

# An Optimization Method of Re-scheduling for Urban Rail Transit

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**Abstract.** Urban rail transit systems are often characterized by high traffic density which is very sensitive to the disturbances. A re-scheduling method using routing graph and time interval between two adjacent trains at station as the objective function to minimise the consequences of disturbances is presented here. Computational results from experiments are presented and the practical strengths and limitations are discussed.

## Introduction

As of end of 2014, the mileage of urban rail transit (URT) has reached 2886 Km and the number of city with URT is 22 in china. The ridership of Beijing urban rail transit (BURT) network is about 9,000,000 person-trips, which is still increased quickly. The BURT network is characterised by high traffic density and very sensitive to the disturbances. Minimising the consequence of disturbances and improving the train punctuality is a challenging problem which has been studied in different ways over the years. Although slack times are used by timetable to manage train delays, but they can only absorb small disturbances. When disturbances are severe, train delays are inevitable and the delays can be propagated over the entire network to cause secondary delays and route conflicts.

Following the instruction of timetable strictly is still the way used by China's dispatching system, even in the circumstance of severe interference. In fact, even some disturbances are not so severe, they also can cause domino effect of secondary delays and significant deviation from timetable. If the system still make decisions based on the timetable at this point, a series of problems may be caused in train dispatching activities, such as serious delays and train blocking.

In this paper, we propose a re-scheduling method which takes the time interval between two adjacent trains at station as the objective function, to reduce the waiting time of passengers and maintain the traffic order when severe disturbances happen.

## Problem Description

In URT, trains arrive at a station usually in a fix time interval in a train routing mode. Passengers do not care about whether the train following the timetable or not, they are only sensitive to the time spent waiting for the train. So it is feasible to do train regulation without timetable and achieve the smooth transition from chaos to order in case of interference.

We denote by  $G := (V, A)$  the routing graph with  $V$  as the set of platforms and the  $A$  as the set of travel arcs which denote the train travel from one platform to the next. The reversal track is also considered as a virtual platform, such as C1, C2, C3 in the Fig. 1.

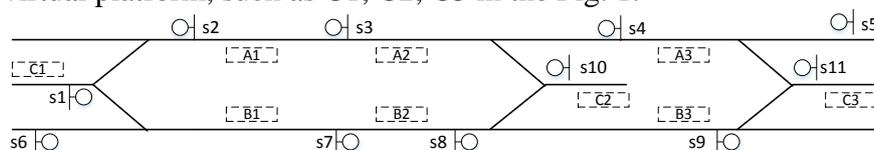


Fig. 1 Example URT network

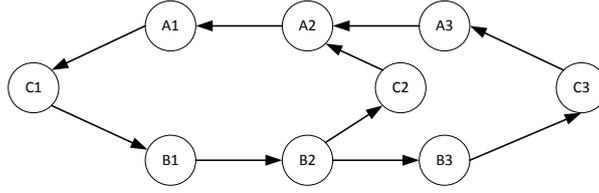


Fig. 2 Routing graph of the example network. There are two routings in the graph: (B1, B2, B3, C3, A3, A2, A1, C1) and (B1, B2, C2, A2, A1, C1).

The operation organization in URT always adopts several routing modes, such as full-length & short-turn operation modes. Let  $R$  be the set of routing and  $R \subseteq G$ ,  $K$  be the cycle time of routing,  $M$  be the number of train using the routing. So the time interval  $d_i$  between two adjacent train arriving at platform  $v_i$  can be calculated as Eq. (1)

$$d_i = \frac{k_i}{\left( \sum_{v_i \subset (r_i \cap r_j)} \frac{k_i \times m_j}{k_j} \right) + m_i} \quad (1).$$

In URT, generally train arrives at a station or a platform in a fix time interval. For convenience, we assume that platforms, which share exactly same routings, has the same interval for train arriving. For example, train arrives at platform A1 and A2 every 3 minutes, and platform A3 and B3 every 6 minutes. This assumption does not make difference for the operation organization, but simplifies the mathematic model. So the interval can be calculated by Eq. (1).

Let  $t_{i,j}^a$  denotes the time when train  $j$  arrives at  $v_i$ ;  $t_{i,j}^d$  denotes the time when train  $j$  departs from  $v_i$ ;  $t_{i,j}^r$ ,  $T_{i,j}^{r,\min}$ ,  $T_{i,j}^{r,\max}$  denotes the interval, the minimum interval and the maximum interval the train  $j$  running from  $v_{i-1}$  to  $v_i$  respectively;  $t_{i,j}^w$ ,  $T_{i,j}^{w,\min}$ ,  $T_{i,j}^{w,\max}$  denotes the dwell time, the minimum dwell time and the maximum dwell time the train  $j$  stops at  $v_i$  respectively;  $T^v$  denotes the minimum turnaround time;  $T^f$  denotes the minimum train headway. Then the rescheduling mathematic model can be written as mixed integer programming as follows:

$$f = \min \sum_{i,j} |t_{i,j}^a - t_{i,j-1}^a - d_i| \quad (2)$$

$$\begin{cases} t_{i,j}^a = t_{i-1,j}^d + t_{i,j}^r \\ t_{i,j}^d = t_{i,j}^a + t_{i,j}^w \\ T_{i,j}^{r,\min} \leq t_{i,j}^r \leq T_{i,j}^{r,\max} \\ T_{i,j}^{w,\min} \leq t_{i,j}^w \leq T_{i,j}^{w,\max} \\ d_i > \max(T^v, T^f + T_{i,j}^{w,\min}) \\ t_{i,j}^a, t_{i,j}^d, t_{i,j}^w, t_{i,j}^r \in N \end{cases} \quad (3)$$

The Eq. (2) is the objective function which ensures the train arrival interval is fixed for the platform. The Eq. (3) is the constraints which applies the single routing mode for the URT operation organization. The additional constraints should be added if there are exist several routings.

Take the routing graph of Fig. 1.2 as example, there are two routings  $r_1$ ,  $r_2$  in the graph, and  $r_1 \cap r_2 \neq \emptyset$ . The train from A3 to A2 will conflict with the train from C2 to A2 if two train leave for the A2 in a certain period of time. We can use Eq. (4) to detect if some conflict between two trains or not. For the train should not stop in the section between two platforms.

$$\begin{cases} |t_{i,k}^a - t_{i,j}^a| < T^f + t_{i,j}^w \\ v_{i-1}^j \neq v_{i-1}^k \end{cases} \quad (4)$$

In Eq. (4),  $v_i^j$  denotes the train j arrives at platform i. We can use the last platform the train passing to determining whether two train share same routing or not.

When the conflict is detected, we can use the relative delay time as the standard to decide which train should depart for he platform first, as the Eq. (1.5) presents, and  $t_{i,j}^{a'}$  denotes the estimated time train j arrives at vi if the train k is prioritized for vi,  $\Delta t_{i,j}$  is the relative delay time.

$$\begin{cases} t_{i,j}^{a'} = \max((t_{i,k}^a + t_{i,k}^w + T_{\min}^f), (t_{i,k}^a + d_i)) \\ \Delta t_{i,j} = t_{i,j}^{a'} - t_{i,j}^a \end{cases} \quad (5)$$

The priority for the vi is given to the train with shorter relative delay time.

## Experiments and results

The data we used for experiments is based on the Chongqing Metro line 3 (CML3). The mileage of CML3 is 55 KM and the ridership is more than 680,000 person-trips in 2014. There are 39 stations and 80 platforms. The operation organization uses 3 routings: Yudong - Longtousi, Jiugongli – Longtousi and Sigongli – Jiangbeijichang, which are also used in our expetiments. We construct the interference scenario with several trains broken down and other trains delay seriously. The results of experiments are shown in table 1.



Fig. 3 Chongqing Metro line 3

Table 1 the results of experiments

Routing	Train number	time interval	Broken trains	Recover time
Yudong - Longtousi	20	4 minutes	3	20 minutes
Yudong – Longtousi	10	8 minutes	5	30 minutes
Jiugongli – Longtousi	15	4 minutes		
Yudong – Longtousi	10	8 minutes	6	45 minutes
Jiugongli – Longtousi	15	4 minutes		
Sigongli – Jiangbeijichang	10	6 minutes		

Form the results, we can conclusion that the re-scheduling method is effective and the recovering time is proportional to the used routings, broken trains and the traffic density.

## Summary

This paper introduced an effective re-scheduling method based on routing graph representation of a URT system. The model takes into account train arrival intervals at station to recover from serious disturbances, in case of which it is impossible to follow the primary timetable. The conflict resolution algorithm takes the relative delays of two conflict train as the criteria to decide the train priority, which is feasible, but may not be the most optimal. The conflict resolution algorithm should be improved in the future research.

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