Terminal Sliding Mode Control of MIMO Linear Time Delay Systems

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Abstract. A new terminal sliding mode control of MIMO linear time delay system is proposed in this paper. By inserting a time delay term in sliding mode, a new terminal sliding mode is developed for MIMO linear time delay systems. A terminal sliding mode controller is designed to drive the system state variables to reach the terminal sliding mode with a desired finite time and stay there forever. By suitably designing the parameter matrices of the sliding mode, the system state variables reach the system origin in finite time.

Introduction

Recently, a new technique called terminal sliding mode control has been developed in [1]-[5]. It offers some superior properties such as finite time convergence of the system dynamics in the terminal sliding mode and less steady state errors. In [1]-[3], the first-order terminal sliding mode control technique is developed and is applied into second-order nonlinear system and nth-order nonlinear rigid robotic manipulator. In [4], the terminal sliding mode technique for high-order single-input and single-output linear system is studied. In [5], the terminal sliding mode control of MIMO systems is investigated and a new terminal sliding mode structure is got.

The systems studied in the papers presented above haven't including time delay. However time delay systems can be found in various industrial systems, such as electrical networks, rolling mill, chemical processes, and hydraulic systems. In such a situation, designing a controller without taking into account the delay may lead to unstable or poor performance. The study of time delay systems has received considerable attention over the past years. The variable structure control of the time delay systems has been well studied in many literatures [6]-[8]. The results about independent of the delay and delay dependent have been got. However the terminal sliding mode control for linear time delay system is rare. It needs to be investigated.

This article is organized as follow. Section 2 describes the proposed new terminal sliding mode control scheme for MIMO linear time delay systems. Section 3 A simulation example is studied and a comparison is made. Section 4 gives conclusion.

Terminal sliding mode control for MIMO linear time delay systems

Consider the following MIMO linear time delay systems:

$$\dot{X}_1(t) = A_{11}X_1(t) + A_{12}X_2(t) + A_{d11}X_1(t-h) \tag{1}$$

$$\dot{X}_2(t) = A_{21}X_1(t) + A_{22}X_2(t) + A_{d21}X_1(t-h) + A_{d22}X_2(t-h) + B_2U(t) \tag{2}$$

where $X_1(t) \in R^{n-m}$ and $X_2(t) \in R^m$ are system states, $A_{11} \in R^{(n-m)\times(n-m)}$, $A_{12} \in R^{(n-m)\times m}$, $A_{d11} \in R^{(n-m)\times(n-m)}$, $A_{21} \in R^{m\times(n-m)}$, $A_{22} \in R^{m\times m}$, $A_{d21} \in R^{m\times(n-m)}$, $A_{d22} \in R^{m\times m}$ and $B_2 \in R^{m\times m}$ are the system matrices. B_2 is non-singular, and $n-m \le m$. $h = const > 0$ is pure time delay.

A new terminal sliding variable vector for the above MIMO linear time delay system is defined as follows:

$$S = C_1 X_1(t) + X_2(t) + C_2 X_1(t)^{\frac{q}{p}} + C_3 X_1(t-h) \tag{3}$$

where C_1, C_2, $C_3 \in R^{m\times(n-m)}$ are the parameter matrices of the terminal sliding mode. The odd

integer p and q satisfy the following condition.

$$p > q, \text{ and } 2q > p \tag{4}$$

$X_1(t)^{\frac{q}{p}}$ is a vector defined as

$$X_1(t)^{\frac{q}{p}} = \left[x_1(t)^{\frac{q}{p}}, \quad x_2(t)^{\frac{q}{p}}, \quad \cdots, \quad x_{n-m}(t)^{\frac{q}{p}} \right]^T \tag{5}$$

The terminal sliding mode is then defined as $S = 0$ or

$$C_1 X_1(t) + X_2(t) + C_2 X_1(t)^{\frac{q}{p}} + C_3 X_1(t-h) = 0 \tag{6}$$

Remark 1: It can be seen from (6) that the structure of the terminal sliding mode is different from the one in [5] due to the fact that the term $C_3 X_1(t-h)$ is inserted in the sliding mode.

To obtain the sufficient condition of the existence of the terminal sliding mode (6) for the MIMO linear tine delay system in (1) and (2), we have the following theorem.

Theorem 1:

Consider the MIMO linear time delay system in (1) and (2). If the control input is designed such as (7),

$$U = \begin{cases} -B_2^{-1}[(C_1 A_{11} + A_{21} + \phi A_{11}) X_1(t) + (C_1 A_{12} + A_{22} + \phi A_{12}) X_2(t) + (C_1 A_{d11} + A_{d21} + \phi A_{d11} + C_3 A_{11}) X_1(t-h) \\ \qquad + (A_{d22} + C_3 A_{12}) X_2(t-h) + C_3 A_{d11} X_1(t-2h) + k_1 \dfrac{S}{\|S\|}], \qquad \text{for } S \neq 0 \text{ and } \text{ any } x_i(t) \neq 0 \quad (i = 1, \cdots, n-m) \\ B_2^{-1}[-A_{21} X_1 - A_{22} X_2 - A_{d21} X_1(t-h) - A_{d22} X_2(t-h) - k_2 X_2], \qquad \text{for } S \neq 0 \text{ and } \text{ any } x_i(t) = 0 \quad (i = 1, \cdots, n-m) \end{cases} \tag{7}$$

where $\phi = \dfrac{q}{p} C_2 diag\left(x_i(t)^{\frac{(q-p)}{p}} \right)$, and

$$diag\left(x_i(t)^{\frac{(q-p)}{p}} \right) = \begin{bmatrix} x_1(t)^{\frac{(q-p)}{p}} & & \\ & \ddots & \\ & & x_{n-m}(t)^{\frac{(q-p)}{p}} \end{bmatrix} \tag{8}$$

k_1 and k_2 are positive numbers, then the system state variables will reach the terminal sliding mode $S = 0$ in finite time.

Proof: Defining a Lyapunov function

$$V = \frac{1}{2} S^T S \tag{9}$$

and differentiating V with respect to time, we have

$\dot{V} = S^T \dot{S}$

$$= S^T [C_1 \dot{X}_1(t) + \dot{X}_2(t) + \frac{q}{p} C_2 diag\left(x_i(t)^{\frac{(q-p)}{p}} \right) \dot{X}_1(t) + C_3 \dot{X}_1(t-h)]$$

$$= S^T [(C_1 A_{11} + A_{21} + \varphi A_{11}) X_1(t) + (C_1 A_{12} + A_{22} + \varphi A_{12}) X_2(t) + (C_1 A_{d11} + A_{d21} + \varphi A_{d11} + C_3 A_{11}) X_1(t-h)$$
$$\qquad + (A_{d22} + C_3 A_{12}) X_2(t-h) + C_3 A_{d11} X_1(t-2h) + B_2 U],$$

$$\tag{10}$$

Using (7) in (10), we get

$$\dot{V} = -k_1 \|S\| < 0 \qquad \text{for } S \neq 0 \quad \text{and} \quad x_i(t) \neq 0 \qquad (i = 1, \cdots, n-m) \tag{11}$$

Expression (11) is the sufficient condition for the terminal sliding variable vector S to converge to zero in finite time according to Lyapunov stability theory [1]-[5].

Remark 2: It can be seen from the above theorem that the convergence of the sliding variable vector S is obtained under the condition that $S \neq 0$ and $x_i(t) \neq 0$ $(i = 1, \cdots, n-m)$ because, at the

point $S \neq 0$ and $x_i(t) = 0$ $(i = 1, \cdots, n-m)$, $S^T S$ is unbounded. In order to make the system dymanics get away from these singular points, the second part of the control law in (7) is used, which is rewritten as

$$U = B_2^{-1}[-A_{21}X_1 - A_{22}X_2 - A_{d21}X_1(t-h) - A_{d22}X_2(t-h) - k_2 X_2]. \tag{12}$$

Using (12) in (2), we have

$$\dot{X}_2(t) = -k_2 X_2(t) \tag{13}$$

and the solution of the above differential equation has the following form:

$$X_2(t) = Ke^{-k_2 t} \tag{14}$$

where $K = [k_1, \cdots, k_m]^T$ is a constant vector.

Using (14) in (1), we have

$$\dot{X}_1(t) = A_{11}X_1(t) + A_{12}Ke^{-k_2 t} + A_{d11}X_1(t-h) \tag{15}$$

It is easy to see that, the control signal $Ke^{-k_2 t}$ will drive the elements of $X_1(t)$ away from zero at the beginning, and after that the first part of the terminal sliding mode control law in (7) works again to drive the system dynamics into the terminal sliding mode.

The design of the parameter matrices of the terminal sliding mode in (6) and the convergence property of the MIMO linear tine delay system dynamics in the terminal sliding mode are states in the following theorem.

Theorem2: Consider the MIMO linear time delay system in (1) and (2) with the terminal sliding mode controller in (7). If the terminal sliding mode parameter matrices are designed such that

$$A_{11} - A_{12}C_1 = 0 \tag{16}$$

$$A_{d11} - A_{12}C_3 = 0 \tag{17}$$

$$A_{12}C_2 = diag(\beta_i) \tag{18}$$

where $\beta_i > 0$, $i = 1, \cdots, n-m$

Proof: According the terminal sliding mode (6), the state vector $X_2(t)$ can be expressed as

$$X_2(t) = -C_1 X_1(t) - C_2 X_1(t)^{\frac{q}{p}} - C_3 X_1(t-h) \tag{19}$$

Using (19) in (1), we have

$$\dot{X}_1(t) = (A_{11} - A_{12}C_1)X_1(t) - A_{12}C_2 X_1(t)^{\frac{q}{p}} + (A_{d11} - A_{12}C_3)X_1(t-h) \tag{20}$$

Considering (16), (17) and (18), (20) becomes

$$\dot{X}_1(t) = -diag(\beta_i)X_1(t)^{\frac{q}{p}} \tag{21}$$

or $\quad \dot{x}_i(t) = -\beta_i x_i(t)^{\frac{q}{p}} \quad i = 1, \cdots, n-m \tag{22}$

The relaxation time t_i for a solution of the system (22) is then given by

$$t_i = -\beta_i^{-1}\int_{x_i(0)}^{0} \frac{dx_i(t)}{x_i(t)^{\frac{q}{p}}} = \frac{|x_i(0)|^{\frac{1-q}{p}}}{\beta_i(1 - q/p)} \tag{23}$$

then, $X_1(t)$ reaches the system origin in finite time. According to (19), therefore $X_2(t)$ identically reaches the system origin in finite time.

A simulation example

At first, the following MIMO linear system is considered [5]:

$$\dot{x}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_2 \\ x_3 \end{bmatrix} \tag{24}$$

$$\begin{bmatrix} \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} x_1 + \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{25}$$

In [5], the system is controlled with a good performance showing in Fig 1. The initialize states is $\begin{bmatrix} 0.15 & 0.09 & -0.15 \end{bmatrix}^T$
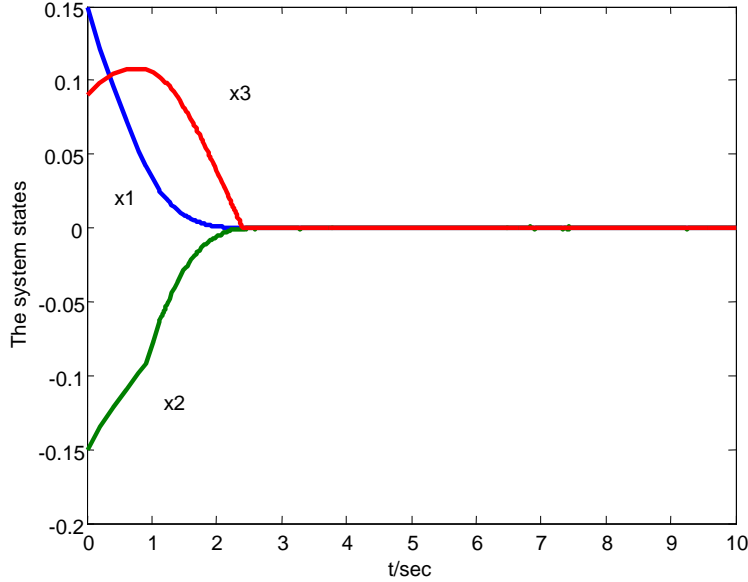

Fig 1. The system states in [5]

When we add the time delay, so the system can be given as follow:

$$\dot{x}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} + x_1(t-h) \tag{26}$$

$$\begin{bmatrix} \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} x_1 + \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} x_1(t-h) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_2(t-h) \\ x_3(t-h) \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{27}$$

h is pure time delay. Adopting the controller in [5], even if $h = 10^{-10}$, the system can't be stabilized. The respond curve in 5 seconds is as in Fig 2.
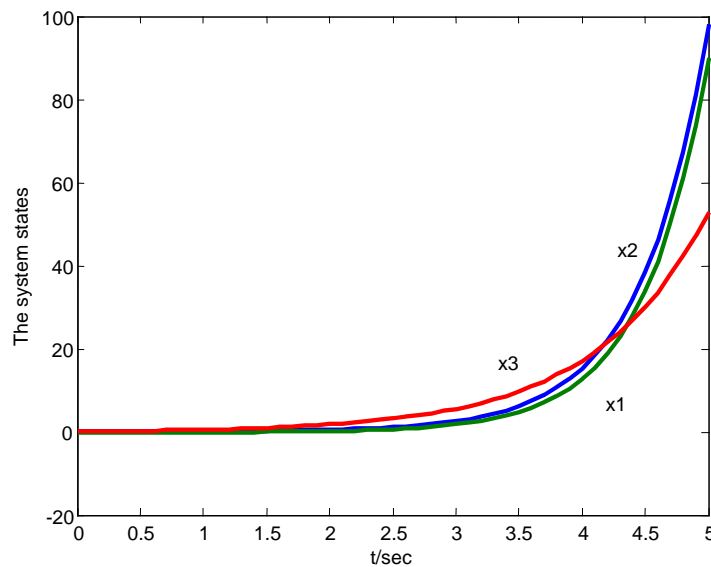

Fig 2. The time delay systems states with controller designed in [5]

The controller in this paper is adopted. where $A_{11} = 0$, $A_{12} = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$, $A_{21} = \begin{bmatrix} 0 & -1 \end{bmatrix}^T$,

$$A_{22} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad A_{d11} = 1, \quad A_{d21} = \begin{bmatrix} 1 & 1 \end{bmatrix}^T, \quad A_{d22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{taking} \quad h=1,$$

$k_1 = k_2 = 0.1$.

The terminal sliding variable vector is define as

$$S = C_1 x_1(t) + x_2(t) + C_2 x_1(t)^{\frac{3}{4}} + C_3 x_1(t-1). \tag{28}$$

where $C_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $C_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

The controller is as follow:

$$\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{cases} \begin{bmatrix} 1 \\ 0 \end{bmatrix} x_1(t) - \begin{bmatrix} -1 & -3 \\ \frac{3}{4} x_1(t)^{-\frac{1}{4}} & 1 \end{bmatrix} \begin{bmatrix} x_2(t) \\ x_3(t) \end{bmatrix} - \begin{bmatrix} 2 \\ \frac{3}{4} x_1(t)^{-\frac{1}{4}} \end{bmatrix} x_1(t-1) - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_2(t-1) \\ x_3(t-1) \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} x_1(t-2) - \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix} \frac{S}{\|S\|} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for} \quad S \neq 0 \quad \text{and} \quad x_1(t) \neq 0 \\ \begin{bmatrix} 1 \\ 0 \end{bmatrix} x_1(t) + \begin{bmatrix} 2 & 3.1 \\ 0.1 & -1 \end{bmatrix} \begin{bmatrix} x_2(t) \\ x_3(t) \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} x_1(t-1) - \begin{bmatrix} x_3(t-1) \\ x_2(t-1) \end{bmatrix} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for} \quad S \neq 0 \quad \text{and} \quad x_1(t) = 0 \end{cases}$$

$$\tag{29}$$

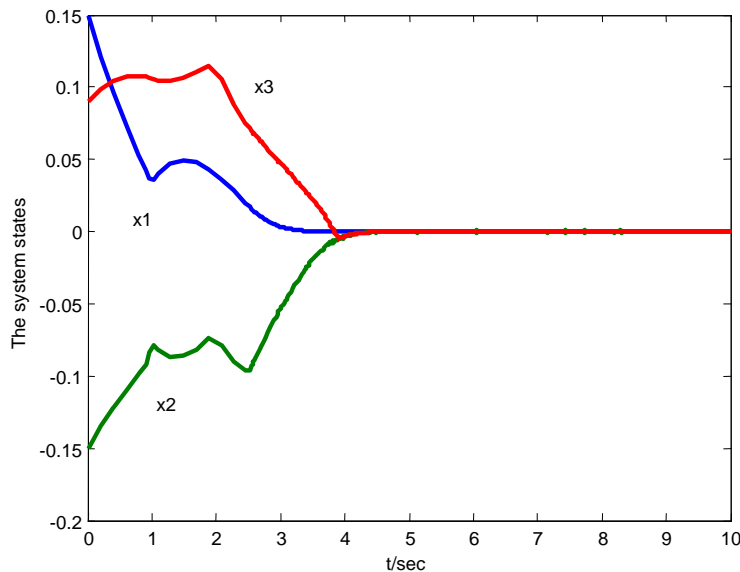The simulation result is as in Fig 3. The system is stable and good performance is got.

Fig 3. The time delay system states using the controller designed in this paper

Conclusion

In this paper, a new terminal sliding mode control scheme for MIMO linear time delay systems is got. The states variable of system can reach the terminal sliding mode in finite time. By suitably designing the parameter matrices of the terminal sliding mode, the states variable of system can reach the system origin in finite time. The simulation example shows the effect of the scheme in this paper. A comparison is made to show the scheme in paper [5] can't control the systems with time delay, even if the time delay is very small.

Acknowledgement

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