Study on the Heat Dissipation of the Bathtub

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Keywords: heat loss, temperature, bathtub, water surface, wall

Abstract. Through the establishment of water on the surface of the heat loss and the bathtub wall heat loss of the two models to analysis heat bath total loss, as the foundation to find a most suitable strategy the person in the bathtub can adopt. Model of heat loss through water surface discuss the heat loss between the water surface and air. Model of heat loss through wall developed by Newton's law[1], analysis the heat loss through bathtub wall. The two reflects the reason why people are difficult to maintain a constant water temperature when taking a bath.

Introduction

Constant temperature and heat loss in the bathing process greatly affected the feeling of comfort. From a business perspective, the various factors in the process of bathtub on the heat will definitely affect its sales. Consider the thermal radiation, heat conduction, and heat convection into two directions of heat dissipation analysis. The process of heat loss of study respectively analysis the wall heat loss, water bathtub of heat loss and heat absorbed by human body, is of great significance.

Model Of Heat Loss Through Water Surface[2]

In this part, we will discuss the heat loss between the water surface and air.

Put the surface as two-dimension plane. When considering the evaporation heat of water, we introduce the concept of flux to help analyze combining the theory of thermal conduction before. The heat exchange flux at the water surface includes the water surface evaporation heat loss flux ϕ_e, the thermal radiation heat flux ϕ_r, as well as the convection heat loss ϕ_c.

Evaporation heat loss flux. The bathtub water temperature is relatively higher than air temperature, it will evaporate. The water vapor takes heat when escaping out of water. For molecules of a liquid to evaporate, they must be located near the surface, they have to be moving in the proper direction, and have sufficient kinetic energy to overcome liquid-phase intermolecular forces. When only a small proportion of the molecules meet these criteria, the rate of evaporation is low. Since the kinetic energy of a molecule is proportional to its temperature, evaporation proceeds more quickly at higher temperatures. As the faster-moving molecules escape, the remaining molecules have lower average kinetic energy, and the temperature of the liquid decreases. This phenomenon is also called evaporative cooling.

$$\phi_e = \beta\left(p_{vw} - p_{va}\right)dS \tag{1}$$

$$\beta = \left[22.0 + 12.5W^2 + 2.0\left(T_a - T_s\right)\right]^{1/2} \tag{2}$$

Where:

- p_{vw} The partial pressure of water vapor near water surface
- p_{va} The partial pressure of water vapor in air
- dS Unite area of the surface
- β Water evaporation coefficient
- W The velocity of flowing wind, 0[m/s]

We'd like to talk more about p_{vw} and p_{va}, and present their expressions. p_{vw} presents the partial pressure of water vapor near the surface. From the point of microcosmic, there is a thin layer of saturated steam having the same temperature with water. The partial pressure of saturated vapor is related to temperature, as below:

$$p_s = 611\exp\left(\frac{17.27T}{T+237.3}\right) \tag{3}$$

In this way,

$$p_{vw} = 611\exp\left(\frac{17.27T_s}{T_s+237.3}\right) \tag{4}$$

As for p_{va}, basing on the definition of relative humidity(the ratio of the partial pressure of water vapor to the equilibrium vapor pressure of water at the same temperature), it is:

$$p_{va} = h\% p_s = 611h\%\exp\left(\frac{17.27T_a}{T_a+237.3}\right) \tag{5}$$

Radiation heat loss flux. According to The Stefan–Boltzmann law, the power radiated from a black body in terms of its temperature. Specifically, the Stefan–Boltzmann law states that the total energy radiated per unit surface area of a black body across all wave lengths per unit time (also known as the black-body radiant exitance or emissive power)is directly proportional to the fourth power of the black body's thermodynamic temperature T:

$$\phi_r = \varepsilon\sigma\left(T_s+273\right)^4 dS \tag{6}$$

Where:

ε Radiance, set as 0.97

σ 5.6×10^{-8} as constant, $[\mathrm{W}/(\mathrm{m^2k^4})]$

T_s The temperature of water surface, K

dS Unite area of the surface

Convection heat loss flux. Convective heat transfer, often referred to simply as convection, is the transfer of heat from one place to another by the movement of fluids. Convection is usually the dominant form of heat transfer(convection) in liquids and gases. Although often discussed as a distinct method of heat transfer, convective heat transfer involves the combined processes of conduction (heat diffusion) and advection (heat transfer by bulk fluid flow).

$$\phi_c = \alpha\left(T_s - T_a\right)dS \tag{7}$$

Where α is the convection coefficient. Based on the Bowen's ratio:

$$\frac{\alpha}{\beta} = \frac{P_a C_p}{0.623L} \tag{8}$$

Where:

$P_a = 1000[\mathrm{hPa}]$ The atmosphere pressure, hPa

$C_p = 1.005[\mathrm{kJ/kg\cdot K}]$ The specific heat capacity of dry air

$L = 2500[\mathrm{kJ/kg}]$ The vaporization heat

Through discussion above, we can get the total heat escape from the surface per second on the unite area:

$$Q_s = \alpha\left(T_s - T_a\right) + \beta\left(p_{vw} - p_{va}\right) + \varepsilon\sigma\left(T_s+273\right)^4 \tag{9}$$

Model Of Heat Loss Through Wall[3]

In the absence of evaporation heat loss, heat loss through bathtub wall is much simpler. Newton's law of cooling states that the rate of heat loss of a body is proportional to the difference in temperatures between the body and its surroundings. As such, it is equivalent to a statement that the heat transfer coefficient, which mediates between heat losses and temperature differences, is a constant. This condition is generally true in thermal conduction (where it is guaranteed by Fourier's law), but it is often only approximately true in conditions of convective heat transfer, where a number of physical processes make effective heat transfer coefficients somewhat dependent on temperature differences. Finally, in the case of heat transfer by thermal radiation, Newton's law of cooling is not true. But in this case, Newton's law of cooling suits well for we assume the water is stationary without thermal radiation. The rate of heat transfer in such circumstances is derived below:

$$Q_w = \frac{h\left(T_w - T_{env}\right)dS}{d} = \frac{h\Delta T dS}{d} \tag{10}$$

Where

Q	Thermal energy in joules
h	The heat transfer coefficient of wall
d	The thickness of the wall
S	Heat transfer surface area
T_s	The temperature of water surface
T_a	The temperature of air
$\Delta T = T_s - T_a$	The time-dependent thermal gradient between air and water

Convection heat transfer is closely related to the flow of the fluid. If we want to get the surface heat transfer coefficient of convection heat transfer, we must first obtain the distribution of the flow field. Navier and Stokes proposed the famous real viscous fluid motion equation -- NS equations[4].The NS equation cannot be solved directly. So the coefficient of heat transfer is usually expressed by the characteristic numbers. Nusselt number:

$$Nu = \frac{hl}{\lambda} \tag{11}$$

Where

λ	The thermal conductivity of the fluid
h	The surface heat transfer coefficient
l	The characteristic length of the surface

It represents the average dimensionless temperature gradient in the normal direction of the fluid at the wall. Its size shows the Intensity level of convection heat transfer.Reynolds number:

$$\mathrm{Re} = \frac{ul}{v} \tag{12}$$

Where

u	Thecharacteristic velocity
v	The kinematic viscosity of fluid
l	The characteristic length of the surface

It represents the relative magnitude of fluid inertia force and viscous force. The greater the Re, the greater the influence of inertia force.Prandtl number:

$$P_r = \frac{v}{a} \tag{13}$$

It is the number of physical characteristics of the fluid and represents the relative size of fluid diffusion capacity and heat diffusion capacity.Convert the forced convection heat transfer characteristic number into to the form of power function:

$$Nu = c\,\mathrm{Re}^n\,\mathrm{Pr}^m \tag{14}$$

When the end of the plate is in laminar flow, we have:

$$Nu = 0.664\,\mathrm{Re}^{\frac{1}{2}}\,\mathrm{Pr}^{\frac{1}{3}} \quad (15)$$

And the heat transfer coefficient on the surface of the convective heat transfer can be derived:

$$h=\frac{\lambda}{l}Nu=0.664\frac{\lambda}{l}\mathrm{Re}^{\frac{1}{2}}\mathrm{Pr}^{\frac{1}{3}}=0.664\frac{\lambda}{l}(\frac{ul}{v})^{\frac{1}{2}}(\frac{v}{a})^{\frac{1}{3}} \tag{16}$$

So when considering the human behaviors lead to water and wall of forced commutation, the reduced caloric value is:

$$Q_{\mathrm{wall}}=\int_{t_1}^{t_2}-0.664\frac{\lambda}{l}(\frac{ul}{v})^{\frac{1}{2}}(\frac{v}{a})^{\frac{1}{3}}A\theta dt \tag{17}$$

$$\theta=\theta_0 e^{-\frac{hA}{\rho cV}t} \tag{18}$$

Model Of Heat Absorbed By Human Body

In this part, we mainly analyzed the heat absorbed by human body from water bath. Assume that the whole body is immersed in water except head, is the water temperature difference between human body and bathtub.

Tab.1 specific definitions

Abbreviation	Definition
Δu	The temperature difference between skin and water
V	The volume of body
A_d	The total body surface area
A_r	The effective area of skin contacted with water in bathtub
ρ_{hm}	The density of human body
M_{hm}	The mass of human body
L_{hm}	The length of human body
c_{hm}	The specific heat capacity of the human body
h_{hm}	Thermal conductivity of human skin

When the person sits in the bathtub, the water temperature is higher than the temperature of his skin. In order to maintain a comfortable feeling and physiological needs of the human body, the body involuntarily takes heat and moisture exchange with the surrounding environment. From a macro point of view, the person feels comfortable when the heat generation inside body and the heat exchange with the surrounding environment reach a good balance. Due to the heat absorption of skin, body's temperature will certainly raise when the water temperature is higher than the skin temperature. Thanks to the body's blood circulation, the fluctuation of temperature is very small, and the temperature won't deviate from the normal much.

There is great difference between one person to another in weight, height and body density, which diversify the heat absorbed by human body. Now we take human body mass, human body density, the effective area of skin contacted with water in bathtub and the specific heat capacity of the human body into consideration.

If the body density ρ_{hm} varies, the heat absorbing ability of skin tissue varies. The person with larger density has higher proportion of muscle and lower proportion of fat in skin tissue. That means the heat conduction efficiency is higher in contact with hot water. So its thermal conductivity h_{hm} is larger. On the contrary, smaller density of individual means smaller thermal conductivity.

Define[5]:

$$h_{hm}/\rho_{hm}=0.013 \tag{19}$$

The total area of a person's body refers to the naked area of the human body. We adopt the method proposed by DuBois:

$$A_d=0.202M_{hm}{}^{0.425}L_{hm}{}^{0.725} \tag{20}$$

The action of the human in the bathtub will affect the effective area A_r. Different degrees of intense movement will lead to different ratio of A_r and A_d. And the fiercer the movement is, the smaller A_r / A_d is. The behavior of the person in bathtub is divided into three levels.

behavior	A_r / A_d
Static lying	0.9
Normal bath	0.4
Stand up	0.1

In this case, person is slightly active in the bathtub. So we take $A / A_d = 0.4$.Therefore, we conclude that the heat absorbed through the skin(in unit time) is:

$$Q_{hm} = \frac{h_{hm}}{\rho_m}\frac{A_r}{A_d}\mathrm{M}_{hm}\ c_{hm} \cdot \Delta u = 0.4\frac{h_{hm}}{\rho_m}\mathrm{M}_{hm}\ c_{hm} \cdot \Delta u \tag{21}$$

Summary

Through a series of analysis, the final heat loss Q_{wall}, Q_s and Q_{hm} is obtained. Heat loss through water surface relates to the thermal radiation, heat conduction and the heat convection, the heat loss of expression is relatively fixed. Heat loss through wall, conduction account for a substantial majority of the heat loss. In addition, such relationship is relative to the shape of the bathtub, material, size, etc.

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