

# Prestack Synthesized Areal Shot Record Reverse-time Migration

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**Abstract.** Reverse time migration allows steep dip imaging, any lateral velocity variation and correct amplitude and phase information at the same time. However, it is time consuming and requires large hard disk memories. To shorten the running time of prestack reverse time migration, we use synthesized areal shot source and record as the substitution of shot source and record in the prestack reverse time migration. The rightness of the method is derived. A numerical example on the Salt model shows the method works well.

## Introduction

As a two way wave equation technique using the acoustic wave equation, reverse time migration does not suffer from dip limitation of one way downward continuation algorithms such as phase shift [1], split step Fourier [2], Fourier finite difference method [3]. Reverse time migration can handle not only multi-arrivals but steep dips and overturned reflections also. However, prestack reverse time migration is computationally expensive because it requires consecutive forward and backward wavefield modeling. To improve the efficiency of prestack reverse time migration, we employ the areal shot record method proposed by Berkhout [4]. In his approach the point shot records are synthesized into an areal source response prior to shot-record migration, an enormous data reduction can be obtained. The resulting areal shot record can be migrated using the same scheme as the one for conventional shot-record migration. The main advantage of areal shot-record migration is that the extrapolations need not be done for all the individual shot records, but for the areal shot record only. This makes the areal shot record approach very attractive for the processing of 3-D data. The synthesized plane wave idea is also used in other cases such as velocity analysis [5] and modeling. In this paper, the synthesized plane wave method is used in the prestack reverse time migration. The property of the solution of wave equation claims the rightness of the method. In this paper, first we give a demonstration of the rightness of the plane wave method. Next, the efficiency is analyzed. Finally, results are shown.

## Areal shot source modeling

When the density is constant, the acoustic wave equation reads

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{1}{v^2} \frac{\partial^2 p}{\partial t^2} + s, \quad (1)$$

where  $p = p(x, z, t)$  represents pressure,  $\omega$  denotes angular frequency,  $v(x, z)$  represents the velocity and  $s = s(x, z, t)$  represents the source item which equals the divergence of the body forces. Let

$$L = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - \frac{\omega^2}{v^2}, \quad (2)$$

Eq.1 can be expressed as  $s = Lp$ . When a source is excited, a serious wavefields  $p = p(x, z, t)$  are obtained. In frequency domain,

$$L = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - \frac{\omega^2}{v^2}, \quad (3)$$

Suppose the record  $r(x, t)$  is received on the surface and the source is also excited on the surface.

Then

$$r(x, t) = \sum_{i=1}^m r_i^* = \sum_{i=1}^m C_i p(x, z, i\Delta t) \quad (4)$$

where  $r_i^*$  represents a  $nt \times nx$  matrix whose items are all zeroes except those in the  $i$ th row is the same as  $r(x, t)$ ,  $C_i$  has the form:

$$C_i = \begin{bmatrix} \mathbf{0}, \mathbf{0}, \dots, \mathbf{0} \\ \mathbf{0}, \mathbf{0}, \dots, \mathbf{0} \\ \dots\dots\dots \\ \mathbf{1}, \mathbf{0}, \dots, \mathbf{0} \\ \dots\dots\dots \\ \mathbf{0}, \mathbf{0}, \dots, \mathbf{0} \end{bmatrix}, \quad (5)$$

i.e., the only item 1 is located at the  $i$ th row and the first column. The whole process of recording data is expressed as

$$s = Lp \rightarrow r \quad (6)$$

The form in the frequency domain is

$$S = LP \rightarrow R \quad (7)$$

Assume there are  $nshot$  records  $R_i = R_i(x, \omega)$  and sources  $S_i = S_i(x, z = 0, \omega)$  satisfying  $S_i \rightarrow R_i \quad i = 1, 2, \dots, nshot$ , using  $j$  to denote  $\sqrt{-1}$ , it is easy to prove  $L$  has the following properties:

(1) Linearity

$$\sum_{i=1}^{nshot} a_i S_i(x, \omega) = L \left( \sum_{i=1}^{nshot} a_i P_i(x, z, \omega) \right) \rightarrow \sum_{i=1}^{nshot} a_i R_i(x, \omega), \quad \alpha_k \text{ could be any real or complex}$$

numbers.

(2) Shifting

$$S_k(x, z = 0, \omega) \exp(\pm j\omega t_1) = L \left[ P_k(x, z, \omega) \exp(\pm j\omega t_1) \right] \rightarrow R_k(x, \omega) \exp(\pm j\omega t_1), \quad t_1 \text{ could be any real numbers denoting delayed time.}$$

On the surface, the sources and the recording data are expressed as:

$$S_{syn}(x, z = 0, \omega) = \sum_{i=1}^{nshot} \exp(-j\omega t_i) S_i(x, \omega) \quad (8)$$

$$R_{syn}(x, z = 0, \omega) = \sum_{i=1}^{nshot} \exp(-j\omega t_i) R_i(x, \omega) \quad (9)$$

where  $t_i = (x_i - x_c) \frac{\sin \alpha}{v_0}$  is the delayed time for the source located at  $x_i$ ,  $\alpha$  is the incident angle of the

source wavefield along  $x$  axis,  $x_c$  represents the lateral positions with zero time shift,  $\frac{\sin \alpha}{v_0} = p_r$  is

often referred as the ray parameter,  $v_0$  is the velocity of the model on the surface. With the property of  $L$ ,

$$\begin{aligned} S_{syn}(x, z = 0, \omega) &= \sum_{i=1}^{nshot} \exp(-j\omega t_i) S_i(x, z=0, \omega) \\ &= \sum_{i=1}^{nshot} \exp(-j\omega t_i) L P_i(x, z, \omega) \\ &= L \sum_{i=1}^{nshot} \exp(-j\omega t_i) P_i(x, z, \omega) \rightarrow \sum_{i=1}^{nshot} \exp(-j\omega t_i) R_i(x, \omega) = R_{syn}(x, \omega), \end{aligned} \quad (10)$$

It means that the record data  $R_{syn}(x, z = 0, \omega)$  can be obtained by exciting the synthesized areal shot source. Then the synthesized areal shot records and sources can be used in reverse time migration just as a single shot record and source. A single shot-source  $S_i(x, z = 0, \omega)$  has the following matrix form:

$$S_i(x, z = 0, \omega) = \begin{bmatrix} 0, 0, \dots, wavelet_i(\omega_1), \dots, 0 \\ 0, 0, \dots, wavelet_i(\omega_2), \dots, 0 \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ 0, 0, \dots, wavelet_i(\omega_{nt}), \dots, 0 \end{bmatrix}, \quad (11)$$

where a source wavelet is located at the  $i$ th column. So a synthesized areal shot source matrix has the form:

$$S_{syn}(x, z = 0, \omega) = \begin{bmatrix} wavelet_1(\omega_1), wavelet_2(\omega_1), \dots, wavelet_i(\omega_1), \dots, wavelet_{nx}(\omega_1) \\ wavelet_1(\omega_2), wavelet_2(\omega_2), \dots, wavelet_i(\omega_2), \dots, wavelet_{nx}(\omega_2) \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ wavelet_1(\omega_{nt}), wavelet_2(\omega_{nt}), \dots, wavelet_i(\omega_{nt}), \dots, wavelet_{nt}(\omega_{nt}) \end{bmatrix}. \quad (12)$$

To avoid boundary reflections, an absorbing boundary as a 20-point weighting function [6] is applied to  $P$  on the bottom and lateral edges of the spatial grid.

### Numerical example

We have tested the methods for the synthesized areal shot-record reverse time migration using the 2-D SEG/EAGE salt model (Fig.1). Depth and distance are measured in kilo feet. The plane waves are synthesized on the surface and the incidence angles of the waves are from  $-45^\circ$  to  $+45^\circ$  with interval  $1^\circ$ .

The migrated sections for the salt model using the acoustic wave equation are depicted in Fig.2a-b with crosscorrelation and the wavefield decomposition imaging condition, respectively. Fig.2c-d are the results after a performance of Laplace filtering on the migration results shown in Fig.2a-b.

### Summary

The synthesized areal shot source and record are employed in prestack reverse time migration. The feasibility of the method has been derived and tested on the 2-D Salt model. The numerical example shows the method works well.

### Acknowledgements

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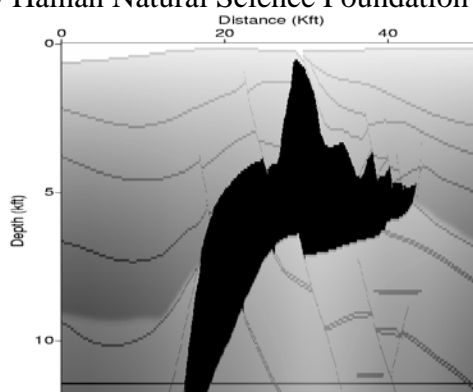


Fig.1 2-D Salt model.

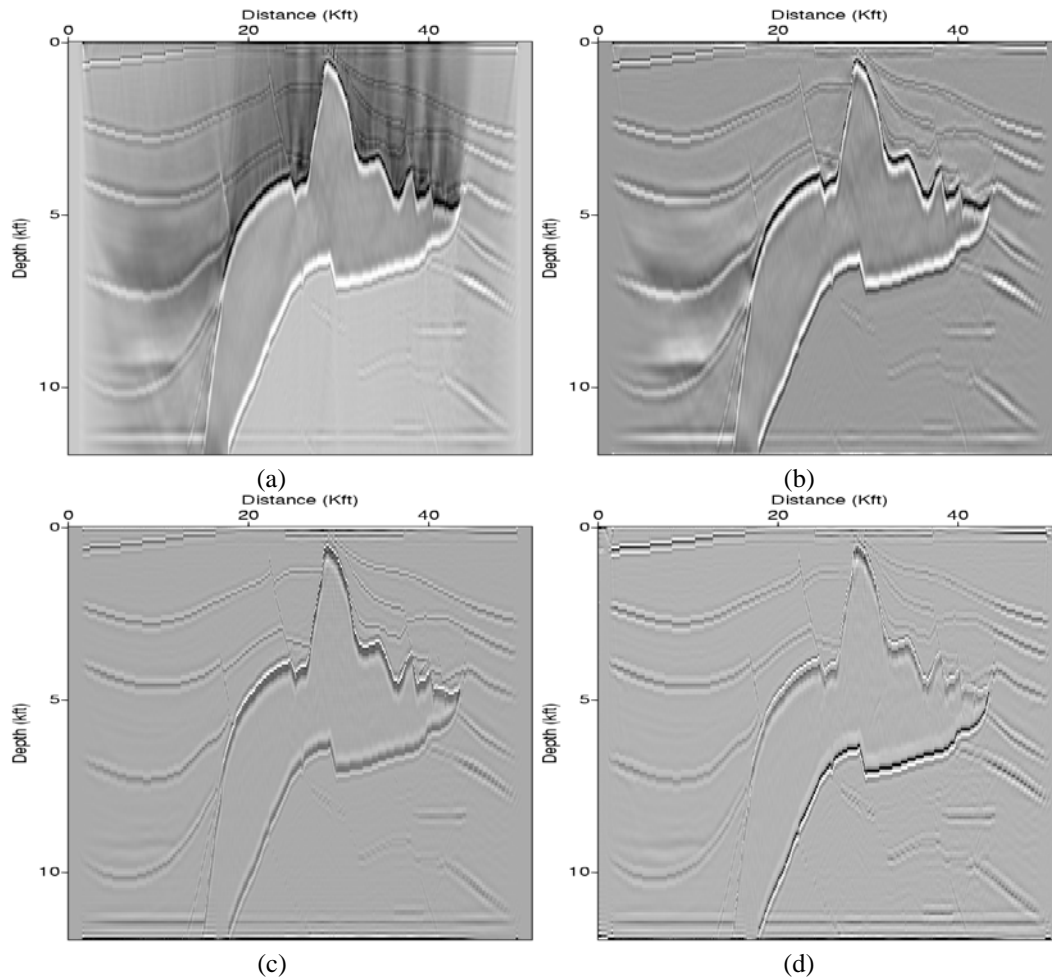


Fig. 2 Migration results.

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