

Dynamical Behavior of a Predator-prey System with Delayed Stage Structure for the Prey and Impulsive Control

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Abstract: Based on the biological control strategy in the pest management, a delayed stage-structured predator-prey model with impulsive control is considered. Using the theories and methods of impulsive delayed differential equations, sufficient conditions are obtained, which guarantee the global attractivity of pest-eradication periodic solution and permanence of the system.

Introduction

China is a great agricultural country whose agricultural development level has a profound impact on the development of the country. The outbreak of pest will cause the harm for agricultural production and development of the country. Therefore the prevention of pest cannot be ignored. Currently, there are two methods to control pest: chemical control and biological control. Though chemical control effect quickly, it pollutes the environment and does harm to human and other beneficial insects. While biological control takes advantage of interactions of biology to eradicate pest which is safe and environmental. In order to control pest and eliminate pest in a short time, impulsive releasing natural enemies can be considered a better method [1-3]. However, in the natural world, many species usually go through two distinct life stages from birth to death, immature and mature. Therefore it is practical to introduce and consider the stage-structured model [4,5].

In this paper, we propose the following impulsive controlling predator-prey system with B-D function and delayed stage-structure in prey species

$$\left\{ \begin{array}{l} \dot{x}_1(t) = rx_2(t) - re^{-d_1t}x_2(t-t) - d_1x_1(t) - \frac{ax_1(t)y(t)}{1+k_1x_1(t)+k_2y(t)}, \\ \dot{x}_2(t) = re^{-d_1t}x_2(t-t) - d_2x_2(t) - dx_2^2(t), \\ \dot{y}(t) = \frac{bx_1(t)y(t)}{1+kx_1(t)+k_2y(t)} - d_3y(t) \end{array} \right. \quad t \neq nT$$

$$\left\{ \begin{array}{l} x_1(t^+) = x_1(t), x_2(t^+) = x_2(t), y(t^+) = y(t) + p, \end{array} \right. \quad t = nT \quad (1)$$

where $x_1(t)$ and $x_2(t)$ represent the densities of the immature and mature pest at time t , respectively; $y(t)$ represents the density of the predator (natural enemies) at time t ; r is the birth rate for the immature pest and d_1 is its death rate; d_2 is the death rate of mature pest; d_3 is the death of predator; t is the maturity of the pest; d is the self-regulation constant of the prey; a denotes the predation rate of predation and b/a is the conversion rate of prey into predator, T is the period of the impulsive; p represents the release amount. All the parameters mentioned above are positive constants. The initial conditions for system (1) take the form

$$x_i(t) = j_i(t) \geq 0, i = 1, 2, y(t) = j_3(t) \geq 0, t \in [-t, 0], j_j(0) > 0, j = 1, 2, 3. x_1(0) = \int_{-t}^0 re^{d_1V} j_2(V) dV. \quad (2)$$

where $(j_1(q), j_2(q), j_3(q)) \in C_+ = C([-t, 0], R_+^3)$, $R_+^3 = \{x \in R^3 \mid x \geq 0\}$.

Preliminary

Lemma 1 [6] For the impulsive differential system

$$\begin{cases} \dot{u}(t) = -du(t), & t \neq nT \\ u(t^+) = u(t) + p, & t = nT \\ u(0^+) = u_0 \end{cases} \quad n = 1, 2, 3, \dots \quad (3)$$

where $d, p > 0$, then system (3) has a globally asymptotically stable positive periodic solution

$$\tilde{u}(t) = \frac{pe^{-d(t-nT)}}{1-e^{-dT}}, \quad nT < t \leq (n+1)T, \quad n \in N.$$

Lemma 2 [7] Consider the following equation

$$\dot{u}(t) = au(t-t) - bu(t) - cu^2(t),$$

where $a, b, c, t > 0$, $u(t) > 0$ for $t \in [-t, 0]$. We have: (i) If $a \leq b$, then $\lim_{t \rightarrow +\infty} u(t) = 0$; (ii) If $a > b$,

$$\text{then } \lim_{t \rightarrow +\infty} u(t) = \frac{a-b}{c}.$$

Lemma 3 All positive solutions of system (1) satisfying initial conditions (2) are ultimately bounded, that is, there exists a constant $M > 0$ such that $x_1(t) \leq M/b$, $x_2(t) \leq M/b$, $y(t) \leq M/a$ for all t large enough.

Existence and global attractivity of the pest-eradication periodic solution

If $x_1(t) = x_2(t) = 0$, from Lemma 1 we know that system (1) has a pest-eradication periodic solution $(0, 0, \tilde{y}(t))$, where $\tilde{y}(t) = \frac{pe^{-d_3(t-nT)}}{1-e^{-d_3T}}$, $t \in (nT, (n+1)T]$, $n \in N$.

Theorem 1 Let $(x_1(t), x_2(t), y(t))$ be any solution of system (1). If $re^{-d_1t} < d_2$ holds, then the pest-eradication periodic solution $(0, 0, \tilde{y}(t))$ of system (1) is global attractive.

Proof. With the third equation of system (1), we have

$$\begin{cases} \dot{y}(t) \geq -d_3y(t), \\ y(t^+) = y(t) + p. \end{cases}$$

Consider the auxiliary impulsive differential equation, we have

$$\begin{cases} \dot{u}(t) = -d_3u(t), & t \neq nT, \\ u(t^+) = u(t) + p, & t = nT, \\ u(0^+) = y(0^+) = u_0. \end{cases}$$

By Lemma 1 we know that $\lim_{t \rightarrow +\infty} u(t) = \tilde{u}(t) = \frac{pe^{-d_3(t-nT)}}{1-e^{-d_3T}} = \tilde{y}(t)$. With the comparison theorem, we

obtain that for all t large enough $y(t) \geq u(t) > \tilde{y}(t) - e = \frac{pe^{-d_3(t-nT)}}{1-e^{-d_3T}} - e \geq \frac{pe^{-d_3T}}{1-e^{-d_3T}} - e \stackrel{\Delta}{=} m_3$.

With the second equation of system (1), we have

$$\dot{x}_2(t) \leq re^{-d_1t}x_2(t-t) - d_2x_2(t) - dx_2^2(t).$$

Consider the following equation

$$\dot{u}(t) = re^{-d_1t}u_2(t-t) - d_2u_2(t) - bu_2^2(t).$$

From Lemma 2, we know that, if $re^{-d_1t} < d_2$, then $\lim_{t \rightarrow +\infty} u(t) = 0$. With the comparison theorem $x_2(t) \leq u(t) \rightarrow 0$, $t \rightarrow +\infty$. With the first equation of system (1), we have

$$\dot{x}_1(t) = rx_2(t) - re^{-d_1t}x_2(t-t) - d_1x_1(t) - \frac{ax_1(t)y(t)}{1+k_1x_1(t)+k_2y(t)} \quad (4)$$

Since $x_2(t) \rightarrow 0$, $t \rightarrow +\infty$, then we obtain the limit system of system (4) $\dot{x}_1(t) = -d_1x_1(t)$. Therefore $x_1(t) \rightarrow 0$, $t \rightarrow +\infty$.

For any $e_1 \in (0, d_3)$, we obtain $x_1 < \frac{e_1}{b}$ holding for all t large enough. With the third equation of system (1), we have

$$\begin{cases} \dot{y}(t) = -d_3y(t) + \frac{bx_1(t)y(t)}{1+k_1x_1(t)+k_2y(t)} \leq -d_3y(t) + e_1y(t) = -(d_3 - e_1)y(t), \\ y(t^+) = y(t) + p. \end{cases}$$

Consider the auxiliary impulsive differential equation

$$\begin{cases} \dot{u}(t) = -(d_3 - e_1)u(t), & t \neq nT, \\ u(t^+) = u(t) + p, & t = nT, \\ u(0^+) = y(0^+) = u_0. \end{cases} \quad (5)$$

By Lemma 1 we know that system (5) has a globally asymptotically stable solution

$$\tilde{u}_{e_1}(t) = \frac{pe^{-(d_3-e_1)(t-nT)}}{1-e^{-(d_3-e_1)T}}, \quad t \in [nT, (n+1)T]$$

By comparison theorem and Lemma 1, let $e_1 \rightarrow 0$, then we have $y(t) \leq u(t) < \tilde{u}_{e_1} + e_1$. Therefore we have $\lim_{t \rightarrow +\infty} y(t) = \tilde{y}(t)$ for all t large enough, which implies that $y(t) \rightarrow \tilde{y}(t)$ as $t \rightarrow +\infty$, that is, the pest-eradication periodic solution $(0, 0, \tilde{y}(t))$ of (1) is global attractive. This completes the proof.

Permanence

Definition 1 System (1) is said to be permanent if there exist positive constants m and \overline{M} such that each positive solution $(x_1(t), x_2(t), y(t))$ of system (1) satisfies $m \leq x_1(t), x_2(t), y(t) \leq \overline{M}$ for t sufficiently large enough.

Theorem 2 Assume that $re^{-d_1t} > d_2$ and $m_2 > \frac{Me^{-d_1t}}{b}$ hold, then system (1) is permanent, where M and m_2 are defined in Lemma 4 and (6), respectively.

Proof. Let $x(t) = (x_1(t), x_2(t), y(t))$ be any solution of system (1) with $x(0) > 0$, from Lemma 3 we know that $\exists M > 0$ and $x_1(t) \leq M/b, x_2(t) \leq M/b, y(t) \leq M/a$ for all t large enough.

By the proof of Theorem 1, we have $y(t) > m_3$ for all t large enough.

Next we shall find a $m_2 > 0$, such that $x_2(t) > m_2$ for all t large enough. With the second equation of system (1), we have

$$\dot{x}_2(t) = re^{-d_1t} x_2(t-t) - d_2 x_2(t) - dx_2^2(t).$$

By the Lemma 2, we have that $\lim_{t \rightarrow +\infty} x_2(t) = \frac{re^{-d_1t} - d_2}{d}$ as $t \rightarrow +\infty$, than we obtain

$x_2(t) > \frac{re^{-d_1t} - d_2}{d} - e \stackrel{\Delta}{=} m_2$ for all t large enough. With the first equation of system(1), we have

$$\dot{x}_1(t) \geq r(m_2 - e^{-d_1t} \frac{M}{b}) - (d_1 + \frac{a}{k_2})x_1(t)$$

Consider the following auxiliary equation

$$\dot{u}(t) = -\left(d_1 + \frac{a}{k_2}\right)u(t) + r\left(m_2 - \frac{Me^{-d_1t}}{b}\right).$$

When $m_2 > \frac{Me^{-d_1t}}{b}$ holds, we obtain $\lim_{t \rightarrow +\infty} u(t) = \frac{rk_2}{d_1k_2 + a} \left(m_2 - \frac{Me^{-d_1t}}{b}\right)$. By comparison theorem, we have

$$x_1(t) \geq u(t) > \frac{rk_2}{d_1k_2 + a} \left(m_2 - \frac{Me^{-d_1t}}{b}\right) - e \stackrel{\Delta}{=} m_1. \quad (6)$$

Define $m' = \min\{m_1, m_2, m_3\}$, hence $x_1(t) > m', x_2(t) > m', y(t) > m'$ for all t large enough. Therefore system (1) is permanent under the conditions of Theorem 2. This completes the proof.

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