# Space Debris Removal Scheme Simulation

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**Abstract.** This paper develops a discrete time-dependent model to design space debris removal scheme based on risk, cost, incomes and time. Especially, a collision model was established to consider the collision between debris. The economically attractive scheme was chosen according to the model simulation at last.

## 1. Introduction

The amount of debris in orbit around earth has been a growing concern. It is estimated that more than 500,000 pieces of space debris, also called orbital debris, are currently being tracked as potential hazards to space craft.

Space debris larger than 10 centimeters is classified as large debris, that between 10 centimeters and 1 centimeters as medium debris and that smaller than 1 centimeters as small debris. Only large debris and small debris could be observed and predicted [1]. As a result of it, only large debris and small debris removal are considered in this paper.

Currently there is about six metric tons of space debris in earth orbit and about most of that is in low-earth orbit, geosynchronous orbit and middle-high orbit where the threat of collisions continues to increase. This process can lead to an escalating cascade of more and more debris [2] and it is shown in Fig. 1:
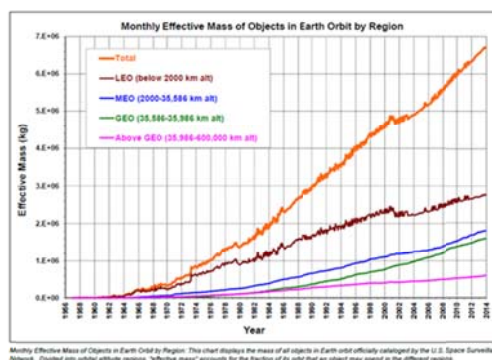


Figure .1. The Monthly Effective Mass of Objects in Earth Orbit by Region [3].

High-energy pulsed laser radiation may be the most feasible means to mitigate the threat of collision of a space station or other valuable space assets with orbital debris in the size range of 1–10 centimeters. Building and deploying a giant net on the satellite is an effective way to catch large debris. We chose this two kinds of way to simulate the space debris Removal.

Our work is to develop a time-dependent model which includes quantitative and/or qualitative estimates of costs, risks, benefits, as well as other important factors to determine the best alternative or combination of alternatives. And, using ours model, determine whether an economically attractive opportunity exists or no such opportunity is possible.

# 2. Model parameter description

## 2.1 The risk

The paper regard the risk $R'$ as a comprehensive index multiplied by several risk factors simply. And the risk $R'$ is a constant coefficient including:

***The Loss of Fire Deviation:***

Refer to the central-limit theorem, the relative position $P_1(x, y)$ between the target position of lasers and the central position of space debris subject to the two-dimensional normal distribution, that is

$$P_1 \sim N(0, 0, \sigma_1{}^2, \sigma_2{}^2, 0)$$

Taking the loss generated from the fire deviation of lasers into consideration, we get the loss function $f$ is:

$$f = k(x^2 + y^2)$$

Then we get the expectation p1 of the loss of fire deviation:

$$p_1 = E(f)$$

***The Failed Rate of Launching Satellites:***

It's well-known that the failed rate of launching satellites depends on the current level of science and technology. Then, we regard it as a constant coefficient which is denoted by $p_2$.

In conclusion, we get the risk $R'$ is:

$$R' = p_1 p_2$$

## 2.2 The income

Giving a definition of the incomes that the incomes is multiplied by the number of the eliminated space debris and the reward for each piece of eliminated space debris. Then we have:

$$B = \sum_i c_i n_i$$

where $c_i$ is the reward that firms can gain from each piece of space debris.

The incomes of each piece of small space debris could be same because the number and the size of the small space debris are roughly same.

## 2.3 Launch costs

It's acknowledged that the cost of launching is related to the cost of the fuel and the cost of the equipment.

The cost of fuel is related to the orbit altitude. The unit cost of launching is:

$$t = kE = k\left(\frac{GM_e m_f}{2R} + m_f gR\right)$$

where $k$ is the unit price of fuel and $E$ is the energy provided for launching into the orbit. Each laser costs is almost 20 million dollars and each of satellites with rocket costs 180 million dollars approximately.

Based on the data, the unit cost $l'$ of space shuttle is 120 million dollars approximately [4].

The emitting cost of small space debris is $D_s = (t + l)m$, and the emitting cost of large space debris is $D_l = (t + l')m$.

where $m$ is the number of the satellites launched.

# 3. The Model of Disposing of Small Debris

Because of the large interval between the different orbits and the areas out of laser-shooting range, we can get the overall optimal scheme by optimal schemes of each orbit without considering orbital transfer. So we make the set $H_i$ show the optimal scheme of an orbit and make the set $I = \{H_i\}$ show the final optimal scheme.

## 3.1 Motion State in Two-Body Problem

Assume that a single object and the Earth form a two-body problem without considering the effects of other objects. For the object, the simplified equation of motion in the basic coordinate

system is

$$\vec{r}(t) = -\frac{G}{[\vec{r}(t)]^3}\vec{r}(t)$$

The Runge-Kutta method and the variable-step Euler's polygonal arc method do good in solving the three-dimensional position of the object at each observation time. Although the Runge-Kutta method has high computational accuracy, in order to simplify the algorithm, it's reasonable to take the variable-step Euler's polygonal arc method.

Making use of the difference quotient to replace the derivation is the basic thinking of Euler's polygonal arc method which is

$$\dot{r}(t_n) \approx \frac{r(t_n) - r(t_{n-1})}{h}$$

where $h$ is step size and h subjects to the condition that is $h = t_n - t_{n-1}$. While h is very small, it's convenient and proper that taking place of the true motion by its linear approximation. The iterative equations based on the method above is:

$$\begin{cases} \ddot{\vec{r}}(t_{n-1}) = -\frac{G_m}{|\vec{r}(t_{n-1})|^3}\vec{r}(t_{n-1}) \\ \dot{r}(t_n) = \dot{r}(t_{n-1}) + \ddot{r}(t_{n-1})h \\ r(t_n) = r(t_{n-1}) + \dot{r}(t_{n-1})h + \frac{1}{2}\ddot{r}(t_{n-1})h^2 \end{cases}$$

However, it's inevitable that the variable-step Euler's polygonal arc method will bring in certain error. Therefore, it's important to get more effective results with smaller step size.

In the process of calculation, we make the step size h equals to 0.01s. First of all, we get the initial position and velocity in the basic coordinate system randomly and substitute the parameters above into the first equation to get the initial acceleration. Finally, substitute the all parameters into the last equation. And in the case of $t = 50.0$s and $n = 5000$, the process of calculation is shown by Fig. 2.

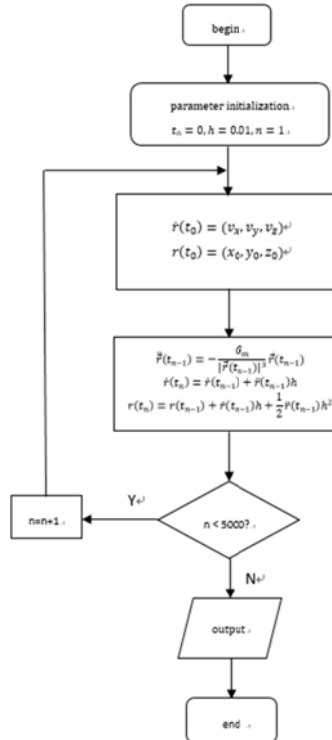where $n$ is the iterations equals to $\frac{t - t_0}{h}$.



Figure 2. The flow chart of the optimal algorithm

In order to demonstrate the orbit of object around the center of the earth intuitionally, we fit the 7-day motion track of the object whose period is more than 150 minutes, the dip of orbital plane is

about 45°, the semi-major axis of elliptical orbit is $9.55910 \times 10^6$m and the semi-minor axis of elliptical orbit is $9.21316 \times 10^6$m.

## 3.2 Coordinate Dimensionality Reduction

As for the two-body problem, it's proved that satellites motion in the same plane. So the motion equation is the plane S:

$$a_1 z + a_2 x + a_3 y = 0$$

Keep the origin of the coordinate system unchanged and get the equation by rotation transformation in the new coordinate system is:

$$S': z' = 0$$

First of all, keep the y-axis unchanged and do rotation transformation:

$$R_y = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

where $\alpha$ is rotation angle.

Then keep the x-axis unchanged and do rotation transformation:

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{pmatrix}$$

where $\beta$ is rotation angle.

Then keep the z-axis unchanged and do rotation transformation:

$$R_z = \begin{pmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Above all, we can know that:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = R_y R_x R_z \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Obviously, if the coordinate of space debris $(x, y, z)$ is known in the original coordinate system, it's easy to get the new coordinate $(x', y', 0)$ in the new coordinate system. That is, the 3D coordinate $(x, y, z)$ could be reduced into 2D coordinate $(x', y')$.

## 3.3 Model Establishment

In addition, based on the motion model above, we optimize the limited conditions to the laser-shooting distance in the improved model.

*Object Function*:

$$min\ min\ A_i = \frac{R'_i}{B_i - D_i} \quad (i = 1, \cdots, M)$$

where, $D_i$ is the launch costs, $R'_i$ is the risk, $B_i$ is the income and $A_i$ is the scheme's comprehensive evaluation index.

*Restrictions*:

- $n_i \epsilon N$
- The number of space debris in orbit finally should less than the maximum capacity of orbital debris:

$$n_i < N$$

- The maximum elapsed time $t$ subjects to:

$$t_i < T$$

where $t_i$ is the total running time of the satellite and $T$ is the maximum satellite running time.

- Space debris only can be eliminated one time.
- The incomes brings more value than the costs:

$$B_i > D_i$$

*Parameter Description*:

Space debris's coordinate in the new coordinate system some time:

1. Get the 3D coordinate $(x_i, y_i, z_i)$ in the original coordinate system at observation time by

means of the variable-step Euler's polygonal arc method.

2. Get the 2D coordinate $(x_i', y_i', 0)$ in the new coordinate system at observation time by the formula of point coordinate transformation.

3. Get the 3D coordinate $(x_i, y_i, z_i)$ reduced into 2D coordinate $(x_i', y_i', 0)$ and discuss about the range of laser-shooting in the plane $z' = 0$.

4. Get the coordinate of satellites in the new coordinate system and reduced to the coordinate $(x_{0i}', y_{0i}')$

Allowed shooting range: $S = \{(x_i', y_i')|(x_i' - x_{0i}')^2 + (y_i' - y_{0i}')^2 \le r^2\}$

*In conclusion*:

$$Objective: min\ min\ A_i = \frac{R'_i}{B_i - D_i} \quad (i = 1, \cdots, M)$$

$$S.t \begin{cases} n_i < N \\ n_i \epsilon N \\ t_i < T \\ D_i < B_i \end{cases}$$

## 4. The Model of Disposing of Large Debris

Large space debris can be disposed of only by capturing. The theorem of capturing is approximately same as the dispose of small space debris. However, the collision between large debris should be taken into consideration which increases the cost of capturing.

### 4.1 Collision Model

The average number of collision $x_p$ of a single debris obeys Poisson distribution in unit time which means $x_p \sim P_1(\lambda_1)$:

$$P_1(x_p = k) = \frac{\lambda_1{}^k}{k!} e^{-\lambda_1} \quad (\lambda_1 = k_4 m)$$

where $k_4$ is a constant.

The average number of debris $x_q$ after the collision of a single debris also obeys Poisson distribution which means $x_q \sim P_2(\lambda_2)$:

$$P_2(x_q = k) = \frac{\lambda_2{}^k}{k!} e^{-\lambda_2} \quad (\lambda_2 = k_5 m)$$

where $k_5$ is a constant coefficient.

New Debris come from collision:

$$x_q = \begin{cases} 0, x_p = 0 \\ x_q, x_p \ne 0 \end{cases}$$

The number of space debris is equal in different collision:

$$n = x_p x_q$$

The side length of large space debris is about 10 times longer than small space debris. We suppose that the space debris is a uniform sphere, so the quality of space debris obeys:

$$m = 1000 m'$$

where m is the quality of large space debris and $m'$ is the quality of small debris

The collision only happen between large debris:

$$m_e > \frac{m_0}{1000}$$

where $m_e$ is the quality of large space debris at the end of the collision and $m_0$ is the initial quality of large space debris.

Suppose each collision comes into being several pieces of space debris in same mass:

$$m_{i+1} = \frac{m_i}{n} \quad (i = 0,1, \cdots, e)$$

The number of new space debris: $Q_i = \frac{m_i}{m_0}$

Through the model, a simulation of the relationship between the number of space debris and time

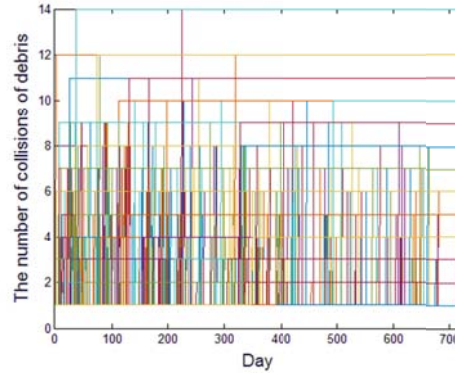is given. So we get 15 groups of result which is shown in Fig. 3:



Figure. 3. One thousand times simulation about the number of new debris comes from a debris in 2 years

## 4.2 Disposing Model Establishment

*Object Function*:

$$min \; min \; A_i = \frac{R'}{B_i - D_{li}' - D_{li}''} \quad (i = 1, \cdots, M)$$

where $D_i'$ is the equipment cost. $D_i''$ is the cost of capturing, $R'_i$ is the risk, $B_i$ is the income and $A_i$ is the scheme's comprehensive evaluation index.

*Restrictions*:

- $n_i \epsilon N$
- Constraint of maximum time:

$$t_i < T$$

where ti   is the total running time and T is the maximum running time.
- Space debris only can be caught within the allowable capture range.
- Space debris only can be caught one time.
- The incomes brings more value than the costs:

$$B_i > D_{li}' + D_{li}''$$

*Parameter Description*:

- Calculating the number of space debris at this time by collision model
- The cost of capturing at this time: $D_i'' = k_5 Q_i$
- Relative angular velocity:

$$\omega_{rel} = \left| \sqrt{\frac{GM}{R_1^3}} - \sqrt{\frac{GM}{R_2^3}} \right|$$

where $R_1$ is the orbital radius of space debris and $R_2$ is the orbital radius of catcher.
- The position between the catcher and the space debris:

$$\vartheta = \vartheta + \omega_{rel} \Delta t$$

where $\vartheta$ is the set of the relative position between a position and all space debris
- Allowed capture range:

$$S = \left\{ \theta \middle| R_1^2 + R_2^2 - 2R_1R_2 \cos \theta \le r^2 \right\}$$

## 5. Simulation and the analysis of the results

We assume that the maximum elapsed time $T$ is $7.776 \times 10^6 s$ (three months), and the maximum capacity of space debris in near Earth orbit $N$ is 500, the initial number of space debris in near Earth orbit $n_0$ is 800. And the radius of near Earth orbit is $2 \times 10^6$ m, the radius middle-high Earth orbit is $2 \times 10^7$ m, the radius of geosynchronous orbit is $3.6 \times 10^7$ m, the initial number of space debris in geosynchronous orbit $n_0$ is 200 and the maximum radius of laser emission $r$ is $1 \times 10^3$ m. Since the problem of missing data of this model, we conducted a simulation while the number of satellites below 7 in a single orbit. We conducts a simulation about initial relative position of the satellite and debris, with uniform distribution U (0,2π). The number of each satellite launched are stochastic simulation 50 times.

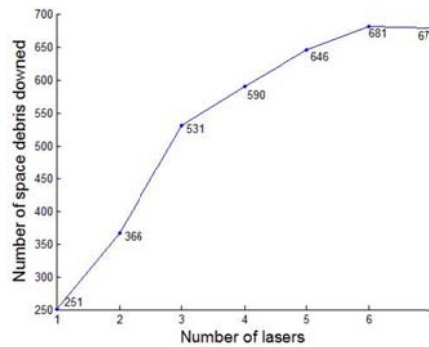We get the results in low Earth orbits which is shown in Fig .4:



Figure. 4.    Line graph about the number of Satellite launched and the average number of space debris downed

In the case of $l = 2 \times 10^9$ and $k_2 = 2 \times 10^7$, launching 3 satellites in low-earth orbit is the optimal scheme. It handles a total of 531 pieces space debris. From this scheme, $4.62 \times 10^8\$$ benefit was gained.

This model can not get the solution in each three orbit which means the best scheme of clearing up large space debris is not clearing up.

## Acknowledgement

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