Hybrid Synchronization of Fractional Order Chaotic Systems

Xue-feng Liang, Li-xin Yang[a]

School of Mathematics and statistics, Tianshui Normal University, Tianshui, China
[a] y0931l@163.com

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Abstract. This paper studies the modified hybrid projective synchronization of fractional order chaotic systems, which generalizes many various synchronization forms. We firstly present this kind synchronization that the response and drive systems have scaling factors at the same time. In addition, based on the stability analysis of fractional order systems and adaptive control technique, a suitable controller and parameter update law can be designed to achieve the modified hybrid projective synchronization for uncertain fractional-order chaotic systems.

Introduction

Recently, synchronization of chaotic fractional-order differential systems have received a significant attention among scientists from various different fields [1]. On one hand, inspired by the pioneering work in 1990 [2], synchronization has attracted increasing attention due to its potential applications in secure communication and signal processing etc, on the other hand, fractional order derivatives provide an excellent instrument for the description of memory properties of various materials and processes. At the same time, it was proved that many fractional-order systems behave chaotically, such as fractional-order Lü system [3], fractional-order Chua's circuit [4] and fractional-order unified system [5]. Among all kinds of chaos synchronization, projective synchronization is the most noticeable one because of its proportional feature [6]. How to effectively realize the modified hybrid projective of two fractional-order chaotic systems with unknown parameters is an important problem for both the theoretical research and practical applications.

Adaptive modified hybrid projective synchronization

Fractional-order Chen system is described as follows：

$$\begin{cases} D_*^q x_1 = a(x_2 - x_1) \\ D_*^q x_2 = (c-a)x_1 - x_1x_3 + cx_2 \\ D_*^q x_3 = x_1x_2 - bx_3, \end{cases} \tag{1}$$

When parameters are set by default as $a = 35, b = 3, c = 28$, and $q = 0.9$, system (1) exhibits chaotic behaviors as shown in Fig.1(a). The fractional-order Lü system is described by the following state equation:

$$\begin{cases} D_*^q y_1 = d(y_2 - y_1) \\ D_*^q y_2 = -y_1y_3 + hy_2 \\ D_*^q y_3 = y_1y_2 - fy_3, \end{cases} \tag{2}$$

Fig.1(b) displays the chaotic attractor of the fractional-order Lü chaotic system.

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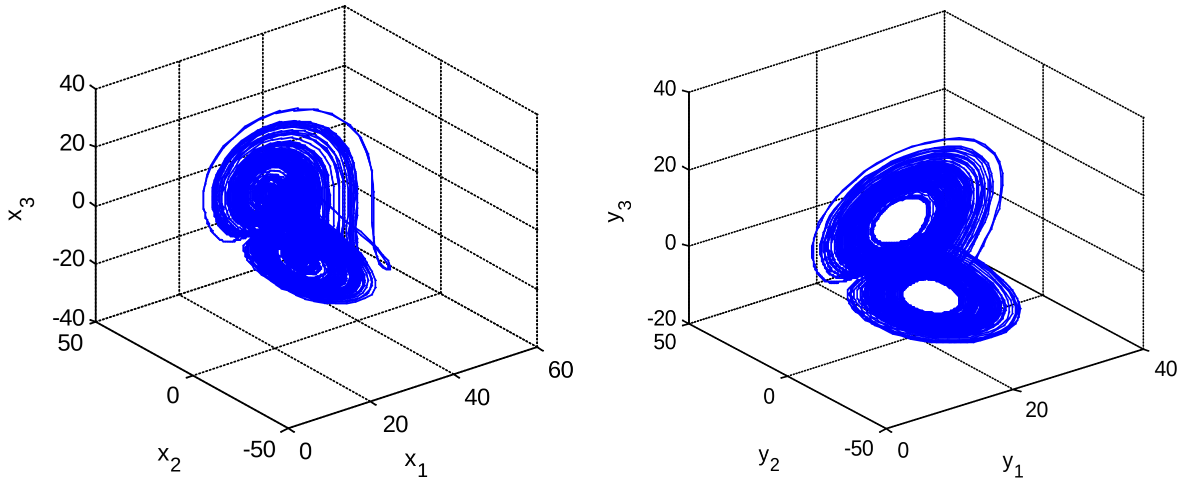

Fig.1 Phase portrait of system (a) Chen system with $q = 0.9$. (b) Lu system with $q = 0.9$.

We assume that fractional-order chaotic Chen system with unknown parameters is the drive system and the response Lu system is given by:

$$\begin{cases} D_*^q y_1(t) = d(y_2 - y_1) + u_1 \\ D_*^q y_2(t) = -y_1 y_3 + h y_2 + u_2 \\ D_*^q y_3(t) = y_1 y_2 - f y_3 + u_3, \end{cases} \tag{3}$$

Denote the state errors as $e_1 = \alpha_1 y_1 - \beta_1 x_1, e_2 = \alpha_2 y_2 - \beta_2 x_2, e_3 = \alpha_3 y_3 - \beta_3 x_3$.

It follows from (1)-(3), Then we have the following error dynamical system

$$\begin{cases} D_*^q e_1(t) = \alpha_1[d(y_2 - y_1) + u_1] - \beta_1[a(x_2 - x_1)] \\ D_*^q e_2(t) = \alpha_2[-y_1 y_3 + h y_2 + u_2] - \beta_2[(c-a)x_1 - x_1 x_3 + c x_2] \\ D_*^q e_3(t) = \alpha_3[y_1 y_2 - f y_3 + u_3] - \beta_3[x_1 x_2 - b x_3]. \end{cases} \tag{4}$$

Theorem For given constant scaling matrices $\alpha_i (i = 1,2,3), \beta_i (i = 1,2,3)$, modified hybrid projective synchronization between system (1) and system (3) will occur by following controller:

$$\begin{cases} u_1 = 1/\alpha_1(\beta_1 x_1 - \alpha_1 y_1 + \beta_1(\tilde{a}(x_2 - x_1)) - \tilde{d}(y_2 - y_1)) \\ u_2 = 1/\alpha_2(\beta_2 x_2 - \alpha_2 y_2 + \beta_2((\tilde{c} - \tilde{a})x_1 - x_1 x_3 + \tilde{c} x_2) + y_1 y_3 - \tilde{h} y_2 \\ u_3 = 1/\alpha_3(\beta_3 x_3 - \alpha_3 y_3 + \beta_3(x_1 x_2 - \tilde{b} x_3) + \tilde{f} y_3 - y_1 y_2, \end{cases} \tag{5}$$

and all the parameter update rule for unknown parameters a, b, c, d, h, f

$$\begin{cases} D_*^q \tilde{a} = \beta_1(x_2 - x_1)e_1 + \beta_1 x_1 e_2 \\ D_*^q \tilde{b} = \beta_3 x_3 e_3 \\ D_*^q \tilde{c} = \beta_2(x_1 + x_2)e_2 \\ D_*^q \tilde{d} = \alpha_1(y_1 - y_2)e_1 \\ D_*^q \tilde{h} = -\alpha_2 y_2 e_2 \\ D_*^q \tilde{f} = \alpha_3 y_3 e_3, \end{cases} \tag{6}$$

Proof: From Eqs.(5)-(7), we can obtain the error dynamical system as below

$$\begin{cases} D_*^q e_1 = \alpha_1(y_2 - y_1)e_d - \beta_1(x_2 - x_1)e_a - e_1 \\ D_*^q e_2 = \alpha_2 y_2 e_h - e_2 + \beta_2 e_c(x_2 + x_1) - \beta_2 x_1 e_a \\ D_*^q e_3 = \alpha_3 y_3 e_f - \beta_3 x_3 e_b - e_3 \end{cases} \tag{7}$$

Combining (6) with (7), one has

$$\begin{aligned} J = {} & e_1 D_*^q e_1 + e_2 D_*^q e_2 + e_3 D_*^q e_3 + e_a D_*^q e_a + e_b D_*^q e_b \\ & + e_c D_*^q e_c + e_d D_*^q e_d + e_h D_*^q e_h + e_f D_*^q e_f. \end{aligned} \tag{8}$$

From Eq.(5-8), we can get that

$$
\begin{aligned}
& e_1 D_*^q e_1 + e_2 D_*^q e_2 + e_3 D_*^q e_3 + e_a D_*^q e_a + e_b D_*^q e_b + e_c D_*^q e_c + e_d D_*^q e_d + e_h D_*^q e_h + e_f D_*^q e_f \\
& = e_1[\alpha_1(y_2 - y_1)e_d - \beta_1(x_2 - x_1)e_a - e_1] + e_2[\alpha_2 y_2 e_h - e_2 + \beta_2 e_c(x_2 + x_1) - \beta_2 x_1 e_a] + e_3[\alpha_3 y_3 e_f - \beta_3 x_3 e_b - e_3] \\
& + e_a[\beta_1(x_2 - x_1)e_1 + \beta_1 x_1 e_2] + e_b[\beta_3 x_3 e_3] + e_c[\beta_2(x_1 + x_2)e_2] + e_d[\alpha_1(y_1 - y_2)e_1] + e_h[-\alpha_2 y_2 e_2] + e_f[\alpha_3 y_3 e_3] \\
& = -e_1^2 - e_2^2 - e_3^2 \le 0.
\end{aligned}
$$

Then the response system (3) can synchronize the drive system (1) globally and asymptotically.

Numerical simulations

We choose the scaling matrices $\alpha=\mathrm{diag}(1,1,1), \beta = diag(-2,-2,-2)$. The values of unknown parameters converge to $\tilde{a} = 35, \tilde{b} = 3, \tilde{c} = 28, \tilde{d} = 36, \tilde{h} = 20, \tilde{f} = 3$ is shown in Fig. 4.

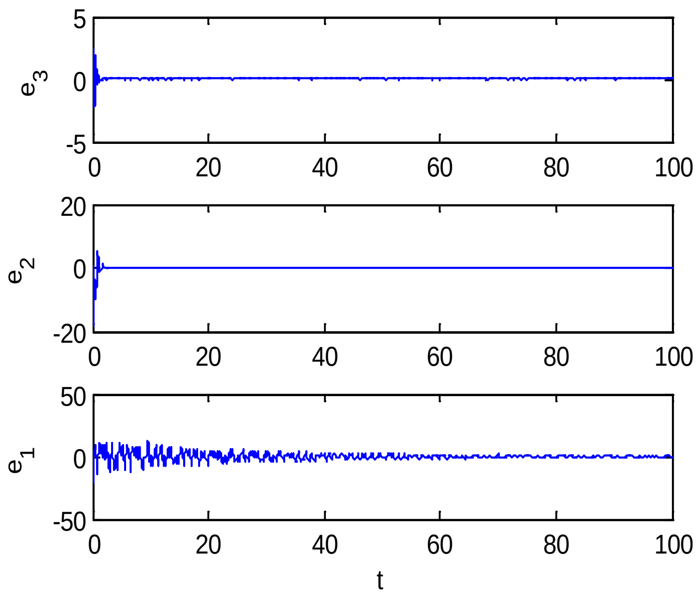

Fig.2. Errors of drive and response system with $\alpha=\mathrm{diag}(1,1,1), \beta = diag(-2,-2,-2)$

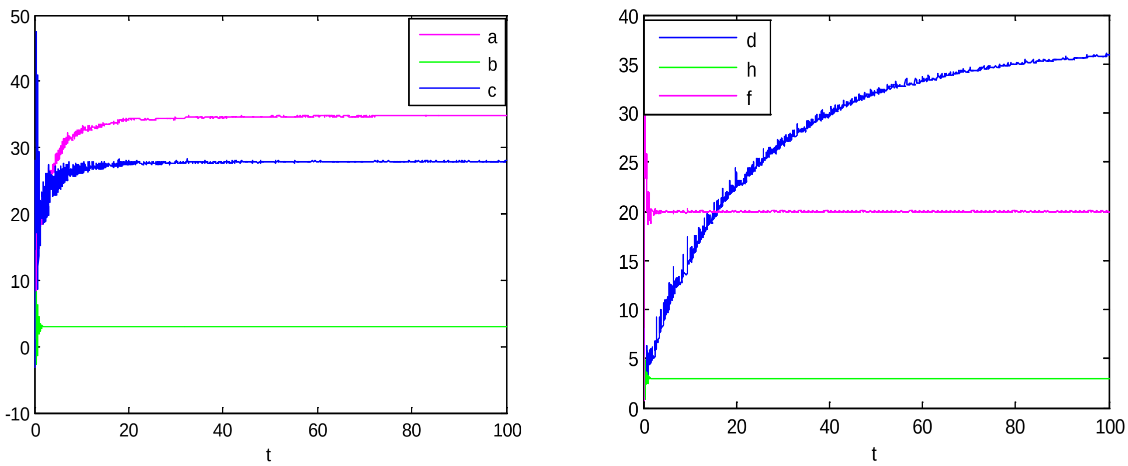

Fig.3. Identification curves of the unknown parameters

Conclusions

Based on the stability theory of fractional-order system, adaptive controller and parameter update law are designed to ensure Chen system synchronize with Lü system up to double scaling matrices. Numeric results show that the proposed scheme is analytically rigorous and practically feasible.

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