Research on the temperature of the bathtub water

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Abstract. When a person fills a bathtub with hot water and settles into the bathtub,the bath gets noticeably cooler.In order to study the temperature even throughout the bathtub and as close as possible to the initial temperature, we need develop a model of the temperature of the bathtub water in space and time.First,we figure out the heat loss,then replace three-dimensional partial differential equations with two two-dimensional equations.Based on the finite difference method,we make time domain and space domain discrete, list discrete node equations by using heat balance mothod and solve discrete node equations.

Introduction

We are discussing the problem basing on the problem A in MCM 2016.

Firstly,both the person and the tub could effect the temperature,we simplify the motions made by the person with rolling motions.Secondly,counting various heat loss,by finite difference mothod,we build and solve two two-dimensional temperature models of different boundary conditions.Last,the results show the distributions of temperature are pretty proper,which proves the rationality of the two models.

Developing the mathematical model

On the basis of the fundamental of heat transfer,the three-dimensional partial differential equation of conduction heat transfer[1],which includes the third class boundary condition,is easily to be developed.

$$\frac{\partial t}{\partial \tau} = a\left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}\right)$$

Due to the complexity of the solution of the three-dimensional partial differential equation,we replace it with two two-dimensional partial differential equations,which include the third class boundary conditions.

$$\text{equation Ⅰ} \quad \frac{\partial t}{\partial \tau} = a\left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial z^2}\right)$$

$$\text{boundary conditions} \begin{cases} -\lambda_1\left(\frac{\partial t}{\partial n}\right) = h_1(t_1 - t_{w1}) + w_1 q_3 \\ -\lambda_2\left(\frac{\partial t}{\partial n}\right) = \frac{Q_2}{S} + w_2 q_3 \end{cases}$$

$$\text{equation Ⅱ} \quad \frac{\partial t}{\partial \tau} = a\left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2}\right)$$

$$\text{boundary conditions} -\lambda_1\left(\frac{\partial t}{\partial n}\right) = h_1(t_1 - t_{w1}) + w_1 q_3$$

By now,our work is to calculate the value of $t_{w1}, h_1, Q_2, Q_3, S, a, \lambda_1, \lambda_2$. During the research,we assume the movement of the person in the bathtub is the same as rolling motion.The intensity of motion made by the person can be expressed in θ_m,which effects heat transfer coefficient of te

forced convection heat transfer by changing $N^{[2]}{}_{u_{av}}$.

$$N_{u_{av}} = 0.01526 R_e^{\ 0.082315} P_r^{\ 0.4} (\frac{\pi r}{uT_0})^{-0.08614} \theta_m^{\ 0.10369}$$

Here we need make a hypothesis:Temperature of inside wall of tub $t_{w1} = 39.78^oC$.

Prove the following:

There is a fundamental theorem[3] saying that the heat leaking out into to atmosphere through the bathtub is equal to the convective heat in the numerical value.We replace the leaking heat with q_1 ,and replace the convective heat with q_2 . $t_{m1} = \frac{t_{w1} + t_1}{2}$, $R_e = u \times r / \nu$, $N_{u_{av}} = 0.01526 R_e^{\ 0.082315} P_r^{\ 0.4} (\frac{\pi r}{uT_0})^{-0.08614} \theta_m^{\ 0.10369}$, $h_1 = \frac{\lambda}{l} N_{u_{av}}$, $q_1 = h_1 \times (t_1 - t_{w1})$,we figure out that $\lambda_1 = 0.6265$ $\nu = 7.32 \times 10^{-7} (m^2 / s)$ $P_r = 4.865$ $\alpha_V = 3.46 \times 10^{-4} (K^{-1})$ and $R_e = 364188.16$ .Then,assuming that $u = 0.3 m/s$ $T_0 = 20s$ $\theta_m = 15$ $r = 0.4m$,we can work out $N_{u_{av}} = 261.92$ $h_1 = 207.90$ $q_1 = 45.738$.

Similarly,the corresponding equations of calculating q_2 are $t_{m2} = \frac{t_{w2} + t_2}{2}$, $t_{w2} = t_{w1} - \frac{q_1 \delta_0}{\lambda}$, $G_r = \frac{g \alpha_V \Delta t l^3}{\nu^2}$, $N_u = C(G_r P_r)^n = CRa^n$, $h_2 = \frac{\lambda}{l} N_u$, $q_2 = \frac{t_1 - t_4}{\frac{1}{h_1} + \frac{1}{h_2} + \frac{\delta}{\lambda}}$. In the same manner,the result is obtained $t_{w2} = 32.56^oC$, $\lambda_2 = 0.02588$, $P_r = 0.7282$, $\nu = 1.608 \times 10^{-5} (m^2 / s)$, $N_u = 108.51$, $G_r = 1.96 \times 10^9$, $\delta_0 = 0.3$, $h_2 = 4.01$, $q_2 = 44.47$.

Comparing q_1 with q_2 ,we found there is no significant difference between them.We can come to the conclusion that the hypothesis is correct $t_{w1} = 39.78^oC$ .Besides, $h_1 = 207.90$.

$$Q_2 = \alpha * y * (0.0174 v_f + 0.0229) * (P_b - P_q) * \frac{760S}{B}$$

The formula is used to calculate the heat loss Q_2 of the water evaporation, which leaks out into atmosphere. the temperature of bathtub water is 40 degrees, $\alpha = 4.1868 / kcal$, $y = 574.2 kcal / kg$, $v_f = 0.3 m/s$, $P_b = 40 mmH_g$, $P_q = 15.6 mmH_g$, $B = 761.84 mmH_g$, $S = 1.36 m^2$,we can get $Q_2 = 621.5919W$ .The method of calculating Q_3 is same as above.We can reckon $q_3 = 1414.52$ by the mothod of calculating q_1 ,we have calculated $\lambda_1 = 0.6265$ $\lambda_2 = 0.02588$ $h_1 = 207.90$ $h_2 = 4.01$ .With the formula $a = \frac{\lambda}{\rho c}$,the final mathematical model can be described as

equation Ⅰ $\frac{\partial t}{\partial \tau} = 0.000151276 (\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial z^2})$

boundary conditions $\begin{cases} -0.6265(\frac{\partial t}{\partial n}) = 207.9(40 - 39.78) + 0.9 \times 1414.52 \\ -0.02588(\frac{\partial t}{\partial n}) = \frac{621.5919}{1.36} + 0.1 \times 1414.52 \end{cases}$

equation Ⅱ $\frac{\partial t}{\partial \tau} = 0.000151276 (\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2})$

boundary conditions $-0.6265(\frac{\partial t}{\partial n}) = 207.9(40-39.78)+0.9\times 1414.52$

Conclusion

On the basis of finite difference method,we need to convert the derivative terms of the mathematical model to a differential expression.In accordance with the energy conservation equation,the discrete equations of internal nodes are listed by the mothod of heat balance.

$$\lambda\Delta y\frac{t^{i}_{m-1,n}-t^{i}_{m,n}}{\Delta x}+\lambda\Delta y\frac{t^{i}_{m+1,n}-t^{i}_{m,n}}{\Delta x}+\lambda\Delta x\frac{t^{i}_{m,n+1}-t^{i}_{m.n}}{\Delta y}+\lambda\Delta x\frac{t^{i}_{m,n-1}-t^{i}_{m,n}}{\Delta y}=\rho c\Delta x\Delta y\frac{t^{i+1}_{m,n}-t^{i}_{m,n}}{\Delta\tau}$$

According to the Newton's iteration method,with the value of node m and its neighboring nodes at some moment,we can reckon the value of node m.

By using Matlab, in the case of selecting $\Delta x = 0.0312$, $\Delta y = 0.0378$, $\Delta t = 0.1$,at the moment of $\tau_1 = 11\text{min}$,the temperature distributions of the two-dimensional plane Ⅰ and plane Ⅱ can be simulated.

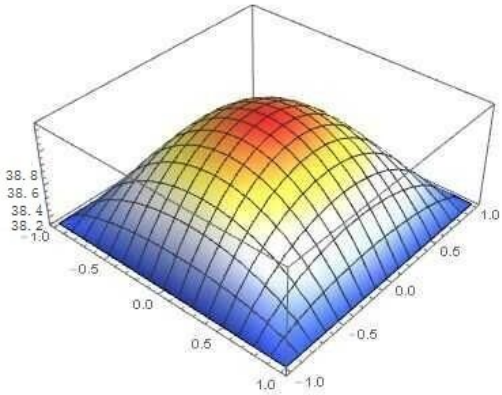

Figure 1.The Temperature Distributions of the Two-dimensional Plane Ⅰ

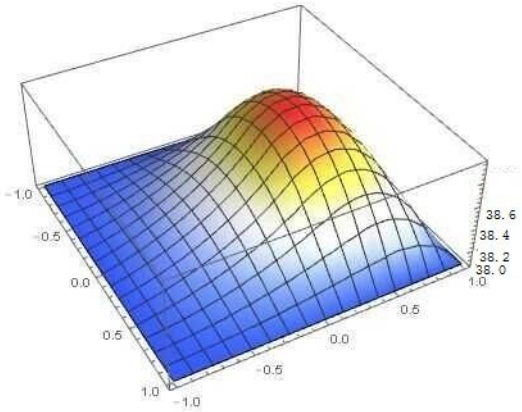

Figure 2.The Temperature Distributions of the Two-dimensional Plane Ⅱ

The two-dimensional temperature distribution can reflect the temperature changes in the three-dimensional space reasonably.

References

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