot of normal forces $f_1(x)$ and $f_2(y)$ on the interface is shown in Fig. 4.

Rectangular concrete-filled steel tube column

Rectangular concrete-filled steel tube column, lengths a= 400mm, b=200mm. Steel tube t=4mm, steel rate ρ =0.075, concrete label is C30, the steel tube is made of Q235 steel.

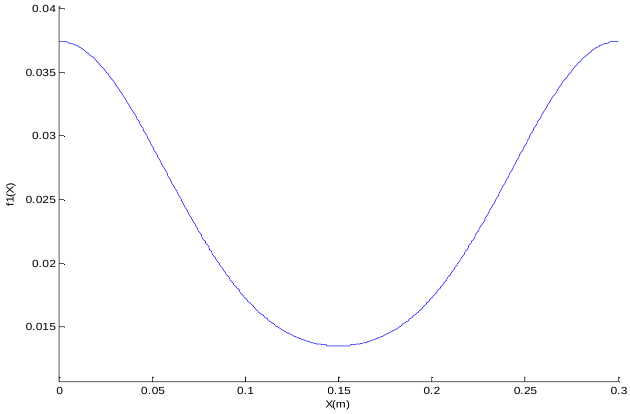


Fig. 4 The distribution of normal force $f_1(x)$ and $f_2(y)$ on the interface of square column

Using Matlab program, setting n=10, m=4, satisfying the convergence of the calculation, N=1(unit force), obtains the normal forces $f_1(x)$ and $f_2(y)$ on the interface for the rectangular CFT.

$$f_1(x) = 0.01638 + 0.01515\cos(5\pi x) + 0.00099\cos(10\pi x). \quad (17)$$

$$f_2(y) = 0.023264 + 0.0081\cos(10\pi y) + 0.001926\cos(20\pi y). \quad (18)$$

The plots of normal forces $f_1(x)$ and $f_2(y)$ on the interface are shown in Fig. 5 and Fig. 6.

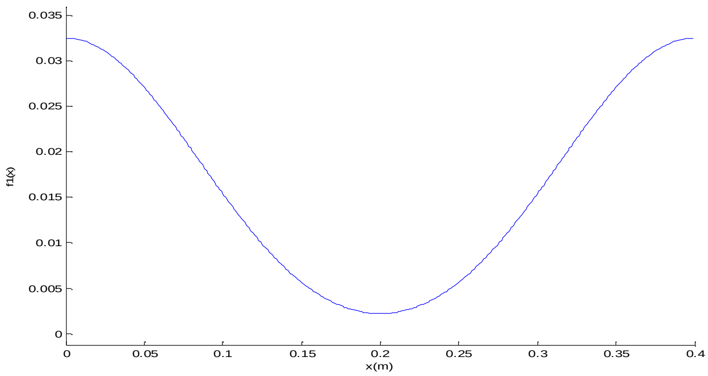


Fig. 5 The distribution of normal force $f_1(x)$ on the interface of rectangular column

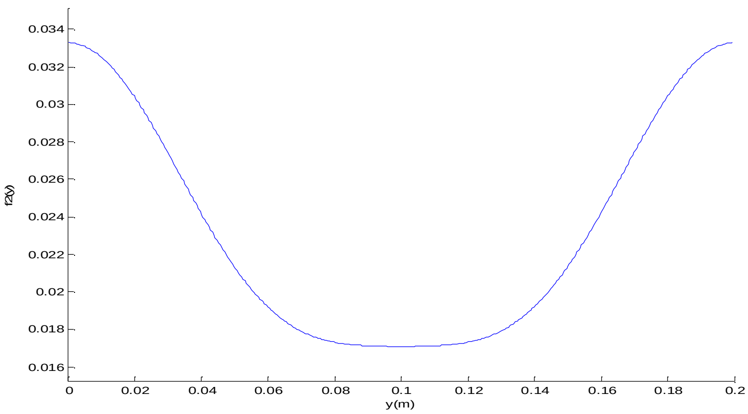


Fig. 6 the distribution of normal force $f_2(y)$ on the interface of rectangular column

According Fig. 4, Fig. 5, and Fig. 6, it concludes that the normal force on the interface of square concrete-filled steel tube column and rectangular concrete-filled steel tube column is not evenly distributed, which behaved like quadratic curve, the hooping force of rectangular cross section is more inhomogeneous than that of square cross section. The corner has the phenomena of the stress concentration. The hooping force is the largest in the corner, and it is the smallest in the central point. For rectangular cross section, the hooping force is considerably small on the longer side comparing to that on the shorter side. The hooping force of square cross section of concrete-filled steel tube is better than that of the rectangular cross section.

Conclusion

Established an elasticity analysis theory of interfacial mechanical problem of concrete-filled steel tube column under small deformed condition; achieved a semi-analytical and semi-numerical solution for the interfacial mechanical problem of the rectangular concrete filled steel tube column.
Used the function expression to describe the characteristics of interfacial mechanics between steel tube and concrete; established the interfacial hooping force distribution functions $f_1(x)$ and $f_2(y)$ for concrete-filled steel tube column; used the plots of those functions conveniently describe the distribution of interfacial hooping force.
Made it possible to add the calculation of the hooping force of steel tube in the calculation of the stiffness of concrete-filled steel tube columns and to improve the stiffness superposition in the current specification.

Acknowledgements

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