Research of the Optimal Shape of Bathtub

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Keywords: Optimal Shape of Bathtub

Abstract: We take heat transfer between water and air and heat dissipation capacity of bathtubs into consideration and discuss the influence of bathtub shapes on temperature. We keep the temperature of bathtub in 39 degrees Celsius.We assume that the bathtub's capacity is the same and try to find the best shape of the bathtub. After calculating, we discover cylindrical bathtub do better on heat preservation.

Introduction

There is heat convection between water and bathtub wall. Due to the fact that the bathroom temperature is generally lower than the water temperature in the bathtub. The heat will dissipate through bath wall. The ability of heat dissipation is related with material and shape of the bathtub. We assume that bathtubs' wall is vertical, the capacity of bathtub is the same and the interface between water and air is the same.

Problem analysis

There are many factors influence water temperature in the bathtub, mainly including:

The liquidity of water: including the heat conduction and diffusion, the geometry of a water, and the location of overflow drain, etc.

The heat exchange at the water surface:

- *Convection:* Water and air heat transfer between two different media mainly by convection
- *Heat conduction:* If the air temperature and water temperature is different, energy exchange can occur through heat conduction.
- *Evaporation:* On the border of water and air, when some water molecules have enough energy, they will escape to the air.

Water and energy exchange in the bathtub: the heat exchange of bathtub with water is by solid heat conduction.

In the bathtub, water temperature is determined by heat exchange between the interior and exterior parts of water. The heat exchange between the water and exterior environment is determined by the temperature brought by inflowing hot water and that taken away by overflow.

The heat exchange in the interior of water including:

- The water vertical flowing caused heat exchange
- The heat exchange caused by water density convection

Besides we also need to consider the shape and volume of the tub.

Basic Assumption

- The initial air temperature and humidity are fixed value in the bathroom.
- The bathroom is an independent system.
- Per unit time, the addition of hot water is constant.
- The temperature of the water flowing into bathtub is 45℃.
- Initial water temperature in the bathtub is 39℃.
- Hot water is supplied through bath all.
- Overfall gap won't be blocked up.

- At first the bathtub is filled with water

The Optimal Shape of Bathtub

The changes of water quality throughout the system are present as following:

$$\begin{cases} m_1 = q_{v1}\rho_w \\ m_2 = q_{v2}\rho_w \\ m_1 - m_2 = M \\ M = h_m(\rho_w - \rho_\infty)A \end{cases}$$

Where ρ_w represents the density of water, m_1 represents mass of inflow water per unit time, m_2 represents mass of overflow water per unit time, q_{v1} represents the flow of inflow water per unit time, and q_{v2} represents the flow of overflow water per unit time.

According to the heat convection equation [2], we get the heat loss of water through the bathtub as follows:

$$\begin{cases} S_1 = Hl \\ Q_3 = h(T_w - T_\infty)S_1 \end{cases}$$

Where S_1 represents the area of the bathtub walls, H represents the height of bathtub, l represents the upper edge length of bathtub, Q_3 represents the heat loss of water through the bath.

Because the bathroom is an independent system, we can keep the heat of whole system constant.

$$c\rho_w q_{v1}T_0 - c\rho_w q_{v2}T_w = Mr + h_m(T_w - T_\infty)A + h(T_w - T_\infty)S_1$$

Where T_0 represents the temperature of hot water, T_w represents the temperature of water in bathtub, T_∞ represents the temperature of the bathroom.

Now, the changes of temperature of water in bathtub can be calculated using the following parameters.

Table 1 Parameters

Physical Quantity	Value	Unit
h_m	2.2×10^{-5}	$W/(m^2 \cdot K)$
ρ_w	1000	kg/m^3
A	1.36	m^2
T_w	312	K
T_∞	297	K
c	4200	$J/(kg \cdot K)$
T_0	318	K
λ	1.5	$W/(m \cdot K)$
δ	0.1	m
H	0.6	m

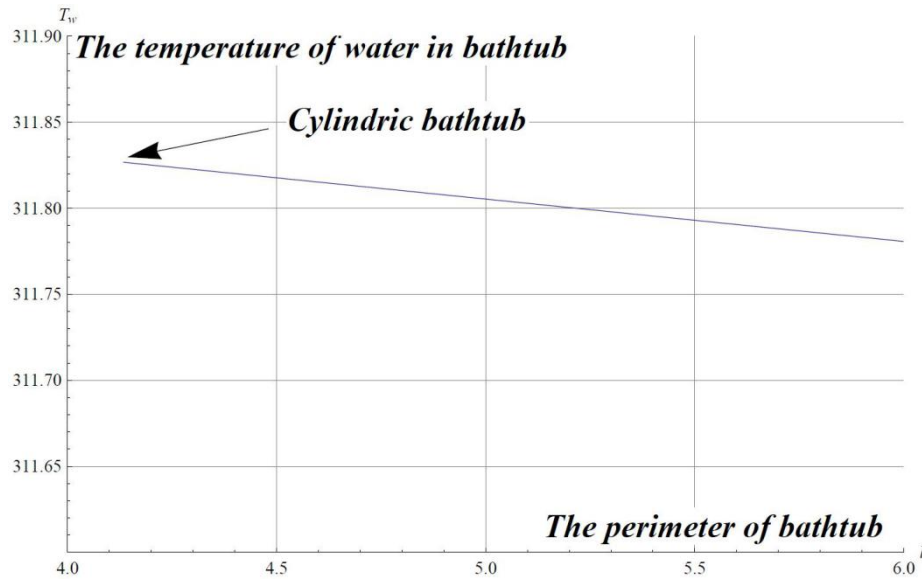The calculation results are presented as follows:

Fig.1 The relation between water temperature and perimeter of bathtub

The relation between water temperature and shape of bathtub is shown in the chart. We assume that the thickness of bathtub wall and the basal area of bathtub are the same. When water area which equal to the floor space of bathtub is determined, changes of temperature relate to shape of bathtub. According to the result, we come to the conclusion that the heat loss of cylindrical bathtubs is minimal.

Summary

Through the analysis of the water heat distribution in the bathtub affected by different factors.We discuss the influence of bathtub shapes on temperature and find the   Optimal Shape of Bathtub.For the best thermal insulation effect, we make the bathtub shape into a cylinder. This makes the size of bathtub as large as possible and contact area minimum to reduce heat loss furthest.

References

[1].Zhiqiang, Ni. The numerical simulation of the reservoir water temperature.

[2].Shiming, Yang and Wenquan, Tao. heat transfer theory.