

## Multi-axial Fatigue Life Prediction of Bellow Expansion Joint

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**Abstract.** A new modeling method of wall thickness is put forward and the multi-axial fatigue life of bellows is acquired. The finite element mode is built considering the varied bellow's wall thickness. The test results show the mode is reasonable and the precise strain is got. Based on the result of the upwards, multi-axial fatigue mode is apply in to life calculation. The results show that the critical-plain model's results are consistent very well with test results. The available of modeling method is validated by the results and the multi-axial fatigue method is formed.

### Introduction

Aerospace engine provide the power for spacecraft. The system is very complex and the working conditions are not very well. Fatigue often occurs when it's working. Metal bellows are widely used on aerospace engine. Bellows are important components especially in the important parts of the engine valve. Also in the pipeline Transporting liquid hydrogen and high temperature gas, bellows are the main part. The working conditions of bellows are often high pressure, high temperature and extreme physical environment. There is a greater probability of failure. According to statistics, space launch mission have failed more than 54 due to an accident caused by the bellow from 1990 to 2002 [1, 2].

At present, the bellows fatigue life studies generally use equivalent strain based on Manson-coffin formula. The author of Document [3] got the stress and strain results based on the finite element method and make a fatigue life prediction analysis using Manson-coffin formula with mean stress correction. The author of Document [4] calculate the fatigue by uniaxial SWT correction Manson-coffin formula using MSC. Fatigue.

Bellows endure complex loads when it is working, such as pressure the axial force, lateral force etc. Under the load, the bellows produce expansion, stretching, twisting, bending deformation which makes the bellows in a complex multiaxial stress state. It is necessary to carry out multiaxial stress fatigue life prediction of the bellows. But a lot of difficulties make it not very easy, such as uneven thickness characteristics, large deformation and the contact. The existing computing are mostly concentrated in stiffness, strength, stability studies [5]. The fatigue life study is relatively small, especially considering the wall thinning and multiaxial stress state prediction is rarely reported.

Considering the non-uniform wall thickness characteristics of bellows, a new method of modeling method of the bellows was proposed and the experiment of stiffness gives a good validation. Taking into account the multiaxial stress stat of bellows, four different multiaxial fatigue models were used to predict the fatigue life. At last, the calculate result make a good consistent with the experimental results. A system method for multiaxial fatigue life prediction of bellows was formed.

## A New Modeling Method

The reasonable finite element model is foundation for accurate life prediction of bellows. Under the hydraulic forming process of bellows, the actual size will result in a deviation to the design. The thickness of the bellows will become not uniform. If we make a finite element analysis according to the thickness of design model, great differences will exist in the experimental results and calculate results. Based on the design size of the bellows, considering the impact of the actual process to its thickness, a finite element model of the bellows was establish by exponentially thinning manner.

**The Design Size of Bellow Structure.** The research object is two bellows in the gas valve, one install in the out and the other in inner. The bellows are double layer unreinforced U-shaped bellows. The geometric model is shown in Fig. 1. The specific parameters were shown in Table 1. Because of its symmetry, we take 1/24 as the calculation model for finite element analysis. The model is shown in Fig. 2.

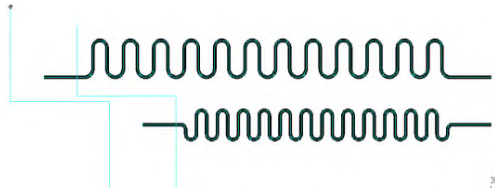


Fig. 1 Axisymmetric model two bellows.

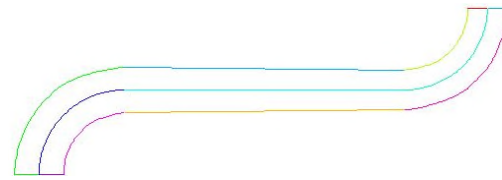


Fig. 2 1/24 model of outer bellows.

Table 1. Structure parameters of bellows.

	material	exterior dia.	interior dia.	Wave length	wall thickness	layer	wave number	Efficiency length
out	GH4169	33.5	25.5	3.2	0.16	2	12	47
inner	GH4169	19.5	13	2.2	0.16	2	13	36.6

(long measure is mm)

The stiffness of bellows with the design size is got by finite element analysis. The simulation results have great different with the experimental results, such as the results shown in Table 2. The author of Document [6] also have to verify the impact of wall thickness. It is necessary to study the wall thickness method and its effect to fatigue.

Table 2. Stiffness values of simulation and experiment.

Stiffness	Experiment (N/mm)	Simulation (N/mm)
out	70	91.8
inner	65	100

**Method of Thickness Reduction** Although lots of wall thickness exists at the present study, the method is generally used in a simplified form. Several standard widely used today is not suitable for the bellows.

American Society of Mechanical Engineers (ASME) standards:

$$t = t_0 \left[ \sqrt{\frac{d}{D}} - 0.04 \right] \quad (1)$$

where, d is the inner diameter of the bellows, D is the outer diameter of the bellows;

American Expansion Joint Manufacturers Association (EJMA) criteria:

$$t = t_0 \sqrt{d_0 / d_p} \quad (2)$$

Japanese Industrial Standards JIS standard:

$$t = t_0 (d_0 / d_p) \quad (3)$$

where,  $d_0$  is the inner diameter of the bellows,  $d$  is the diameter of the measurement point;

The thickness calculated according to ASME formula is the average thickness. The actual thickness should be gradually thinning trend from trough to peak. JIS and EJMA formula can effectively simulate the actual production. A more general form is  $t = (d_0 / d_p)^x$ , where  $x$  is between 0 and 1 according to the actual process situation.

According to the theory above, a gradually thinning method were proposed. In order to ensure the positional relationship of two bellows after thinning, the midline of two layers unchanged. By setting the thinning rate to change the inside and outside layer thickness. The wall thickness is associated with the radius.

Index thinning methods: The new method of thickness reduction is as follows:

$$t = (d_0 / d_p)^x \quad (4)$$

**Finite Element Model.** According to the thickness reduction and modeling techniques, the finite element model without thickness reduction is built first. Then the thickness is adjusted by directly change the nodal coordinates, which avoid modeling complexity and can get high-quality element. The finite element model is shown in Fig. 3.

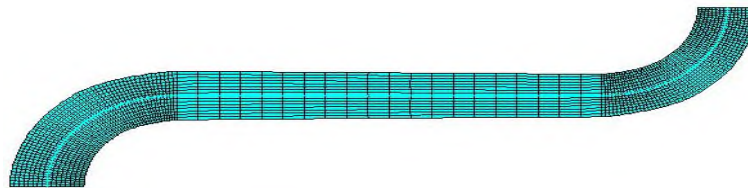


Fig. 3 Finite element model.

The stiffness calculated by four method of thickness reduction are shown in Table 3.

Table 3. Stiffness results of each thinning method.

Stiffness (N/mm)	experiment	ASME	EJMA	JIS	Index		
					x=0.8	x=0.7	x=0.6
out	70	65.54	75.3	61.4	66.3	68.1	71.01
inner	65	58.48	69	54.3	58.7	62.5	66.2

According to the above table, comparing the calculation results with the experimental results, the result calculate by ASME and JIS standard formula is small and EJMA standard formula is large.

When the thinning factor  $x = 0.6$ , the is error 2% between the exponential thinning method and

experimental values with a higher degree of agreement. The results showed that the index thinning method can well simulate the bellows thinning phenomenon caused by the process. The analysis results meet the required accuracy very well. In the subsequent fatigue life calculation, all models adopt the index thinning method.

### Fatigue Life Prediction and Analysis

The finite element models were got by the exponential thinning method. The stress/strain of the dangerous point was get under the actual load conditions. Fatigue life can be obtained by combine the material fatigue characteristics, multiaxial fatigue prediction models. The load conditions and material parameters, the multiaxial fatigue life prediction model and analysis are as follows.

**Load Condition and Material Characteristic.** The Bellows assembly structure is used for two different gas valve. The status of their work in different situations can be grouped into eight different conditions. The exact parameters of stretching, compression displacement and pressure was shown in Table 4. The out bellows only under interior pressure and the inner bellows only under external pressure.

Table 4. Load condition of bellows.

Outer tube					Inner tube			
displacement			pressure		displacement		pressure	
stretch	compression		stretch	compression	stretch	compression	stretching	stretch
1	3.5	3.5	0	7	3.5	3.5	7.2	7.2
2	1	3.8	0	7	1	3.8	7.2	7.2
3	3.5	3.5	0	7	3.5	3.5	6.5	5
4	1	3.8	0	7	1	3.8	5	6.5
5	3.5	3.5	0	6.5	3.5	3.5	2.75	2.75
6	1	3.8	0	6.5	1	3.8	2.75	2.75
7	3.5	3.5	0	6.5	3.5	3.5	2.75	0.1
8	1	3.8	0	6.5	1	3.8	0.1	2.75

(The displacement unit is mm, pressure is measured in MPa)

The materials of bellows are GH4169 alloy, which has a good overall performance at low and high temperatures. The bilinear kinematic hardening guidelines are used to simulate the stress-strain curve of GH4169 in ANSYS. The stress-strain curve is shown below.

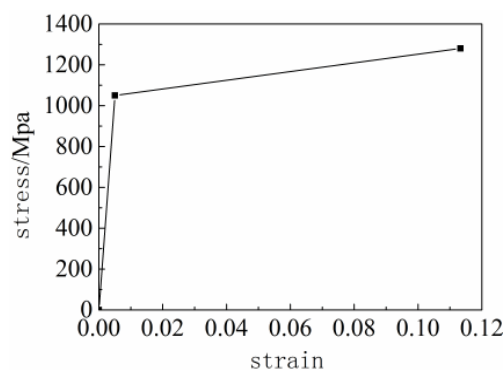


Fig. 4 Stress-strain curve of material GH4169.

**Damage Parameters and Life Predict Equation** How to choose the stress / strain component participate in fatigue analysis? What is the relationship between the fatigue life and stress / strain component? These issues need to be resolved by multiaxial fatigue model. Four models are described in detail, such as maximum principal strain, maximum shear strain model, critical plane model, critical plane correction models.

**The maximum principal strain model.** The maximum principal strain method is equivalent strain method of life prediction. This method considers the maximum principal strain is the main parameter to measure the material damage and fatigue life.

$$\frac{\Delta \varepsilon_{1 \max}}{2} = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c \quad (5)$$

where,  $\Delta \varepsilon_{1 \max}$  is the maximum principal strain range at the danger point,  $\sigma'_f$  is the fatigue strength coefficient,  $\varepsilon'_f$  is the fatigue ductility coefficient,  $b$  is the fatigue strength index,  $c$  is the fatigue ductility exponent,  $N_f$  is the fatigue life,  $E$  is the elastic modulus.

**The maximum shear strain model [7].** The maximum shear strain model considered the maximum shear strain is the main damage parameter, which determines the fatigue life.

$$\frac{\Delta \gamma_{\max}}{2} = \frac{(1+\nu_e)\sigma'_f}{E} (2N_f)^b + (1+\nu_p)\varepsilon'_f (2N_f)^c \quad (6)$$

where,  $\Delta \gamma_{\max} = \Delta(\varepsilon_1 - \varepsilon_3)$ , The elastic modulus  $\nu_e$  take the value 0.3,  $\nu_p$  value is 0.5, the life prediction formula as follows

$$\frac{\Delta \gamma_{\max}}{2} = 1.3 \frac{\sigma'_f}{E} (2N_f)^b + 1.5 \varepsilon'_f (2N_f)^c \quad (7)$$

**Critical plane model.** Early crack is formed along the direction of the plane of maximum shear strain, and then expand approximately along the direction of perpendicularity the plane. Usually critical plane is defined maximum shear strain plane. The damage parameters are mix with maximum shear strain and maximum normal strain of the critical plane. We obtain the following formula for life prediction

$$\sqrt{\Delta \varepsilon_n^2 + \frac{1}{3} \Delta \gamma_{\max}^2} = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c \quad (8)$$

In the above formula,  $\Delta \varepsilon_n = \frac{\Delta(\varepsilon_1 + \varepsilon_3)}{2}$  is the normal strain range at the vertical direction of maximum shear plane,  $\Delta \gamma_{\max} = \Delta(\varepsilon_1 - \varepsilon_3)$  is the maximum shear strain range.

**Critical plane correction model.** The author of document [8] proposed a new damage parameter on the basis of critical plane model:

$$\varepsilon_{eq}^{cr} = (k \varepsilon_n^2 + \gamma_{\max}^2 / 3)^{1/2} \quad (9)$$

Combine the Manson-Coffin equation, the life prediction model as follows:

$$\Delta \varepsilon_n^2 + \Delta \gamma_{\max}^2 / 3)^{1/2} = (1.3 + 0.7k) \frac{\sigma'_f}{E} (2N_f)^b + (1.5 + 0.5k) \varepsilon'_f (2N_f)^c \quad (10)$$

where, k is the influence factor of normal strain,  $0 < k < 1$  [9].

**Fatigue Life Calculation and Results Analysis.** The stress/strain of the dangerous point is the basis of fatigue life prediction. The state of stress and strain at the peaks and valleys is a three-dimensional state. The equivalent rule is often used in the study of the fatigue properties. Because of the stress and strain components is tensor, a single component can not characterize the complex three-dimensional stress-strain state. According to the literature [10], this paper use Von Mises stress to judge by the dangerous points.

Based on the calculation of the finite element analysis, for the outer bellows, the inner surface of the peak (maximum curvature of the structure, but also the thinnest wall thickness) has the highest stress level. When the displacement reaches the maximum, the compression stress has exceeded the material yield limit conditions into plastic state. For the inner bellows, as the internal pressure, the maximum stress and strain may occur in tension and compression conditions. The danger point appears in the surface of the peaks and valleys of the outer surface. For each condition of both bellows, the inner surface of the peaks and the outer surface of the troughs are the dangerous points.

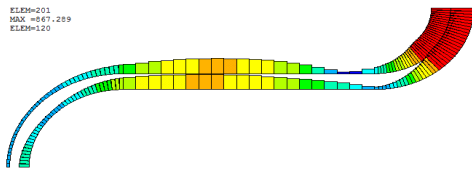


Fig. 5 Equivalent stress of compression.

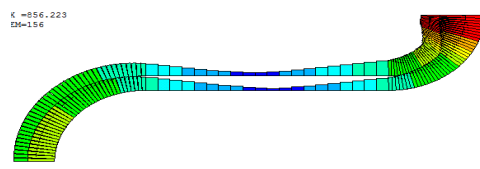


Fig. 6 Equivalent stress of stretch.

The calculated stress and strain result under load condition is shown in Table 5. The fatigue life predict by different model is shown in Table 6.

Table 5. Results of strain.

Outer tube				Inner tube				
maximum principal strain range	maximum shear strain range	critical plane strain parameter	critical plane shear strain parameter	maximum principal strain range	maximum shear strain range	critical plane strain parameter	critical plane shear strain parameter	
1	4.9	7.67	2.13	15.35	5.3	7.3	3.3	14.6
2	4.86	6.91	2.81	13.82	3.65	5.65	1.65	11.3
3	4.9	7.67	2.13	15.35	4.95	7.35	2.55	14.7
4	4.86	6.91	2.81	13.82	4.05	6.25	1.85	12.5
5	4.45	6.7	2.2	13.4	3.4	4.73	2.07	9.45
6	4.6	6.43	2.76	12.87	3.55	5.44	1.66	10.9
7	4.45	6.7	2.2	13.4	3.45	5.57	1.37	11.2
8	4.6	6.43	2.76	12.87	4.05	6.18	1.92	12.4

Table 6. Result of the fatigue life.

	Outer tube				Inner tube			
	maximum principal strain model	maximum shear strain model	critical plane model	critical plane correction model	maximum principal strain model	maximum shear strain model	critical plane model	critical plane correction model
1	2298	1019	154	654	1482	1290	160	720
2	2412	1701	198	1033	17729	5484	486	2212
3	2298	1019	154	654	2166	1248	173	714
4	2412	1701	198	1033	7972	2950	325	1523
5	4167	2002	238	1209	12360	4300	405	2412
6	3370	2498	215	1451	22279	7054	554	3250
7	4167	2002	238	1209	28360	4898	545	2834
8	3370	2498	215	1451	7973	3092	343	1611

In this study, we carried out the fatigue life test of the bellows assembly on the seventh load conditions (Table 4). Experimental system consists of two components: the pressure stabilize system and displacement control system (Fig. 7). The load is control by the displacement. The failure criterion is the bellows leak. At last, the bellows average fatigue life obtained by multiple sets of test is 3000 times. The predict result were compared with the experimental results. It was found the maximum principal strain model predict result is too large, while the critical plane models predict result is small. Therefore, the two models are not suit for the fatigue life prediction of bellows. In the four kinds of life prediction models, the maximum shear strain plane model and the critical correction model predictions match the experimental results very well. In addition, the literature [8] showed that the critical plane modify model predict the life of GH4169 alloy relatively reliable.

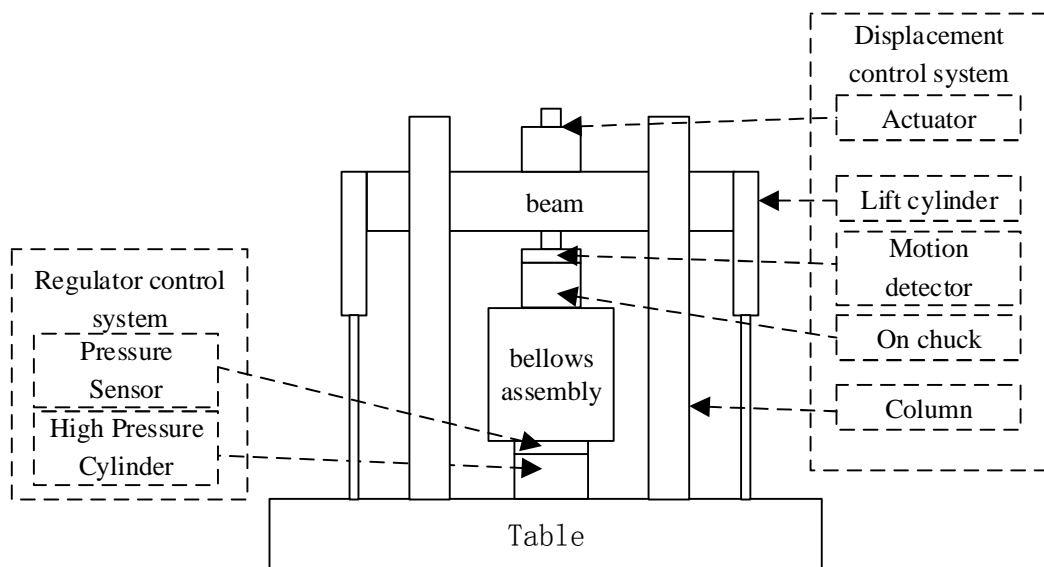


Fig. 7 Bellows assembly fatigue test.

To sum up, the outside bellows assembly most likely to fatigue failure at load condition one, two and the inner is one and three. The best critical correction models have the best applicability of fatigue life prediction, followed by the maximum shear strain model.

## Conclusion

Considering the non-uniform wall thickness of the bellows, precision finite element model was built. The multiaxial fatigue life of bellows is calculate by the maximum principal strain model, maximum shear strain model, the critical plane model, critical plane correction model. The main conclusions are: (1) A new modeling method was proposed. This modeling method combines the features of an international standard formulas ASME, JIS, EJMA. The prediction stiffness was compared with the experimental value. (2) The bellows dangerous point determined by the Von Mises stress is the inner surface of the peaks and the outer surface of troughs. (3) A system multiaxial fatigue life prediction method of bellows is formed. The outer bellows fatigue failure is most likely to occur at load condition one, two and one, three for inner bellows. The critical plane correction models have the best applicability in bellows multiaxial fatigue life prediction.

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